磁場中グラフェンの端状態

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Contents

1. Introduction - STM exp., Motivations, Honeycomb lattice with edge
2. Local charge density
3. Bond order, Entanglement Entropy

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STM/STS Experiments on Graphite

Niimi et al. PRL 97 236804 (2006)

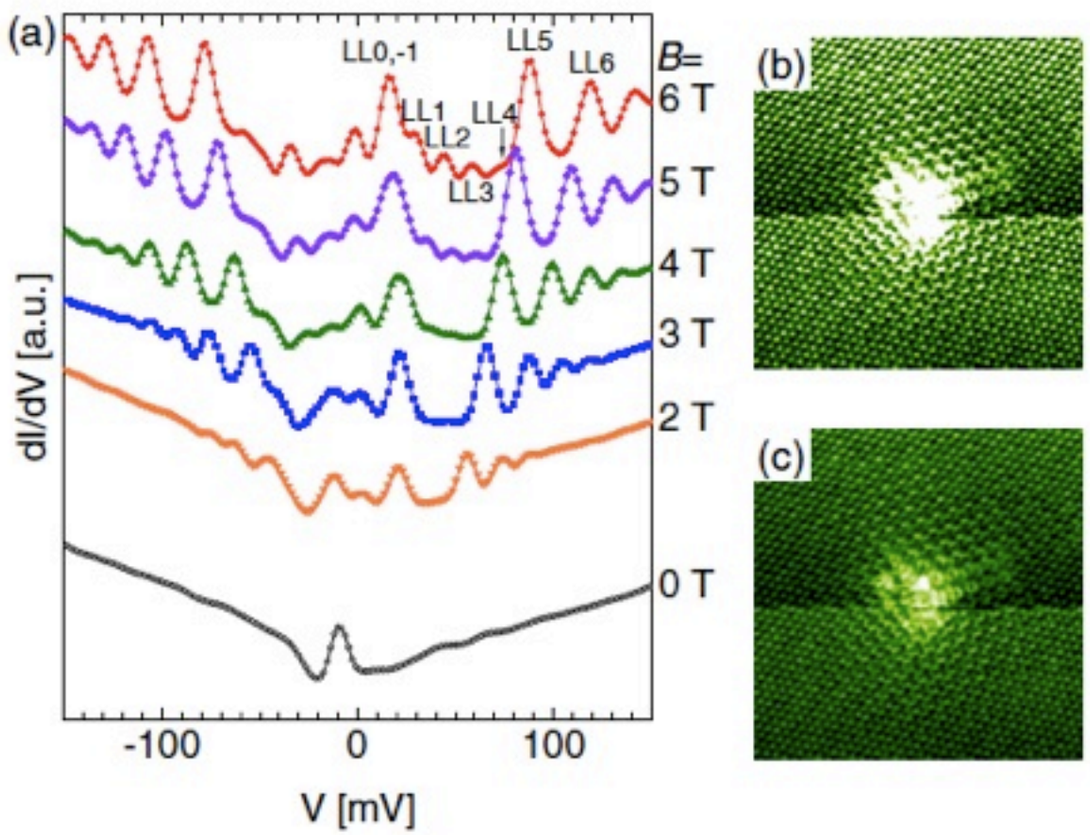
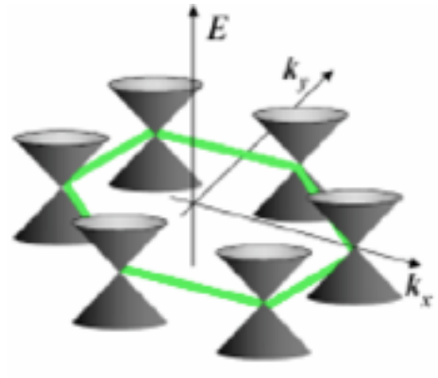


FIG. 1 (color online). (a) Tunnel spectra for a clean HOPG surface at $T = 30$ mK in several different magnetic fields perpendicular to the graphite basal plane ($V = 180$ mV, $I = 0.2$ nA). Each spectrum is vertically shifted for clarity. (b),(c) STM images with the same surface point defects at the center represented with different higher (b) and lower (c) contrasts (8×8 nm^2, $V = 180$ mV, $I = 0.2$ nA, $T = 30$ mK).

Hiroshi Fukuyama group (STM): cf. 23pWH7

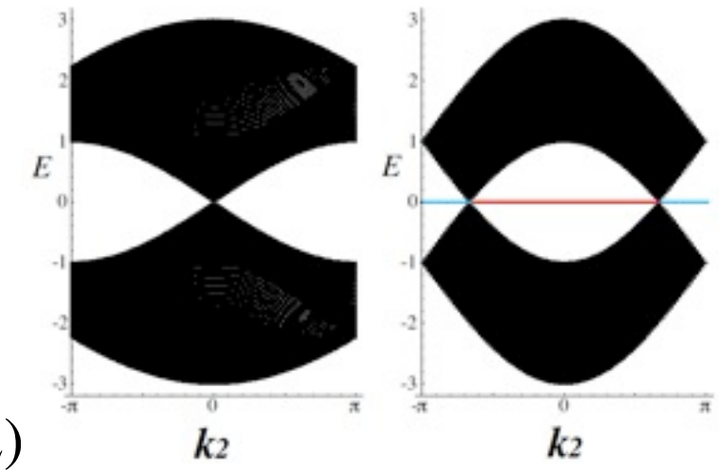
Motivations: New Bulk-Edge correspondence

Dirac cone of Honeycomb lattice



Edge mode (B=0, armchair/zigzag)

Fujita et al('96), Ryu & Hatsugai('02)

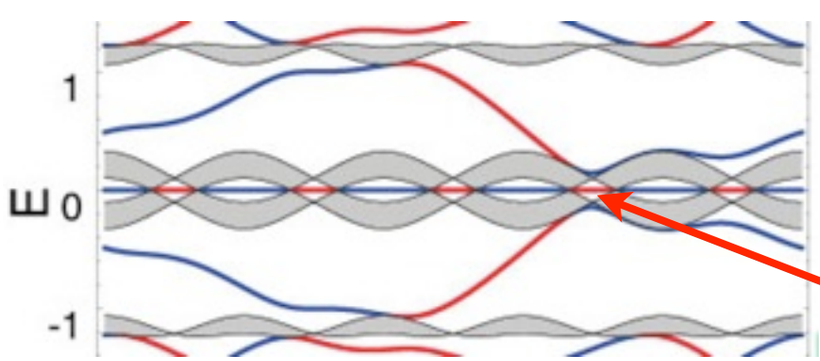


armchair zigzag/bearded

Edge modes play important roles in QHE.

Halperin ('82)

