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I.1

$$\vec{r} = \begin{pmatrix} r\cos\phi \\ r\sin\phi \\ z \end{pmatrix}$$

$$\vec{e}_r = (\vec{e}_1, \vec{e}_2, \vec{e}_3)\, e_r$$

$$e_r = \frac{\partial \vec{r}}{\partial r} = \begin{pmatrix} \cos\phi \\ \sin\phi \\ 0 \end{pmatrix},\quad e_\phi = \begin{pmatrix} -\sin\phi \\ \cos\phi \\ 0 \end{pmatrix}\quad e_z = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$\vec{e}_r = (\vec{e}_1, \vec{e}_2, \vec{e}_3)\begin{pmatrix} \cos\phi \\ \sin\phi \\ 0 \end{pmatrix},\quad \vec{e}_\phi = (\vec{e}_1, \vec{e}_2, \vec{e}_3)\begin{pmatrix} -\sin\phi \\ \cos\phi \\ 0 \end{pmatrix}\quad \vec{e}_z = (\vec{e}_1, \vec{e}_2, \vec{e}_3)\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$\begin{aligned}
\vec{e}_r \times \vec{e}_\phi &= \vec{e}_1\vec{e}_2\cos^2\phi + \vec{e}_2\vec{e}_1(-\sin^2\phi \\
&= \vec{e}_3(\cos^2\phi + \sin^2\phi) \\
&= \vec{e}_3 \\
&= \vec{e}_z
\end{aligned}$$

よって $\vec{e}_z = \vec{e}_r \times \vec{e}_\phi$

2 $(\vec{e}_r, \vec{e}_\phi, \vec{e}_z) = (\vec{e}_1, \vec{e}_2, \vec{e}_3)(e_r, e_\phi, e_z)$

$$= (\vec{e}_1, \vec{e}_2, \vec{e}_3)\begin{pmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$T = \begin{pmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

3. $\vec{v} = \theta v = \underbrace{\theta_{r\phi z}}_{\theta T} v_{r\phi z} = \theta T v_{r\phi z}$

よって $v = T v_{r\phi z}$

位置ベクトル $\vec{r}$ について確認する

$$r = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

$$r' = Tr = \begin{pmatrix} x\cos\phi - y\sin\phi \\ x\sin\phi + y\sin\phi \\ z \end{pmatrix} = \begin{pmatrix} r_r \\ r_\phi \\ r_z \end{pmatrix}$$

$$e_x = \cos\phi\, e_r - \sin e_\phi$$
$$e_y = \sin\phi\, e_r + \cos\phi\, e_\phi$$

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I.4

$$\frac{\partial}{\partial r} = \frac{\partial x}{\partial r}\frac{\partial}{\partial x} + \frac{\partial y}{\partial r}\frac{\partial}{\partial y} + \frac{\partial z}{\partial r}\frac{\partial}{\partial z}$$

$$= \cos\phi\frac{\partial}{\partial x} + \sin\phi\frac{\partial}{\partial y} =$$

$$\frac{\partial}{\partial \phi} = -r\sin\phi\frac{\partial}{\partial x} + r\cos\phi\frac{\partial}{\partial y}$$

$$\frac{\partial}{\partial z} = \frac{\partial}{\partial z}$$

$$\begin{pmatrix} \frac{\partial}{\partial r} \\ \frac{1}{r}\frac{\partial}{\partial \phi} \\ \frac{\partial}{\partial z} \end{pmatrix} = \begin{pmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{pmatrix}$$

$$\begin{pmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{pmatrix} = \begin{pmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \frac{\partial}{\partial r} \\ \frac{1}{r}\frac{\partial}{\partial \phi} \\ \frac{\partial}{\partial z} \end{pmatrix}$$

$$\frac{\partial}{\partial x} = \cos\phi\frac{\partial}{\partial r} + \frac{\sin\phi}{r}\frac{\partial}{\partial \phi}$$

$$\frac{\partial}{\partial y} = -\sin\phi\frac{\partial}{\partial r} + \frac{\cos\phi}{r}\frac{\partial}{\partial \phi}$$

