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STM/STS Experiments on Graphite

Niimi et al. PRL 97 236804 (2006)

FIG. 1 (color online). (a) Tunnel spectra for a clean HOPG surface at $T = 30$ mK in several different magnetic fields perpendicular to the graphite basal plane ($V = 180$ mV, $I = 0.2$ nA). Each spectrum is vertically shifted for clarity. (b),(c) STM images with the same surface point defects at the center represented with different higher (b) and lower (c) contrasts ($8 \times 8$ nm$^2$, $V = 180$ mV, $I = 0.2$ nA, $T = 30$ mK).

Hiroshi Fukuyama group (STM): cf. 23pWH7
Motivations: New Bulk-Edge correspondence

Dirac cone of Honeycomb lattice

Edge mode (B=0, armchair/zigzag)
Fujita et al.’96, Ryu & Hatsugai’02

Edge modes play important roles in QHE.
Halperin ’82

Bulk Landau level (n=0) & Edge mode coexist around $E=0$

$n=0$
$n=1$

$n=0$ edge mode
**Edge modes**

**Ordinary QHE systems**

**Graphene QHE system**

How do edge modes behave here?

ordinary edge modes

How do edge modes behave here?
Topological properties:

(1) Chiral symmetry holds even in finite B and/or for finite systems with edges
(2) Zero mode always exists: $D$ rectangular matrix

$D^\dagger \psi = 0, \exists \psi \neq 0$

Model

Tight-binding Model Calculation

\[ \phi = \frac{B}{\Phi_0}, \quad \Phi_0 = \frac{h}{e} \]

\[ \phi = 1/21 \rightarrow B = 3.7 \times 10^3 \text{ T} \]

(a) Armchair edge

\[ \mathcal{H} = t \sum_j \left[ c_\downarrow^\dagger(j)c_\downarrow(j) + c_\downarrow^\dagger(j + e_1)c_\downarrow(j) + e^{i2\pi\phi j_1}c_\downarrow^\dagger(j + e_1 + e_2)c_\downarrow(j) \right] + \text{H.c.} \]

(b) Zigzag edge

\[ \mathcal{H} = t \sum_j \left[ c_\downarrow^\dagger(j)c_\downarrow(j) + e^{i2\pi\phi j_1}c_\downarrow^\dagger(j)c_\downarrow(j - e_2) + c_\downarrow^\dagger(j + e_1)c_\downarrow(j) \right] + \text{H.c.} \]
No magnetic field case

\[ I(x) = \frac{1}{2\pi} \int_0^{2\pi} dk_2 \sum_{E_1 < E < E_2} |\psi(E, x, k_2)|^2 \]

Area \( \propto \) Charge density

Fujita et al. JPSJ 1996

STM measurements

Local charge density in the Energy window

\[ c_{\alpha}(j) = \frac{1}{\sqrt{L_2}} \sum_{k_2} e^{ik_2j_2} c_{\alpha}(j_1, k_2) \]

\[ |E| < 0.05 \]
No magnetic field case

\[ I(x) = \frac{1}{2\pi} \int_{0}^{2\pi} dk_2 \sum_{E_1 < E < E_2} |\psi(E, x, k_2)|^2 \]

Normalization: \((E_1, E_2) \rightarrow (-\infty, \infty), \ I(x) \rightarrow 1\)

STM measurements

Local charge density in the Energy window

\[ c_\alpha(j) = \frac{1}{\sqrt{L_2}} \sum_{k_2} e^{ik_2j_2} c_\alpha(j_1, k_2) \]

\[ |E| < 0.05 \]
Charge density of $n=0$ Landau Level

$$\phi = \frac{1}{21}$$

Energy window

$$|E| < 0.05$$

Armchair edge

Zigzag edge

Accumulation toward zigzag edge

Area $\propto$ Charge density

$n=0$
Scaled charge density of n=1 LL

Armchair edge  Zigzag edge

charge depleted unlike n=0, but sublattice dependence exists

$\phi = 1/41$

Armchair edge  Zigzag edge

$E$

$n=1$

Armchair edge  Zigzag/Bearded edge
Scaled charge density of LL near band edge

Ordinary QHE

Independent of armchair/zigzag edge

\[ I_0 = \phi / 2 \]
Bond order, Entanglement

Y. Hatsugai, T. Fukui, H Aoki Physica E 40 2008
Energy dispersion

\[ (t_R, t_G, t_B) = (1, 1, r) \]

Armchair

\[ \frac{k_2}{\pi} \]

\[ r = 0.9 \]

\[ r = 1.1 \]

Zigzag

\[ \frac{k_2}{\pi} \]

\[ r = 0.9 \]

\[ r = 1.1 \]
\[ \rho_A = \text{Tr}_B \rho, \quad S_A = -\langle \log \rho_A \rangle_{\rho_A} = -\text{Tr}_A \rho_A \log \rho_A \]

**Eigenvalues of Correlation Matrix**

Armchair

Entanglement Entropy

\[ S \sim -\sum_i (\eta_i \log \eta_i + (1 - \eta_i) \log(1 - \eta_i)) \]

Zigzag