Bulk Landau level(n=0) & Edge mode coexist around E=0

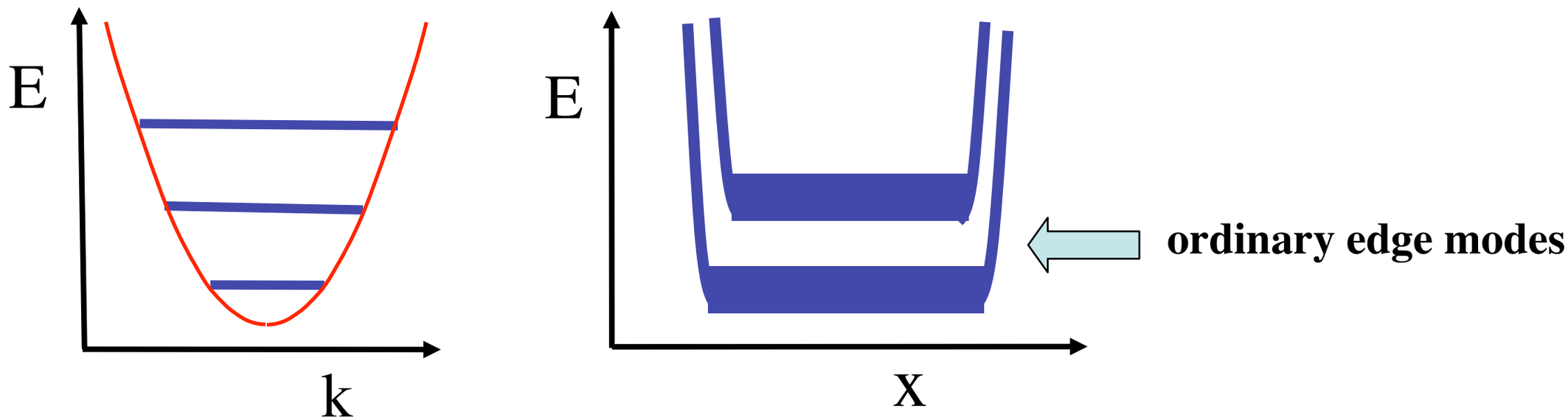


n=1

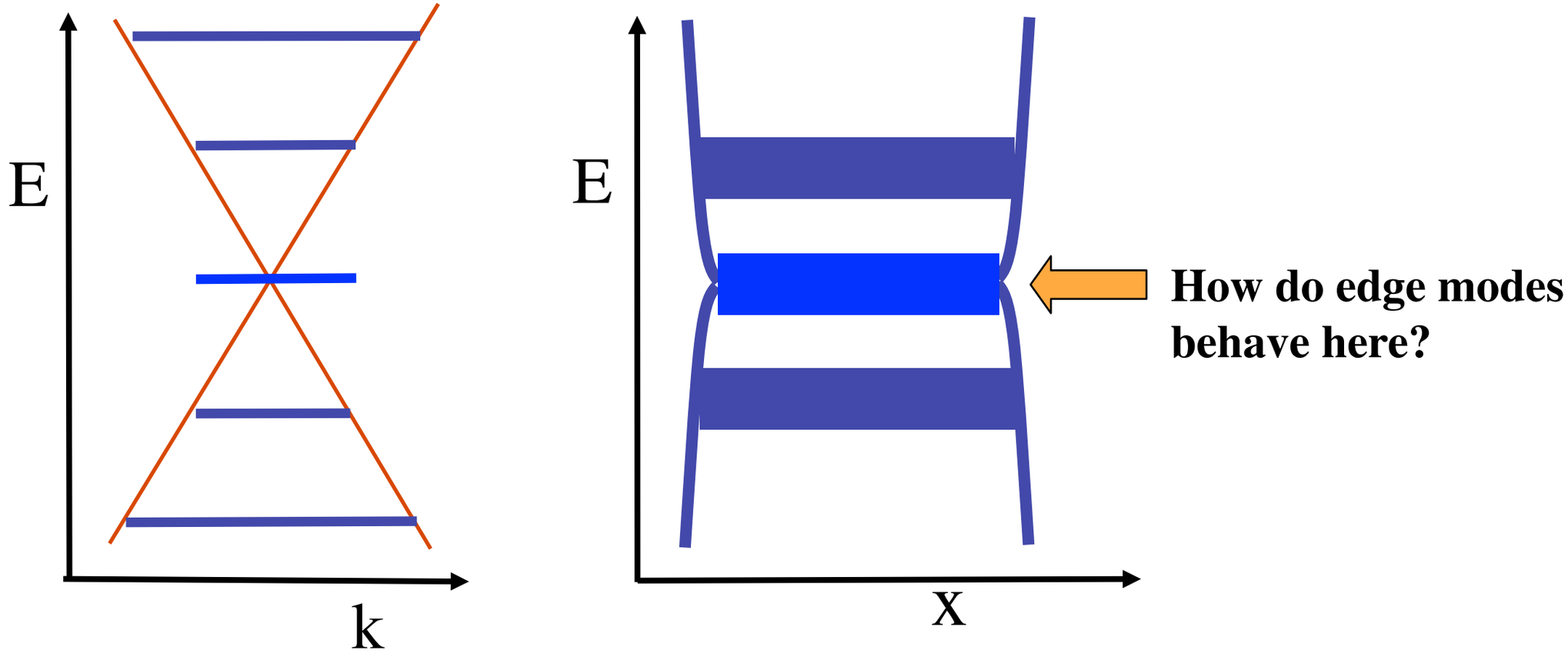
n=0

n=0 edge mode

Edge modes

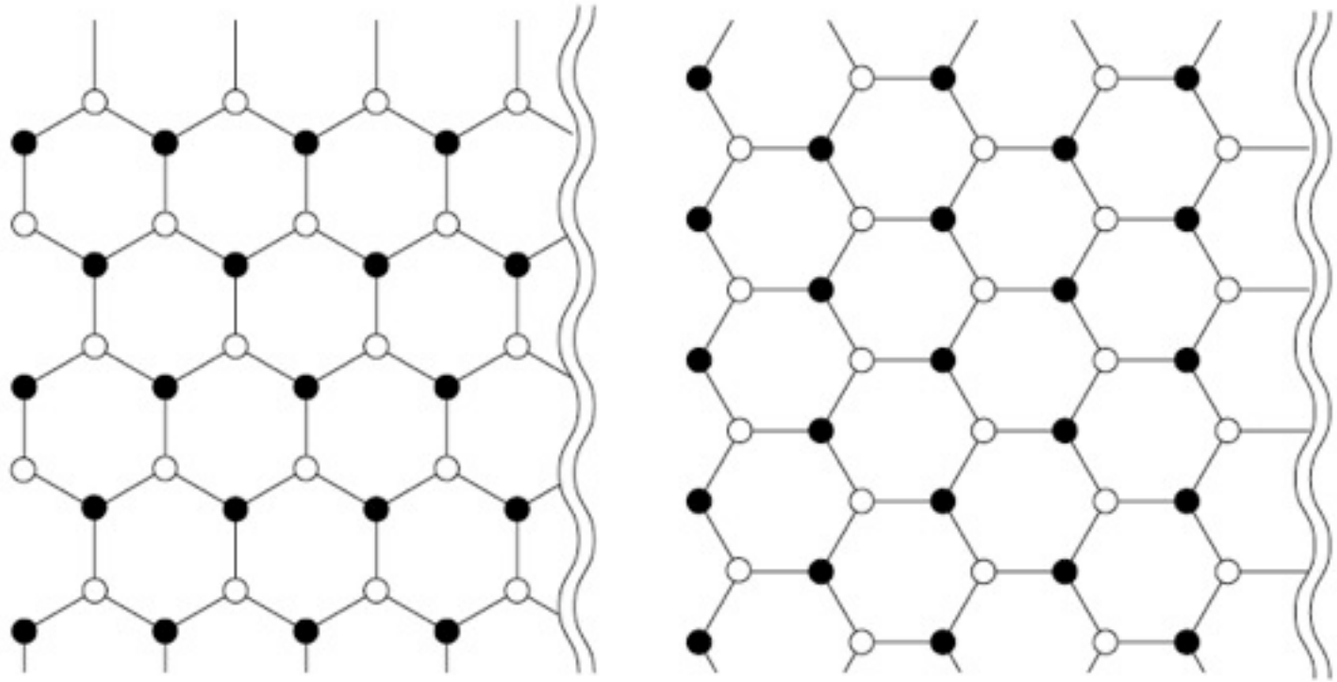


Ordinary QHE systems



Graphene QHE system

Honeycomb lattice with edge



Armchair edge Zigzag edge

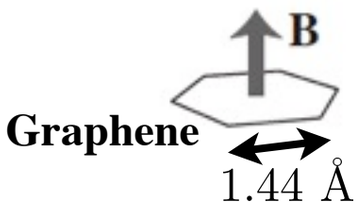
Topological properties: **Y.Hatsugai, T.Fukui, H.Aoki: PRB 2006**

(1)Chiral symmetry holds even in finite B and/or for finite systems with edges
(2)Zero mode always exists: D rectangular matrix

$$\mathcal{H} = \left(\begin{array}{c:c} 0 & D \\ \hdashline D^\dagger & 0 \end{array}\right) \quad D^\dagger \psi = 0, {}^\exists\psi \neq 0$$