$$\frac{\partial}{\partial z} = \frac{\partial}{\partial z}$$

Ⅱ.1 $\vec{F}$が位置ベクトルに比例しているということ

2. $\vec{F} = m\ddot{\vec{r}} = \dot{\vec{p}}$

$\vec{r}\times\vec{F} = \vec{r}\times\dot{\vec{p}}$

$\vec{F} = k\vec{r}$ と書けるから $\vec{r}\times\dot{\vec{p}} = \vec{r}\times k\vec{r} = k\vec{r}\times\vec{r} = 0$

$$\vec{r}\times\dot{\vec{p}} = \frac{d}{dt}(\vec{r}\times\vec{p}) - \dot{\vec{r}}\times\vec{p}$$

$$= \frac{d}{dt}(\vec{r}\times\vec{p}) - \dot{\vec{r}}\times m\dot{\vec{r}}$$

$$= \frac{d}{dt}\vec{L} - m\dot{\vec{r}}\times\dot{\vec{r}}$$

$$= \frac{d}{dt}\vec{L}$$

よって $\vec{L}$ = 一定

3. $\vec{r} = (\vec{e}_1, \vec{e}_2, \vec{e}_3)\begin{pmatrix}0\\s\\s\end{pmatrix}$

$\vec{p} = m\vec{v} = m(\vec{e}_1, \vec{e}_2, \vec{e}_3)\begin{pmatrix}0\\v\\0\end{pmatrix}$

$\vec{r}\times\vec{p} = \vec{e}_3\vec{e}_2\, s\cdot mv$

$= -\vec{e}_1\, s\cdot mv$

よって x軸負方向

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II.4(i)

$\vec{L}$ 方向に z 軸をとるから

$$\vec{r} = O\begin{pmatrix} r\cos\phi \\ r\sin\phi \\ 0 \end{pmatrix} = OT\begin{pmatrix} r \\ \phi \\ z \end{pmatrix}$$

$$\begin{pmatrix} r \\ \phi \\ z \end{pmatrix} = \hat{T}\begin{pmatrix} r\cos\phi \\ r\sin\phi \\ 0 \end{pmatrix} = \begin{pmatrix} r \\ 0 \\ 0 \end{pmatrix},\quad \mathbb{r}_{r\phi z} = \begin{pmatrix} r \\ 0 \\ 0 \end{pmatrix}$$

$$\vec{P} = m\dot{\vec{r}} = mO\begin{pmatrix} \dot{r}\cos\phi - r\dot{\phi}\sin\phi \\ \dot{r}\sin\phi + r\dot{\phi}\cos\phi \\ 0 \end{pmatrix} = mOT\begin{pmatrix} P_r \\ P_\phi \\ P_z \end{pmatrix}$$

$$\begin{pmatrix} P_r \\ P_\phi \\ P_z \end{pmatrix} = \begin{pmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} \dot{r}\cos\phi - r\dot{\phi}\sin\phi \\ \dot{r}\sin\phi + r\dot{\phi}\cos\phi \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} \dot{r} \\ r\dot{\phi} \\ 0 \end{pmatrix}$$

$$\mathbb{P}_{r\phi z} = m\begin{pmatrix} \dot{r} \\ r\dot{\phi} \\ 0 \end{pmatrix}$$

$$\vec{r} \times \vec{P} = \begin{pmatrix} 0 \\ 0 \\ r^2\dot{\phi} \end{pmatrix}$$

$\vec{F}$は中心力だから

$$\frac{d\vec{L}}{dt} = \vec{r}\times\vec{F} = \vec{r}\times k\vec{r} = 0 \quad (\dot{\vec{P}} = m\ddot{\vec{r}} = \vec{F} = k\vec{r}$$

よって $\mathbb{L} = \vec{r}\times\vec{P} =$ 一定

(ii) 単位時間あたり動径が通る面積が一定

III 1. $r : 0 \to \infty$

$\theta : 0 \to \pi$

$\phi : 0 \to 2\pi \ (-\pi \to \pi)$

2. $\vec{e}_r = (\vec{e}_1, \vec{e}_2, \vec{e}_3)\, e_r$

$$e_r = \widehat{\frac{\partial \mathbf{r}}{\partial r}}$$

$$\frac{\partial \mathbf{r}}{\partial r} = \begin{pmatrix} \sin\theta\cos\phi \\ \sin\theta\sin\phi \\ \cos\theta \end{pmatrix} \qquad \left|\frac{\partial \mathbf{r}}{\partial r}\right|^2 = 1$$

$$\vec{e}_r = (\vec{e}_1, \vec{e}_2, \vec{e}_3) \begin{pmatrix} \sin\theta\cos\phi \\ \sin\theta\sin\phi \\ \cos\theta \end{pmatrix}$$

