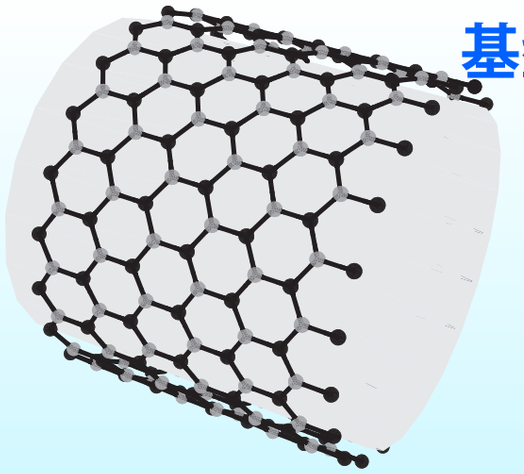
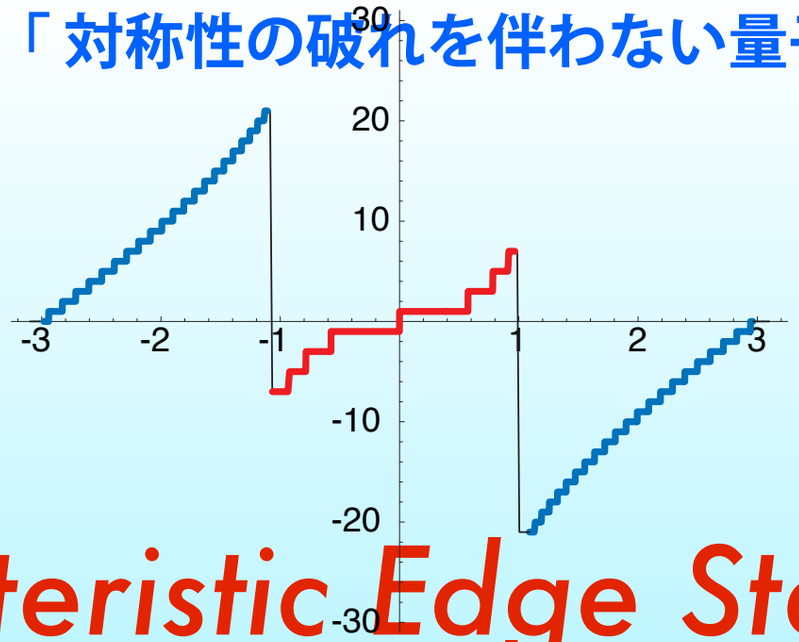
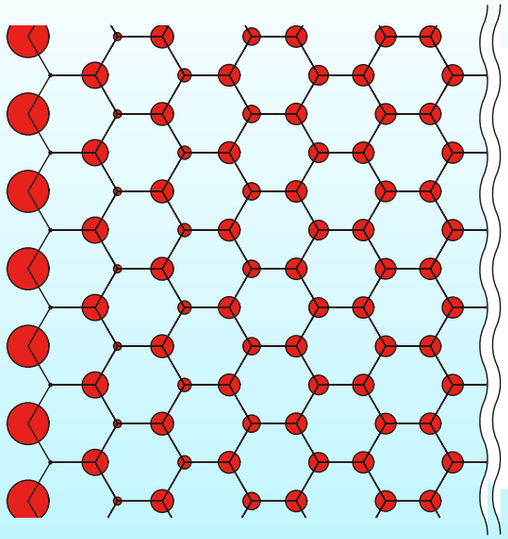
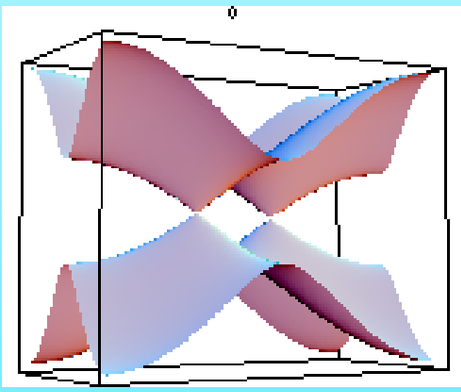