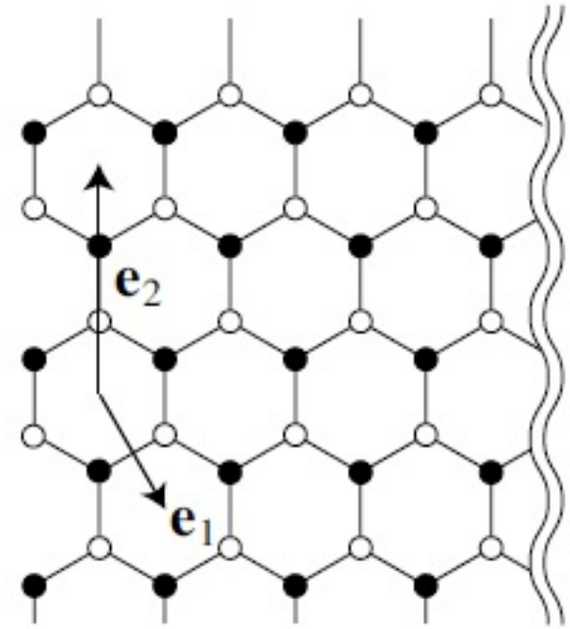
Model

Tight-binding Model Calculation

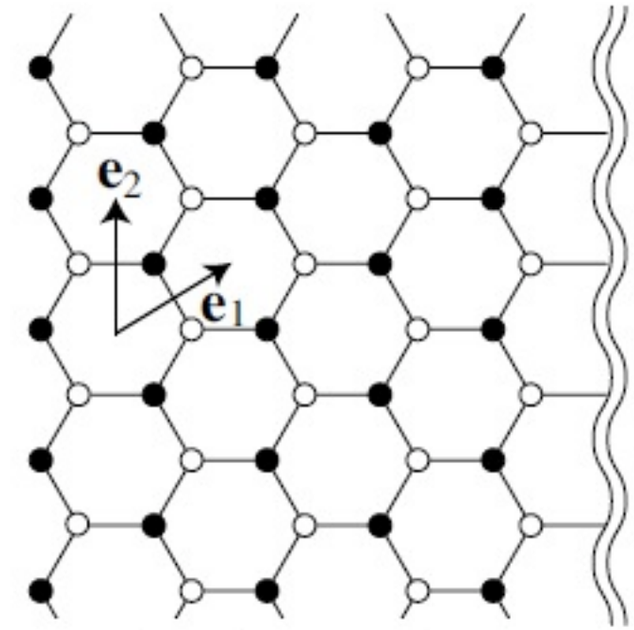


$$\phi = \frac{B\hexagon}{\Phi_0}, \quad \Phi_0 = \frac{h}{e}$$

$$\phi = 1/21 \rightarrow B = 3.7 \times 10^3 \text{ T}$$



(a)Armchair edge



(b)Zigzag edge

(a)Armchair edge

$$\mathcal{H} = t \sum_{\mathbf{j}} \Big[c_\bullet^\dagger(\mathbf{j}) c_\circ(\mathbf{j}) + c_\bullet^\dagger(\mathbf{j}+\mathbf{e}_1) c_\circ(\mathbf{j}) + e^{i2\pi\phi j_1} c_\circ^\dagger(\mathbf{j}+\mathbf{e}_1+\mathbf{e}_2) c_\bullet(\mathbf{j}) \Big] + \text{H.c.}$$

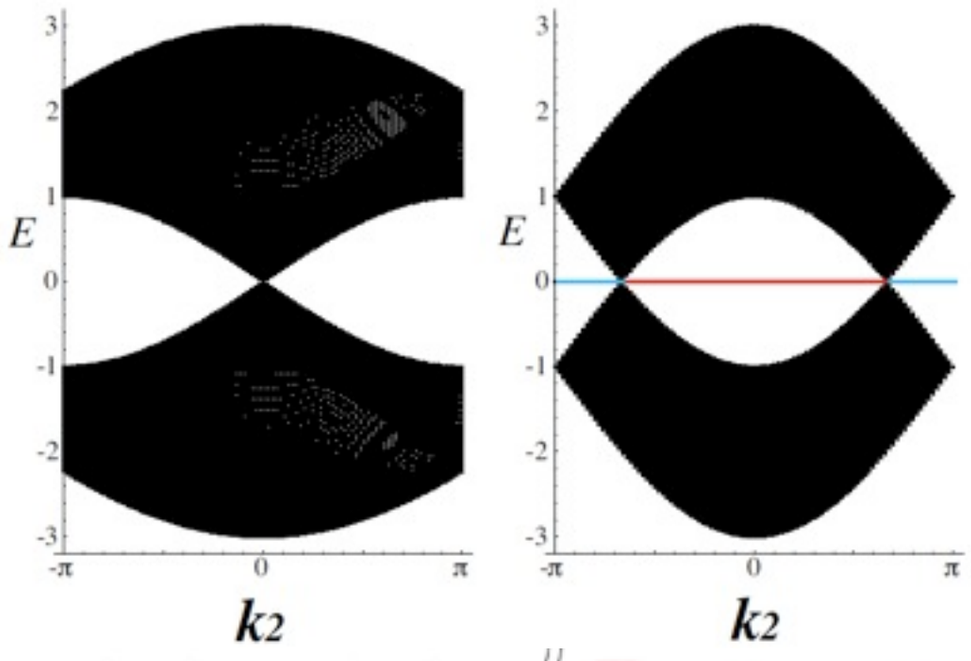
(b)Zigzag edge

$$\mathcal{H} = t \sum_{\mathbf{j}} \Big[c_\bullet^\dagger(\mathbf{j}) c_\circ(\mathbf{j}) + e^{i2\pi\phi j_1} c_\bullet^\dagger(\mathbf{j}) c_\circ(\mathbf{j}-\mathbf{e}_2) + c_\bullet^\dagger(\mathbf{j}+\mathbf{e}_1) c_\circ(\mathbf{j}) \Big] + \text{H.c.},$$

No magnetic field case

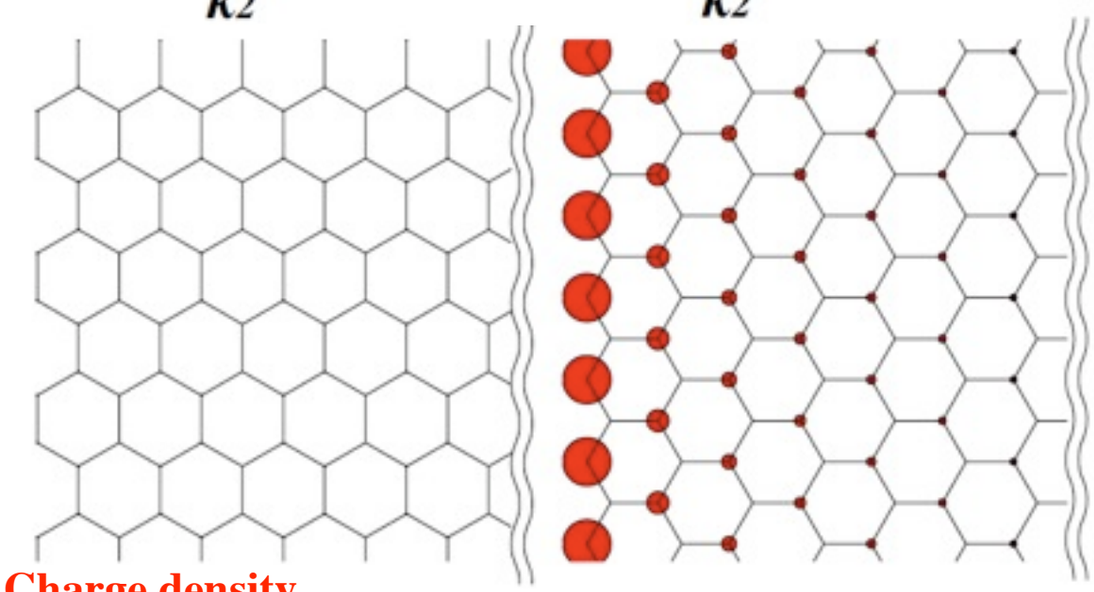
Fujita et al JPSJ 1996

armchair zigzag/bearded



$$c_\alpha(\mathrm{j}) = \frac{1}{\sqrt{L_2}} \sum_{k_2} e^{ik_2 j_2} c_\alpha(j_1, k_2)$$