同様に

$$\vec{e}_\theta = (\vec{e}_1, \vec{e}_2, \vec{e}_3) \begin{pmatrix} \cos\theta\cos\phi \\ \cos\theta\sin\phi \\ -\sin\theta \end{pmatrix}$$

$$\vec{e}_\phi = (\vec{e}_1, \vec{e}_2, \vec{e}_3) \begin{pmatrix} -\sin\phi \\ \cos\phi \\ 0 \end{pmatrix}$$

$$\vec{e}_r \times \vec{e}_\theta = \begin{pmatrix} -\sin^2\theta\sin\phi - \cos^2\phi\sin\phi \\ \cos^2\theta\cos\phi + \sin^2\phi\cos\phi \\ -\sin^2\theta\sin\phi + \sin^2\theta\sin\phi \end{pmatrix} = \begin{pmatrix} -\sin\phi \\ \cos\phi \\ 0 \end{pmatrix}$$

よって $\vec{e}_\phi = \vec{e}_r \times \vec{e}_\theta$

3.

$$\Theta_{r\theta\phi} = (\vec{e}_r, \vec{e}_\theta, \vec{e}_\phi) = (\vec{e}_1, \vec{e}_2, \vec{e}_3) \begin{pmatrix} \sin\theta\cos\phi & \cos\theta\cos\phi & -\sin\phi \\ \sin\theta\sin\phi & \cos\theta\sin\phi & \cos\phi \\ \cos\theta & -\sin\theta & 0 \end{pmatrix}$$

$$T = \begin{pmatrix} \sin\theta\cos\phi & \cos\theta\cos\phi & -\sin\phi \\ \sin\theta\sin\phi & \cos\theta\sin\phi & \cos\phi \\ \cos\theta & -\sin\theta & 0 \end{pmatrix}$$

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III. 4

$$\vec{v}' = O v = O_{r\theta\phi} v_{r\theta\phi} = O T v_{r\theta\phi}$$

$$v = T v_{r\theta\phi}$$

$$r = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

$$r' = T r = \begin{pmatrix} x\sin\theta\cos\phi + y\cos\theta\cos\phi - z\sin\phi \\ x\sin\theta\sin\phi + y\cos\theta\sin\phi + z\cos\phi \\ x\cos\theta - y\sin\theta \end{pmatrix} = \begin{pmatrix} r_r \\ r_\theta \\ r_z \end{pmatrix}$$

5. $\frac{\partial}{\partial r} = \frac{\partial x}{\partial r}\frac{\partial}{\partial x} + \frac{\partial y}{\partial r}\frac{\partial}{\partial y} + \frac{\partial z}{\partial r}\frac{\partial}{\partial z}$

$$= \sin\theta\cos\phi\frac{\partial}{\partial x} + \sin\theta\sin\phi\frac{\partial}{\partial y} + \cos\theta\frac{\partial}{\partial z}$$

$$\frac{1}{r}\frac{\partial}{\partial\theta} = \cos\theta\cos\phi\frac{\partial}{\partial x} + \cos\theta\sin\phi\frac{\partial}{\partial y} - \sin\frac{\partial}{\partial z}$$

$$\frac{1}{r\sin\theta}\frac{\partial}{\partial\phi} = -\sin\phi\frac{\partial}{\partial x} + \cos\phi\frac{\partial}{\partial y} + 0\frac{\partial}{\partial z}$$