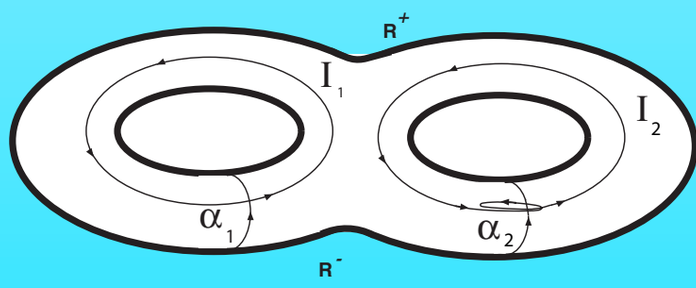
Characteristic Edge States of Quantum Hall effects in Graphene

: Dirac fermions and quantum liquids



Institute of Physics
University of Tsukuba
Y. Hatsugai

with
H. Aoki (Univ. of Tokyo)
M. Arikawa (Univ. of Tsukuba)
T. Fukui (Ibaragi Univ.)



Today's Talk

- Edge States in Condensed Matter Physics
- Quantum Hall effects: stability and topology
  - Fundamental role of Edge states
  - Topological numbers
  - Bulk-Edge Correspondence
- Quantum Hall effects in graphene
  - Graphene as a Filled Dirac Sea
    - Summation of the Chern number for each bands may not work
  - Two types of Edge states
- Topological Zero modes of Graphene
  - quantized Berry phase, chiral symmetry
- Localized charge of graphene n=0 L.L.
  - Topological compensation
- Possible Many body states of graphene as a quantum liquid
  - Bond ordering & Edge states as quasiparticles to condense

Edge States in Condensed Matter Physics

- ☆ Bound states in quantum mechanics
  Levinson's theorem, Friedel sum rule
- ☆ Surface states in Semiconductors
- ☆ Solitons in polyacetylene Su-Schriefer-Heeger '79
- ☆ Edge states in quantum Hall effects Halperin '82 Hatsugai '93
- ☆ Local moments in integer spin chains near the impurities
  Kennedy '90 Hagiwara-Katsumata-Affleck-Halperin-Renard '90
- ☆ Zero bias conductance peaks of the d-wave superconductors
  Hu, '94
- ☆ Zero energy localized states of graphene
  Fujita et al.'96 Ryu-Hatsugai'02
- ☆ Spin Hall Edge states Kane-Mele'95

more possibilities

- ☆ Edge states in 2D cold atoms optical lattices
  Scarola-Das Sarma., PRL 98, 210403 '07
- ☆ One-way edge modes in gyromagnetic photonic crystals
  Wang et al., PRL 100, 013905 '08

Why the Edge States are there??

Accidental ?

NO !

Inevitable reasons

Physical Structures behind:

Bulk determines the edges : Bulk-Edge Correspondence

Protected by

Topological constraints

and also additional Symmetry

Revisit the past and discuss graphene Quantum Hall effects

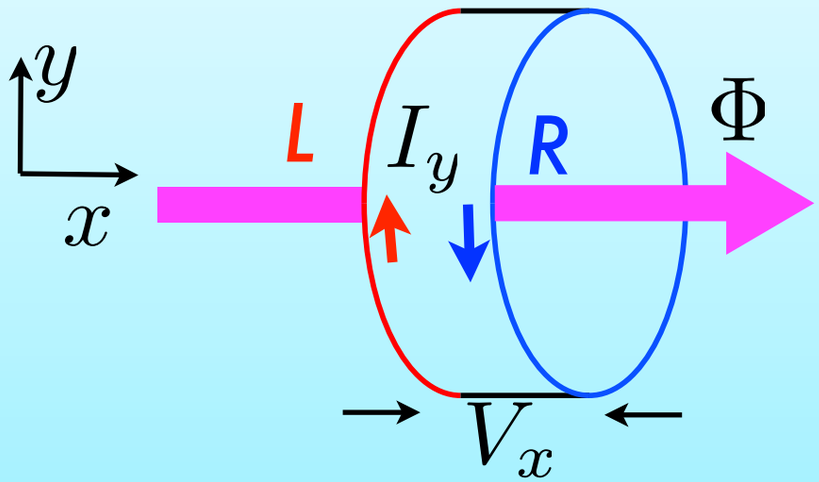
Several New features

They Can Be detected by STM experiments !