$$|E| < 0.05$$



STM measurements

Local charge density in the Energy window

Area ∝ Charge density

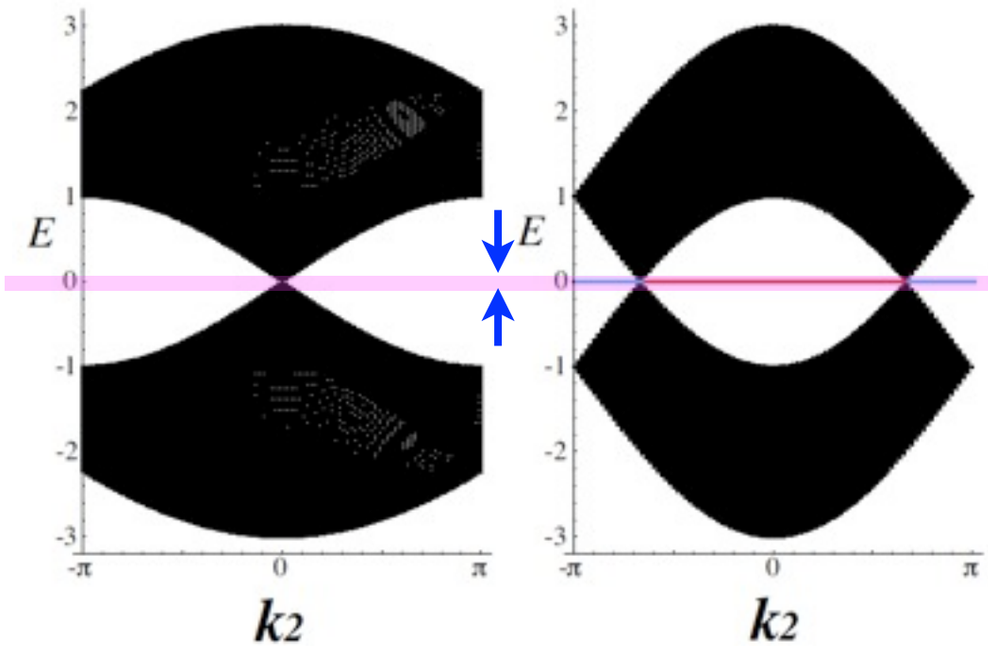
$$I(x) = \frac{1}{2\pi} \int_0^{2\pi} dk_2 \sum_{E_1 < E < E_2} |\psi(E, x, k_2)|^2$$

normalization: $(E_1, E_2) \to (-\infty, \infty),\ I(x) \to 1$

No magnetic field case

Fujita et al JPSJ 1996

armchair zigzag/bearded



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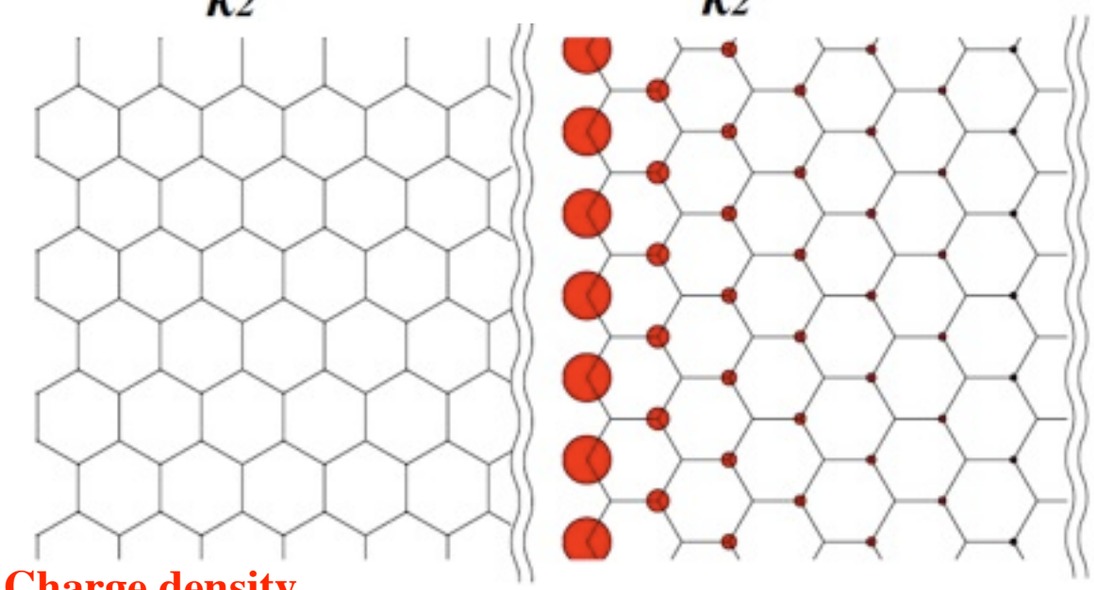
$$|E| < 0.05$$



STM measurements

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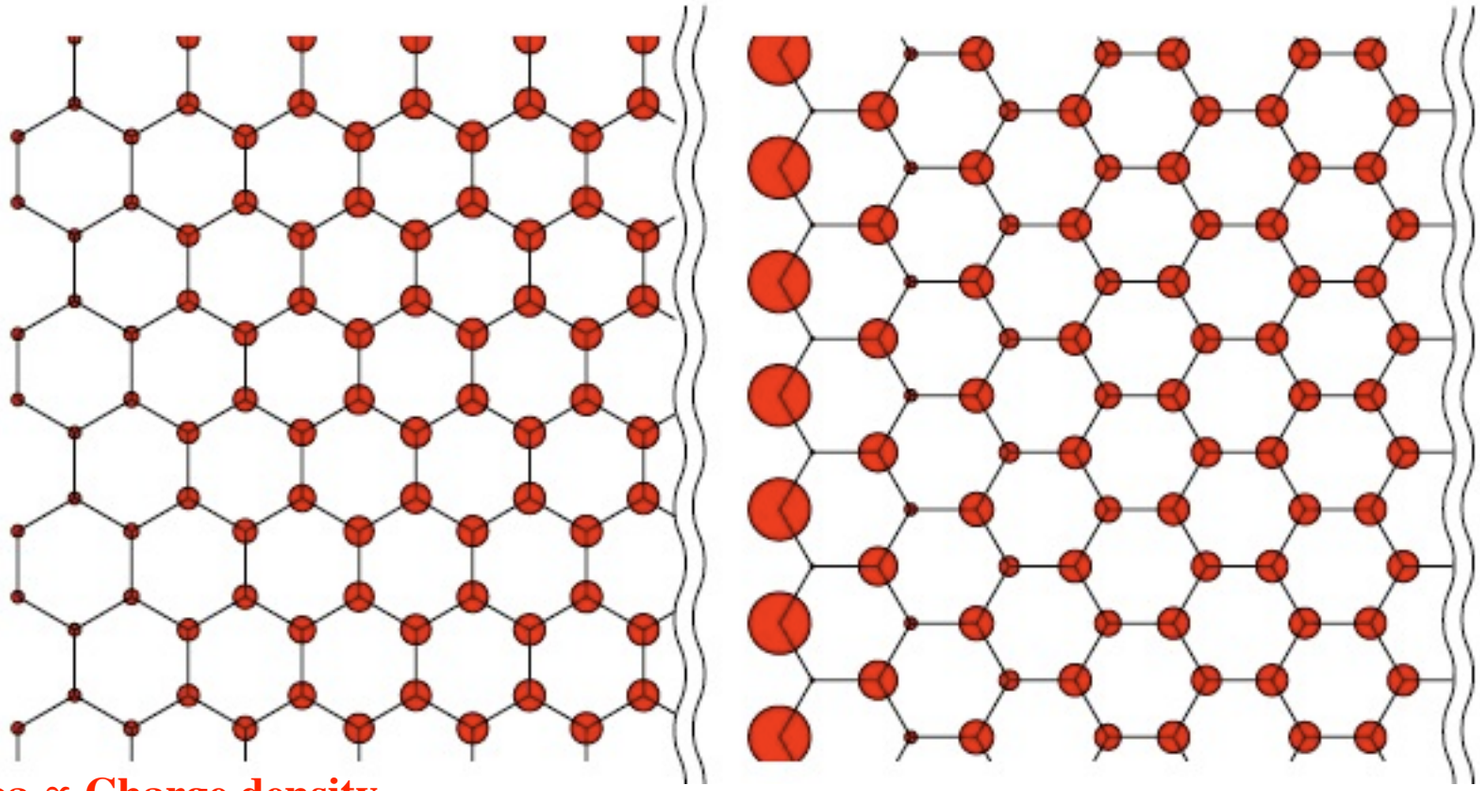
normalization: $(E_1, E_2) \to (-\infty, \infty),\ I(x) \to 1$