よって
$$\begin{pmatrix} \frac{\partial}{\partial r} \\ \frac{1}{r}\frac{\partial}{\partial\theta} \\ \frac{1}{r\sin\theta}\frac{\partial}{\partial\phi} \end{pmatrix} = \hat{T}\begin{pmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{pmatrix}$$

$$\vec{\nabla} = \theta\begin{pmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{pmatrix} = \theta_{r\theta\phi}\hat{T}T\begin{pmatrix} \frac{\partial}{\partial r} \\ \frac{1}{r}\frac{\partial}{\partial\theta} \\ \frac{1}{r\sin\theta}\frac{\partial}{\partial\phi} \end{pmatrix} = \theta_{r\theta\phi}\begin{pmatrix} \frac{\partial}{\partial r} \\ \frac{1}{r}\frac{\partial}{\partial\theta} \\ \frac{1}{r\sin\theta}\frac{\partial}{\partial\phi} \end{pmatrix}$$

$\theta_{r\phi z} = \theta T$

$$\vec{\nabla} = \vec{e}_r\frac{\partial}{\partial r} + \vec{e}_\theta\frac{1}{r}\frac{\partial}{\partial\theta} + \vec{e}_\phi\frac{1}{r\sin\theta}\frac{\partial}{\partial\phi}$$

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IV. 1

$$\delta z = \left(\frac{\partial z}{\partial x}\right)_y \delta x + \left(\frac{\partial z}{\partial y}\right)_x \delta y \ , \ \delta y = \left(\frac{\partial y}{\partial x}\right)_z \delta x + \left(\frac{\partial y}{\partial z}\right) \delta z$$

$\delta z = 0$ より

$$\left(\frac{\partial z}{\partial x}\right)_y \delta x = -\left(\frac{\partial z}{\partial y}\right)_x \delta y \ , \ \delta y = \left(\frac{\partial y}{\partial x}\right)_z \delta x$$

$$-\frac{\left(\frac{\partial z}{\partial x}\right)_y}{\left(\frac{\partial z}{\partial y}\right)_x} = \frac{\delta y}{\delta x} = \left(\frac{\partial y}{\partial x}\right)_z \frac{\delta x}{\delta x} = \left(\frac{\partial y}{\partial x}\right)_z$$

よって

$$\left(\frac{\partial y}{\partial x}\right)_z = -\frac{\left(\frac{\partial z}{\partial x}\right)_y}{\left(\frac{\partial z}{\partial y}\right)_x}$$

2.

$$\left(\frac{\partial y}{\partial z}\right)_x = -\frac{\left(\frac{\partial x}{\partial z}\right)_y}{\left(\frac{\partial x}{\partial y}\right)_z}$$

3. 1より $\left(\frac{\partial z}{\partial x}\right)_y \delta x = -\left(\frac{\partial z}{\partial y}\right)_x \delta y$

$$\left(\frac{\partial z}{\partial x}\right)_y \left(\frac{\partial y}{\partial z}\right)_x \frac{\delta x}{\delta y} = -1$$

$$\left(\frac{\partial x}{\partial y}\right)_z \left(\frac{\partial y}{\partial z}\right)_x \left(\frac{\partial z}{\partial x}\right)_y = -1$$

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V.1 $m\ddot{\vec{r}} = m\vec{g}$

$\ddot{\vec{r}} = \vec{g}$

$\dot{\vec{r}} = \vec{g}t + \vec{v_0}$

$\vec{r} = \frac{1}{2}\vec{g}t^2 + \vec{v_0}t + \vec{r_0}$

2. 原点を東京として

$$\vec{g} = \begin{pmatrix} 0 \\ 0 \\ g \end{pmatrix} \quad \vec{h} = \begin{pmatrix} 0 \\ 0 \\ h \end{pmatrix}$$

$$r(t) = \frac{1}{2}\vec{g}t^2 + \vec{h} = \begin{pmatrix} 0 \\ 0 \\ \frac{1}{2}gt^2 + h \end{pmatrix}$$

x軸を軸とする回転行列T

$$T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{pmatrix}$$

原点をずらすベクトルは $\vec{a}$

$$\vec{a} = \begin{pmatrix} 0 \\ -R\sin\theta_0 \\ -R(1-\cos\theta_0) \end{pmatrix} \quad \theta_0 : 緯度,\ \theta = -\theta_0$$

$$\vec{r'} = T\vec{r} + \vec{a}$$

$$= \begin{pmatrix} 0 \\ \sin\theta_0(\frac{1}{2}gt^2 + h) \\ \cos\theta_0(\frac{1}{2}gt^2 + h) \end{pmatrix} + \vec{a}$$