Stability of the quantized Hall Conductance

☆ Gauge invariance and quantization of σ_{xy} Laughlin '81

adiabatic process to increase Φ



$$A \to A' = A + \nabla\Phi, \quad \delta\Phi = \int_{\bigcirc} (A' - A)$$

$$\psi \to \psi' = e^{i2\pi\delta\Phi/\Phi_0}\ \psi$$ one particle state

$$\delta\Phi = \Phi_0 = \frac{e}{h} \longrightarrow \psi' = \psi$$

flux quantum

Gauge transformation

Byers-Yang formula

$$I_y = \frac{\Delta E}{\Delta\Phi} = \frac{neV_x}{h/e} = \boxed{n\left(\frac{e^2}{h}\right)} V_x = \sigma_{yx} V_x \qquad (\sigma_{yx})$$

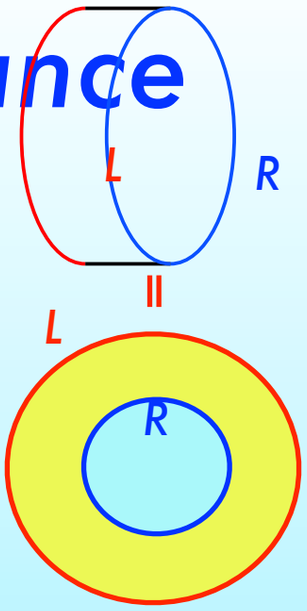
$$\Delta\Phi = \Phi_0 = \frac{h}{e}, \quad \Delta E = n \cdot eV_x$$

All states are invariant up to phase after the process:

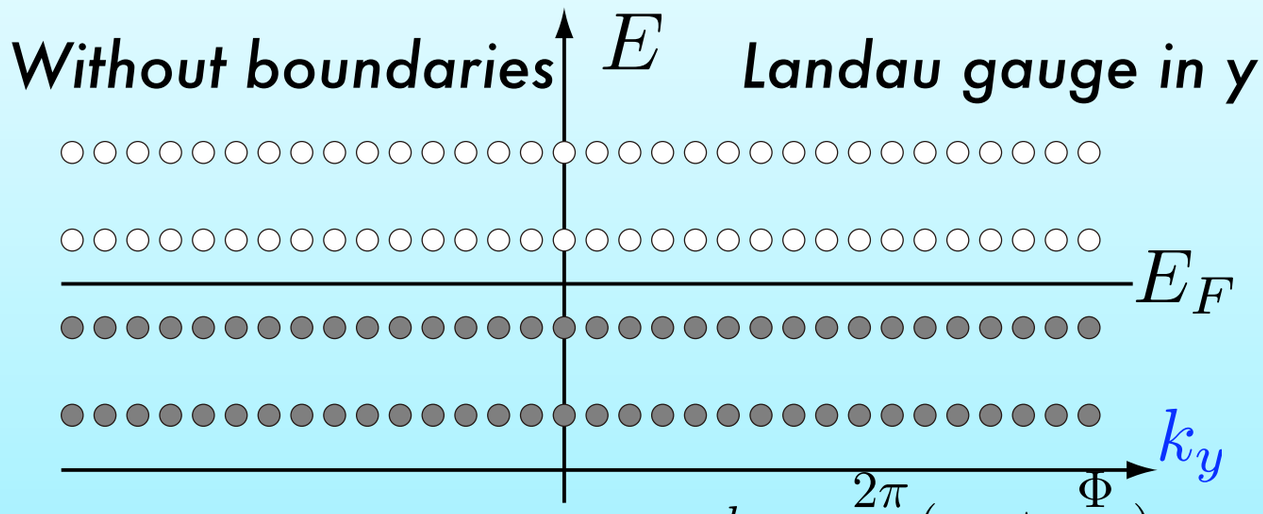
Some n states are carried from L to the R

n : generic integer (but undetermined)