Charge density of n=0 Landau Level

$\phi = 1/21$



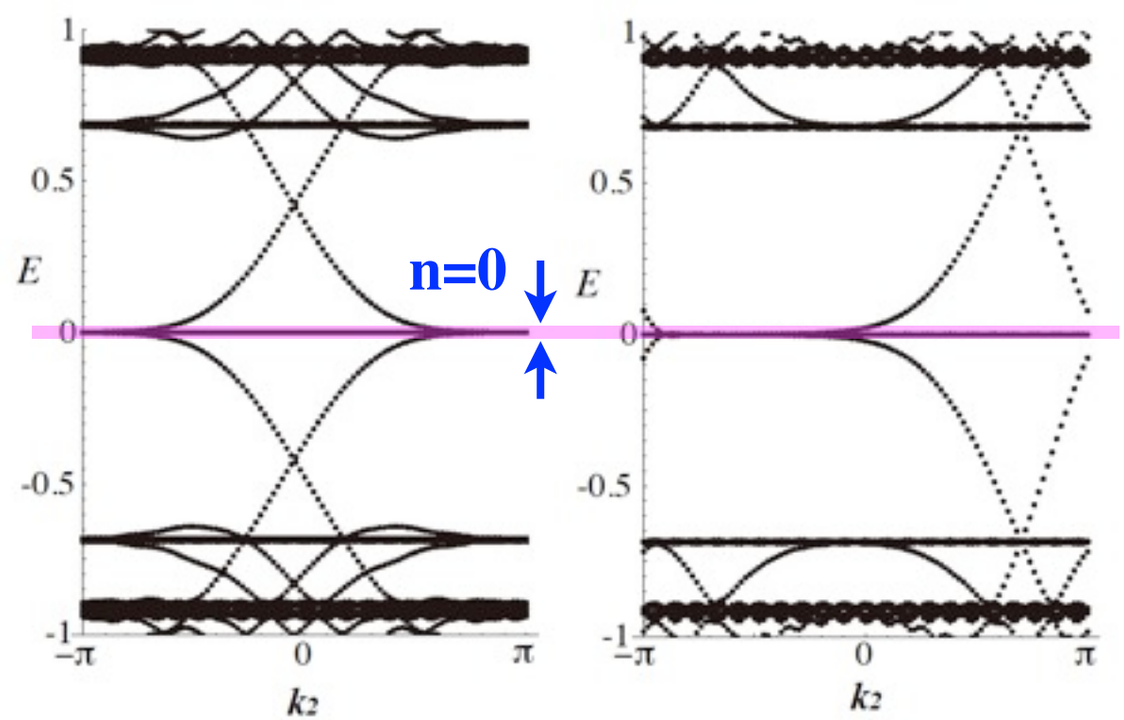
Area ∝ Charge density

Armchair edge Zigzag edge

Energy window

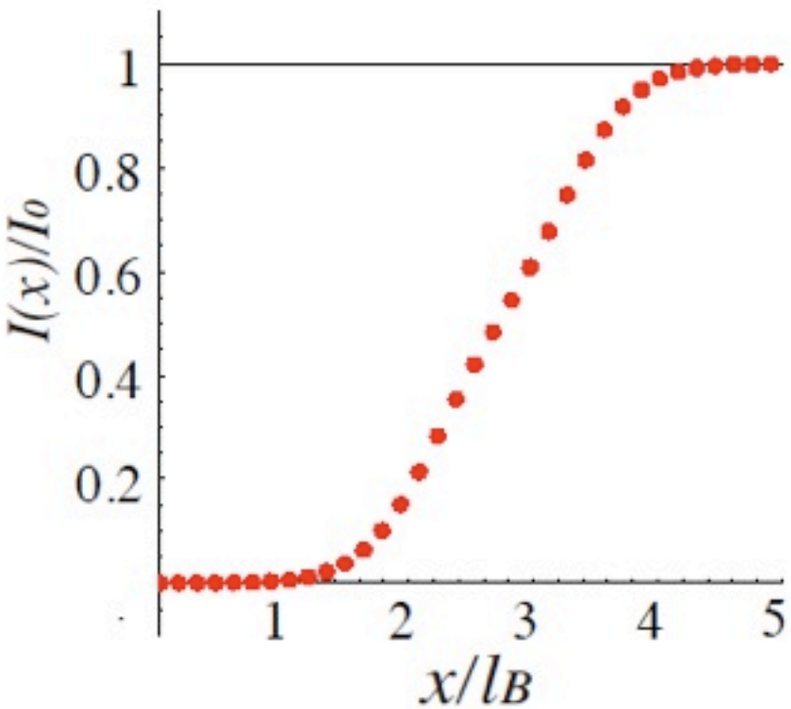
$|E| < 0.05$

Accumulation toward zigzag edge

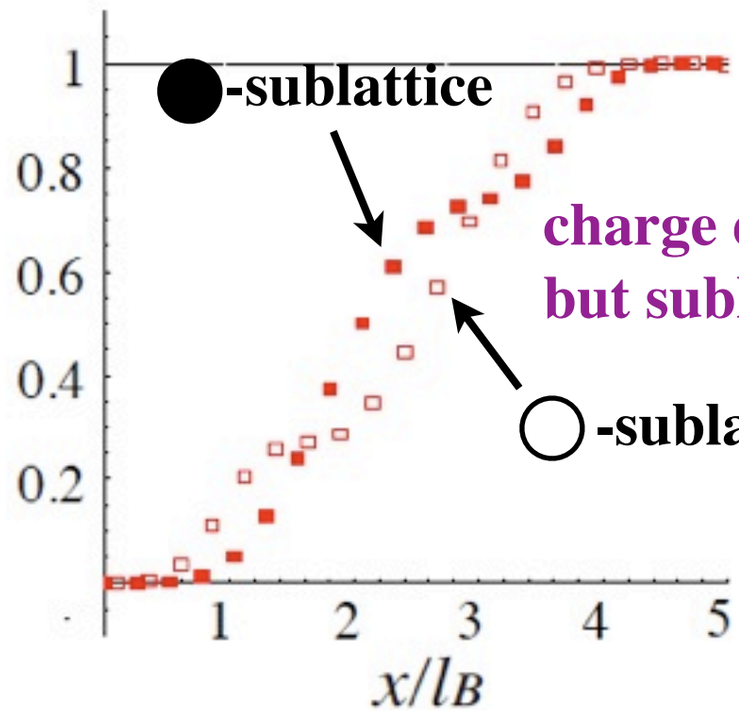


Armchair edge Zigzag/Bearded edge

Scaled charge density of n=1 LL



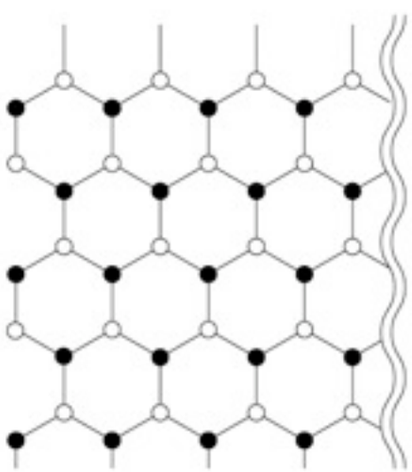
Armchair edge



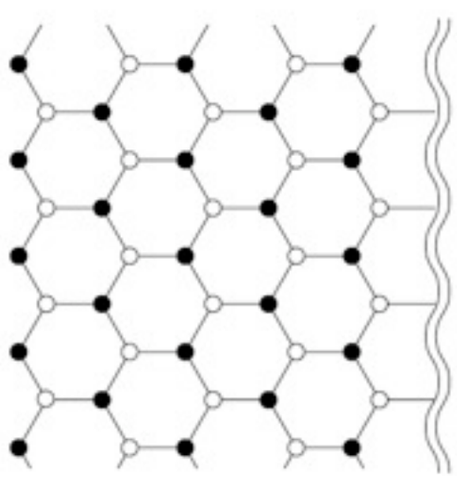
$\phi = 1/41$

charge depledted unlike n=0,
but sublattice dependence exists

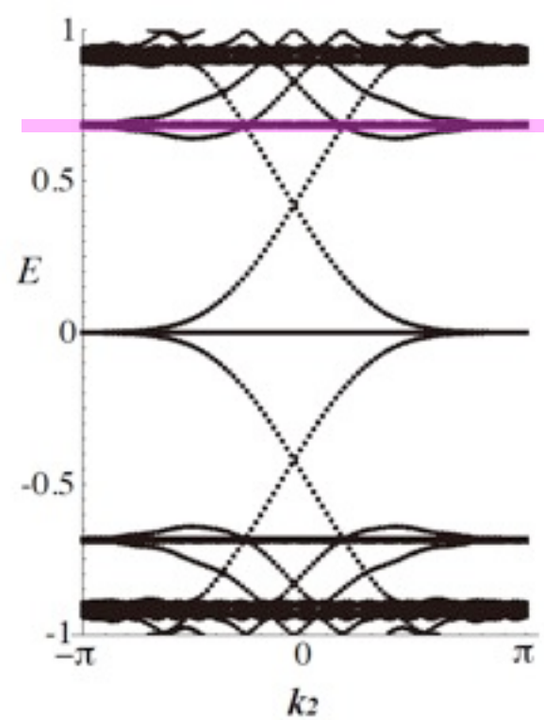
Zigzag edge



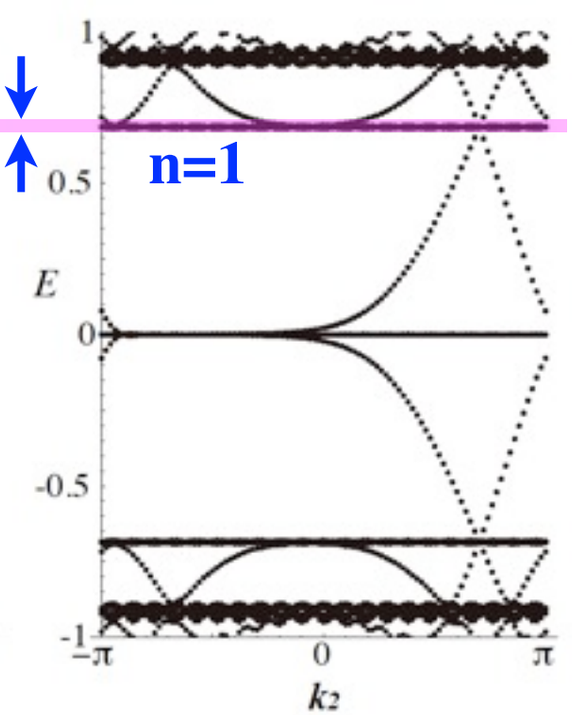
Armchair edge



Zigzag edge



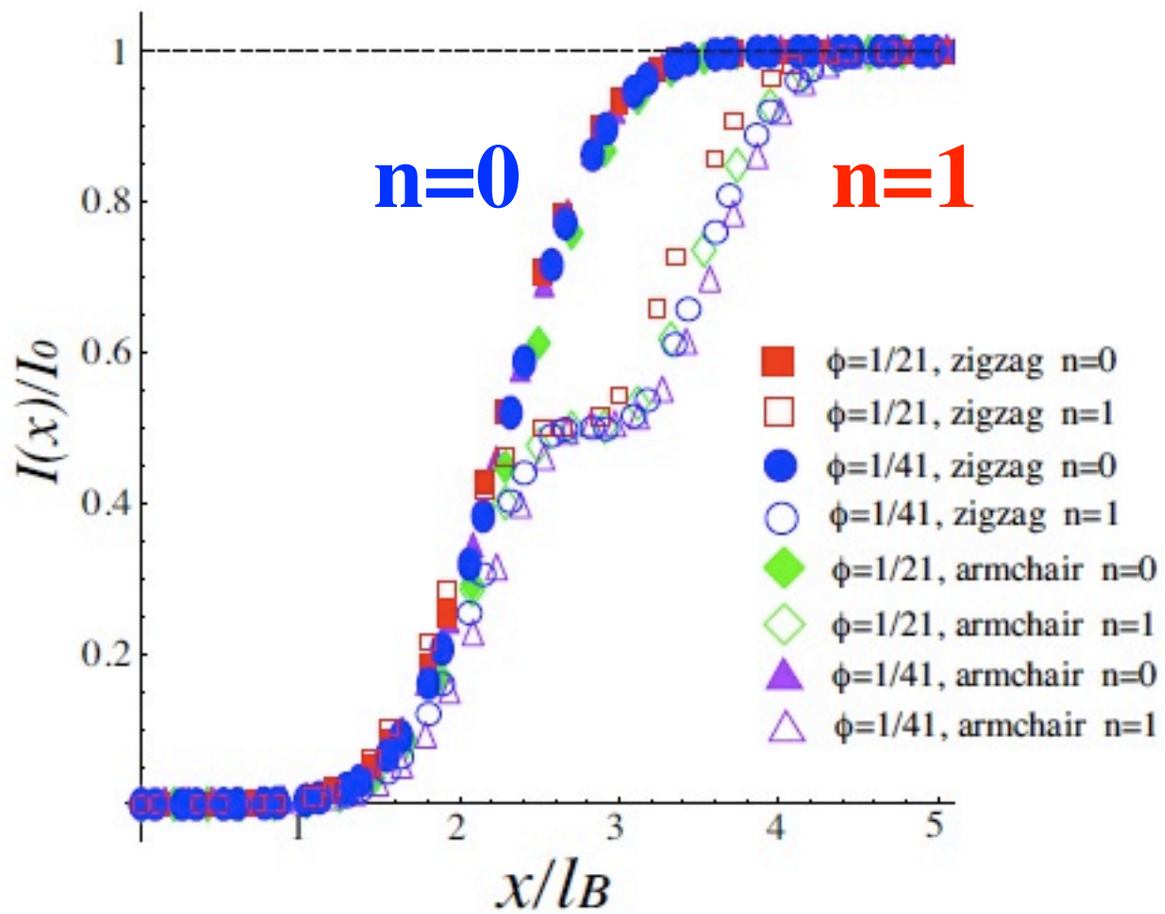
Armchair edge

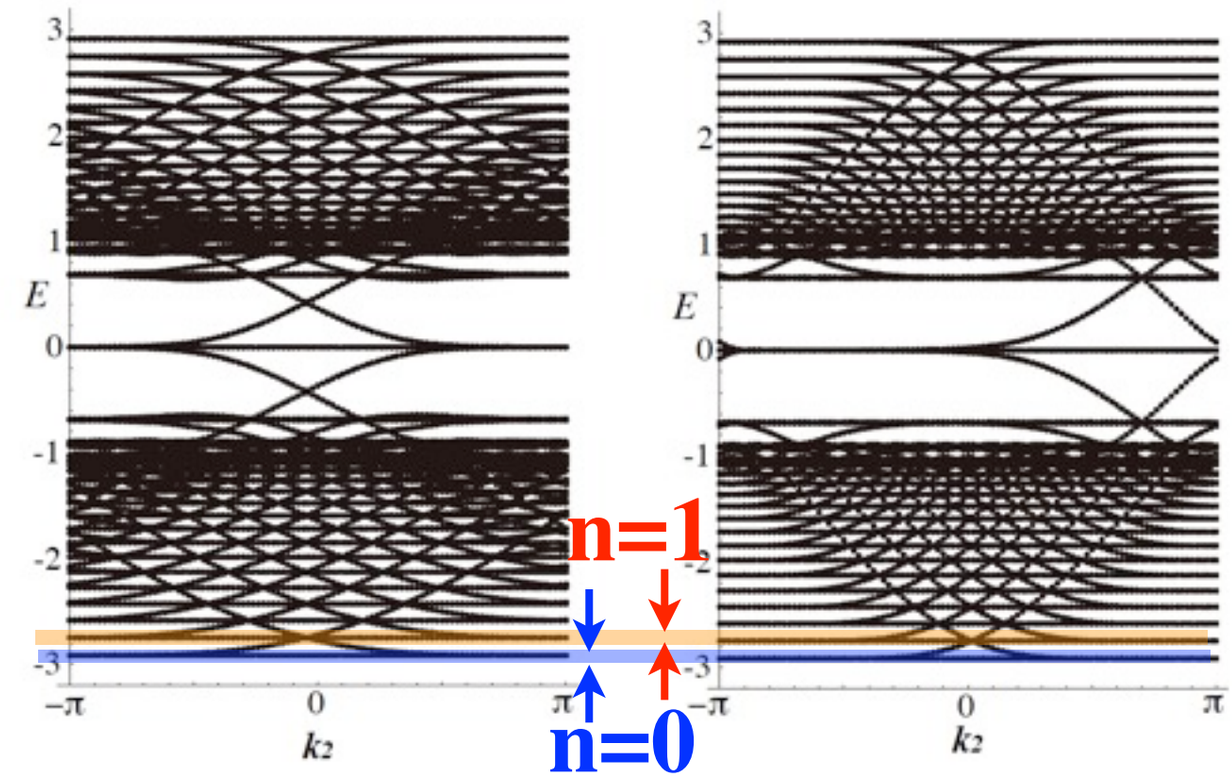