Stability of the quantized Hall Conductance



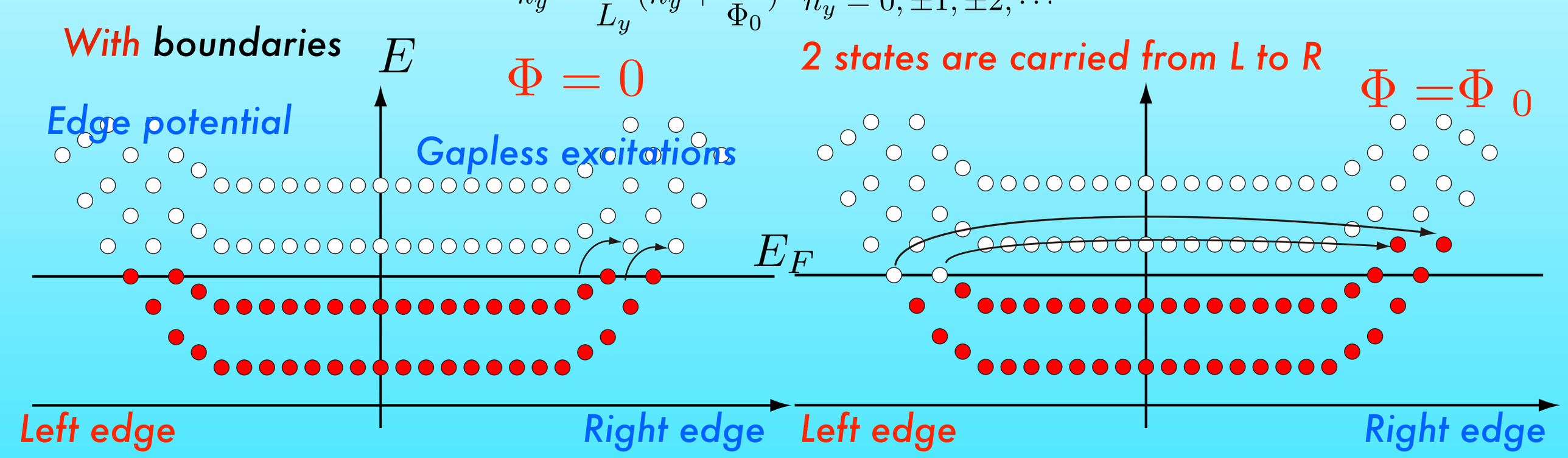
☆ *Edge states and Hall conductance* σ_{xy} *Halperin '82*



$$H_{2D} = \sum_{k_y} H_{1D}(k_y)$$

$H_{1D}(k_y)$: harmonic osc. centered at $\langle x \rangle \sim \ell_B^2 k_y$

$$k_y = \frac{2\pi}{L_y}(n_y + \frac{\Phi}{\Phi_0}) \quad n_y = 0, \pm 1, \pm 2, \cdots$$



Laughlin's undetermined n *: # of Landau Levels below* E_F

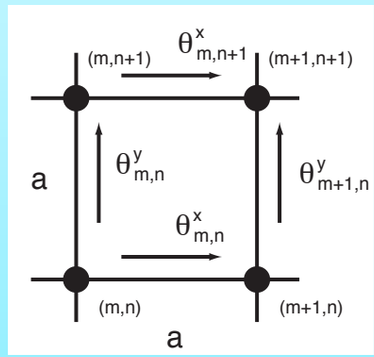
Edge states are essential in the QHE !

Topological meaning of the Hall Conductance

- Discus… …stitution)
  - pres…
  - with…
  - reco… …weak field limit)

$$H = \sum_{\langle i,\ldots}$$

Direct application for the Graphene



…ette

When E_F is in the j-th gap

Two topological quantities

$$\sigma_{xy}^{\text{bulk}} = \frac{e^2}{h} \sum_{\ell:\epsilon_\ell(k)<E_F} C_\ell$$

Sum of the First Chern numbers below E_F

Thouless-Kohmoto-Nightingale-den Nijs '82

$$\sigma_{xy}^{\text{edge}} = \frac{e^2}{h} I(\alpha_j, C^j)$$

Winding number of the edge state in the complex energy surface

Hatsugai '93a

Bulk — Edge Correspondence

Hatsugai '93b

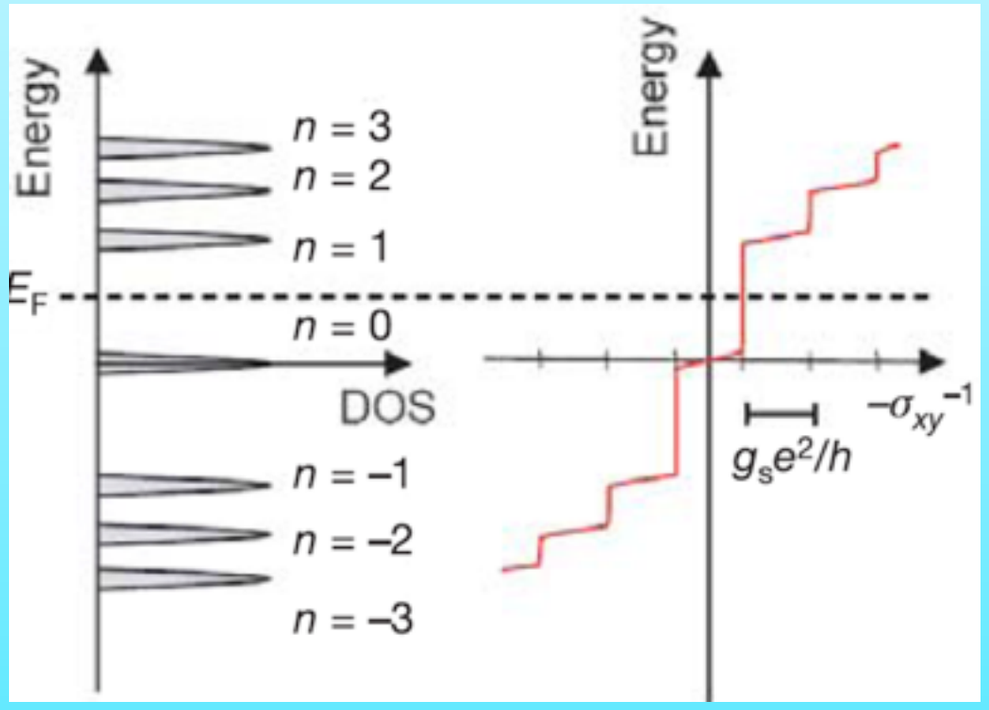
$$\sigma_{xy}^{\text{bulk}} = \sigma_{xy}^{\text{edge}}$$