Zigzag/Bearded edge

Scaled charge density of LL near band edge

Ordinary QHE



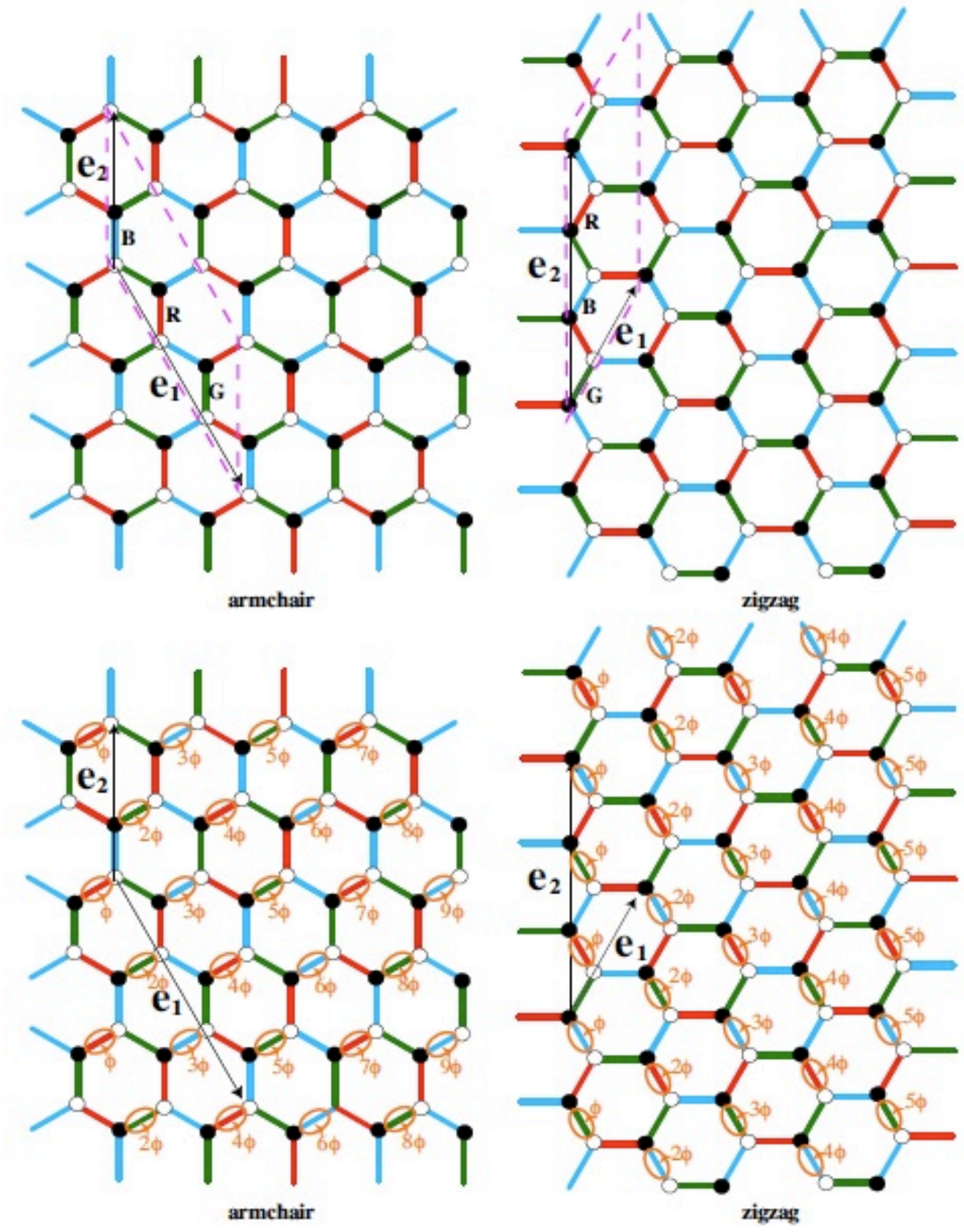


Armchair edge Zigzag/Bearded edge

$$I_0 = \phi/2$$

Independent of armchair/zigzag edge

Bond order, Entanglement



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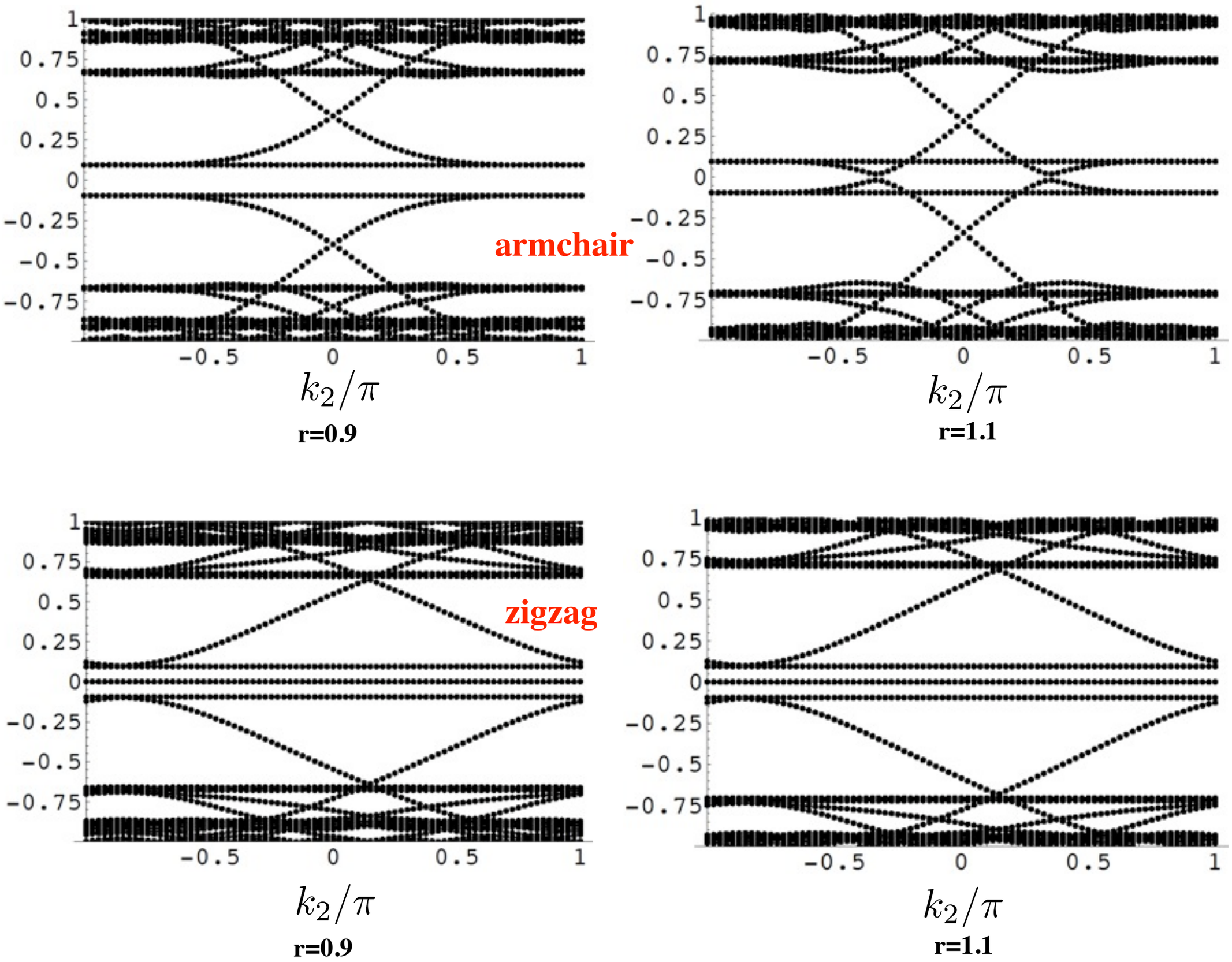
Energy dispersion