Observation of Anomalous QHE in Graphene

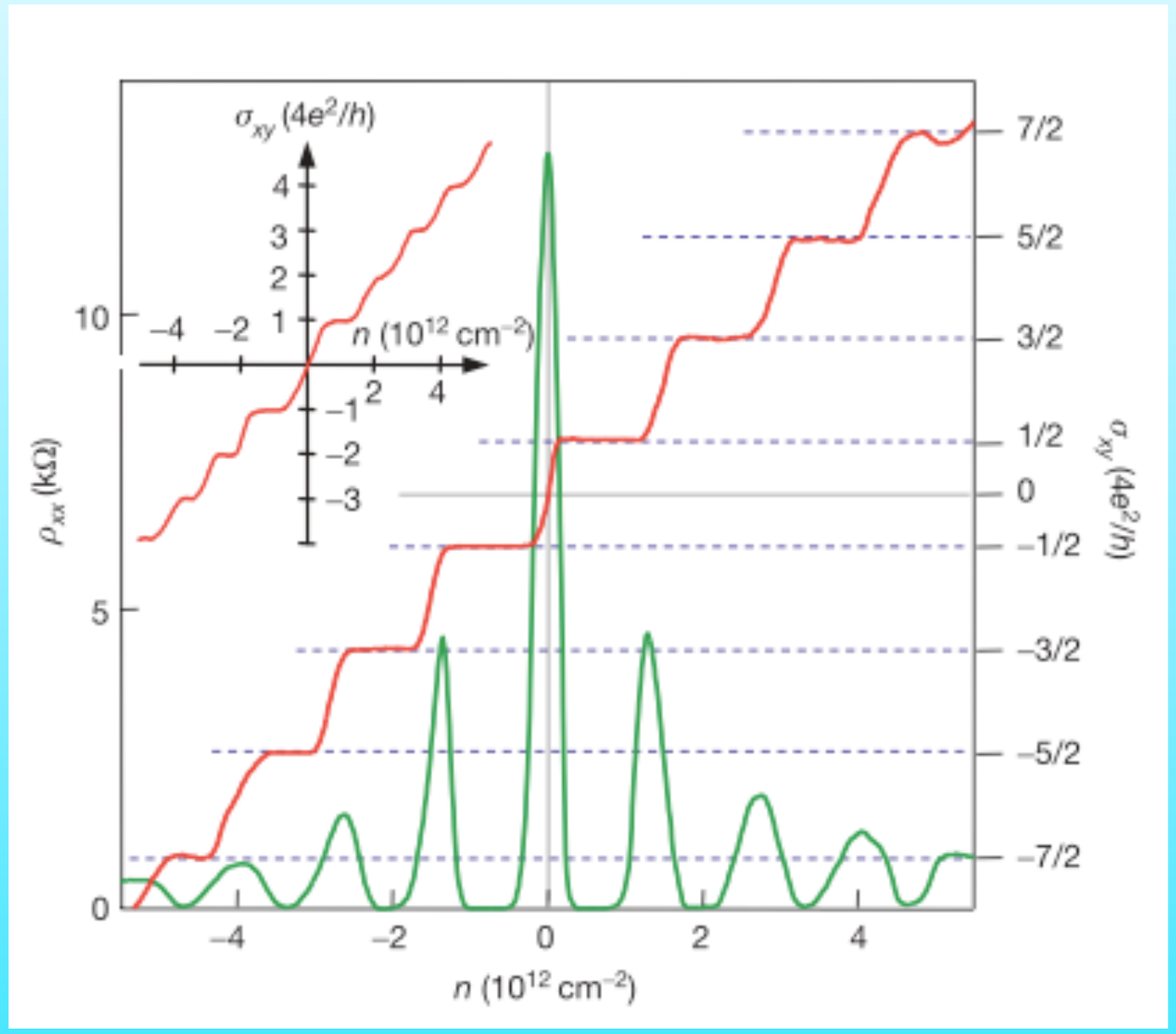
☆ *Anomalous QHE of gapless Dirac Fermions*

$$\sigma_{xy} = \frac{e^2}{h}(2n+1),\ n = 0, \pm 1, \pm 2, \cdots$$

$$= 2\frac{e^2}{h}(n + \frac{1}{2})$$



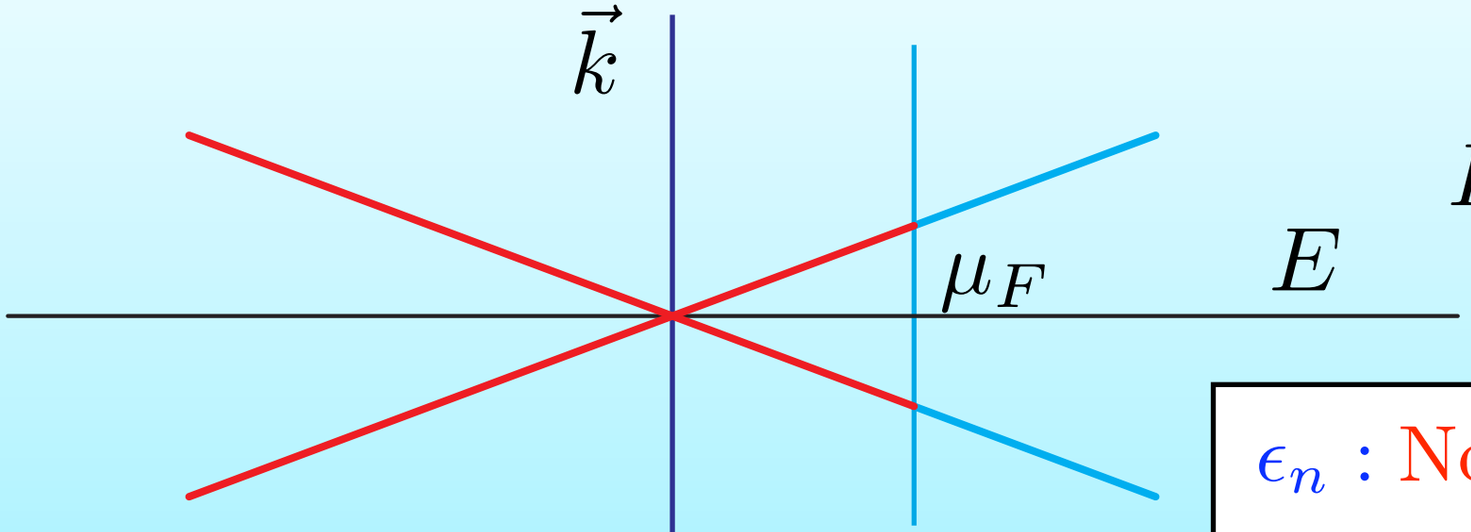
Zhang et al. Nature 2005



Novoselov et al. Nature 2005

QHE of Graphene (Gapless Semiconductor)