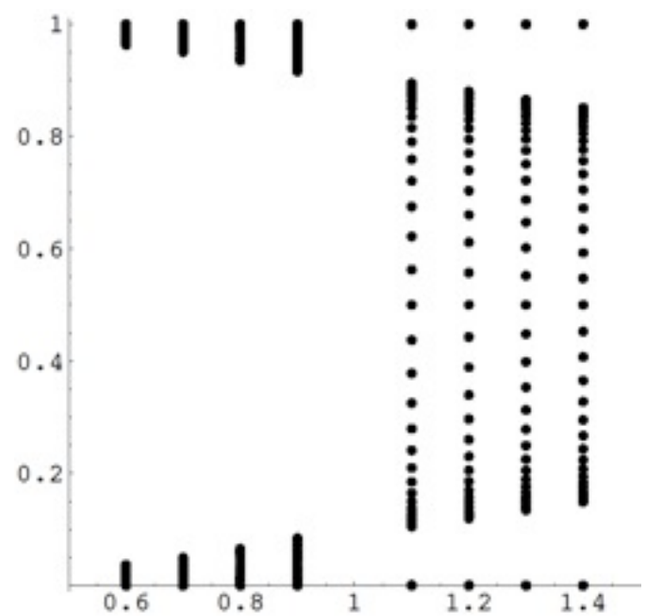
$$(t_R, t_G, t_B) = (1, 1, r)$$

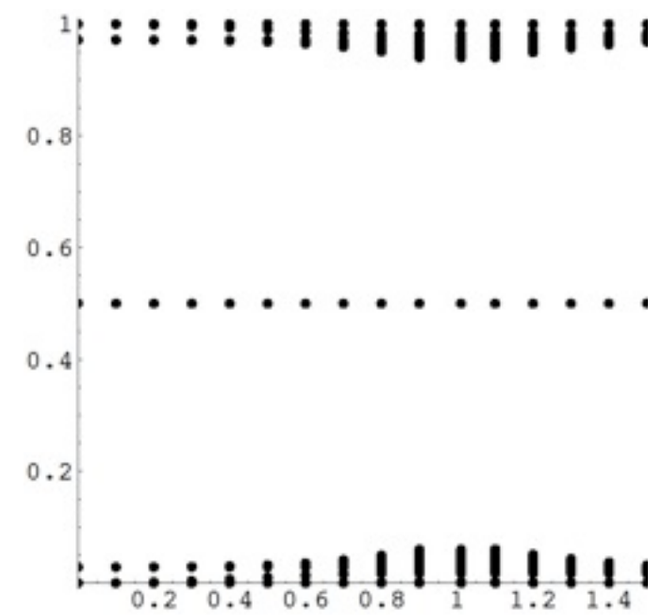


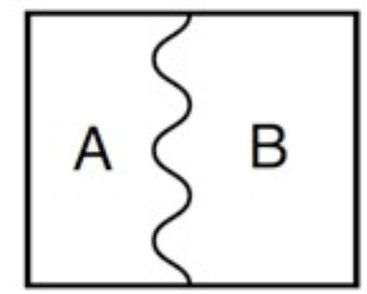
$$\rho_A = \mathrm{Tr}_\mathrm{B}\,\rho, \quad S_A = -\langle \log \rho_A \rangle_{\rho_A} = -\mathrm{Tr}_\mathrm{A}\,\rho_A \log \rho_A$$

Eigenvalues of Correlation Matrix

Eigenvalues of ρ_A $\{\eta_i\}$



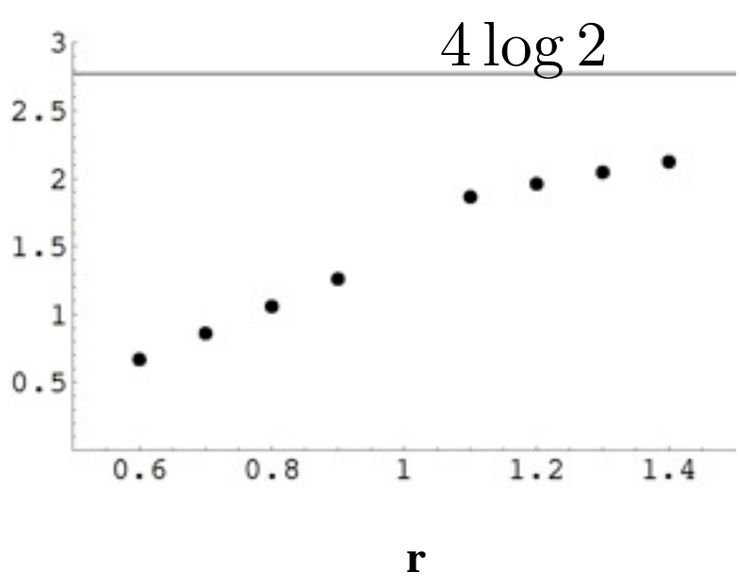


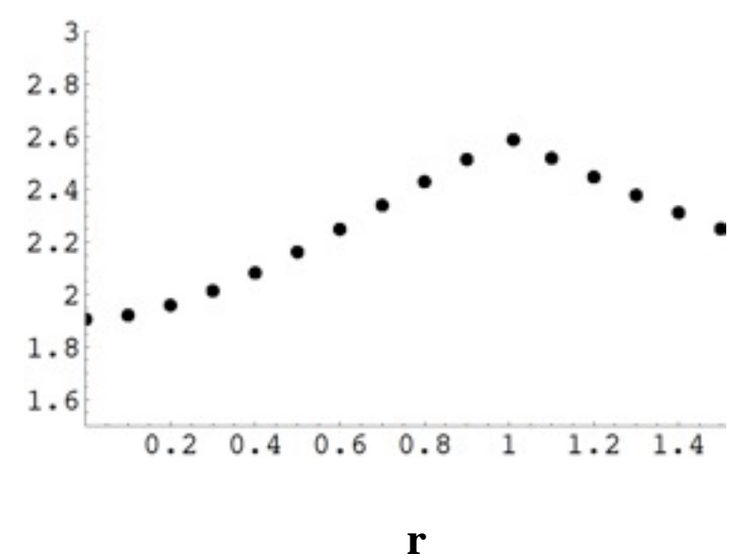


armchair

Entanglement Entropy

zigzag





$$S \sim -\sum_i (\eta_i \log \eta_i + (1-\eta_i)\log(1-\eta_i))$$