☆ Landau Level of Doubled Dirac Fermions

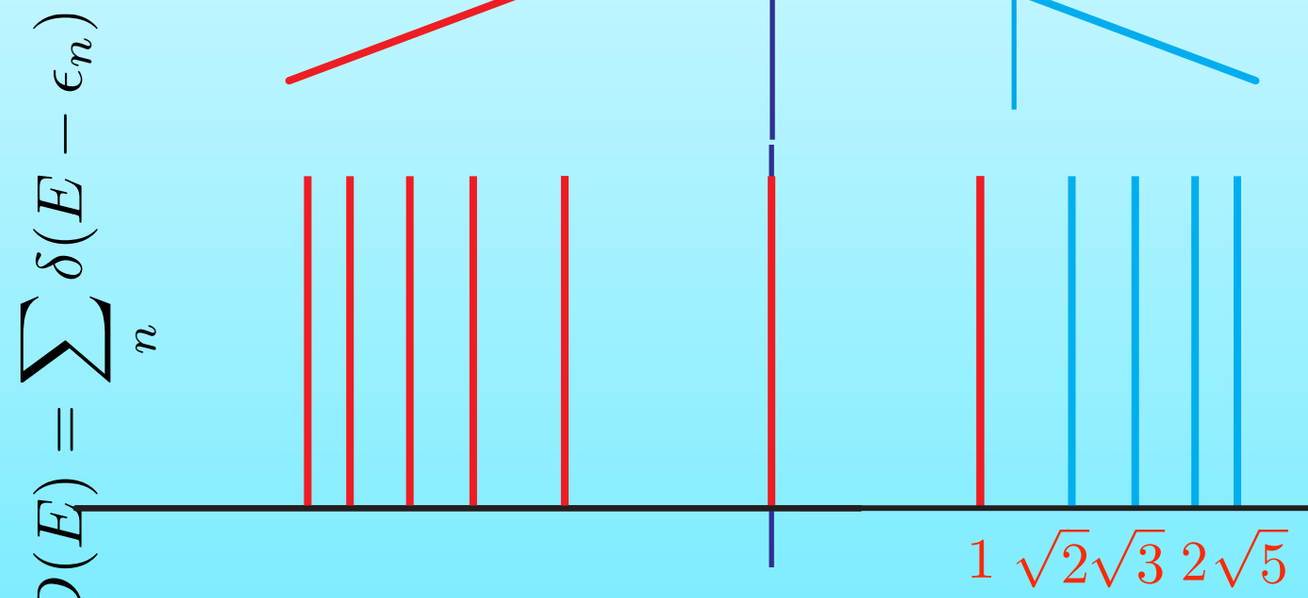


$$E(B=0) = \pm\hbar c|\vec{k}|$$

ϵ_n : No zero point energy shift

$$\epsilon_n = \pm C\sqrt{Bn}$$

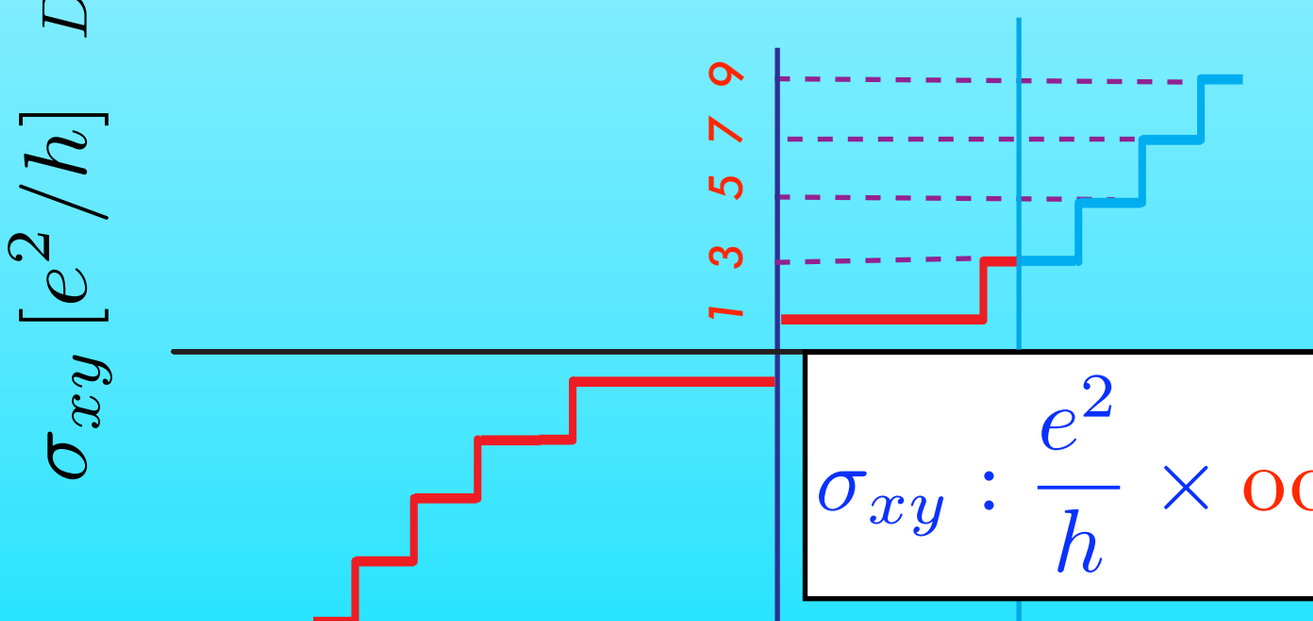
McClure, 1956



$$\sigma_{xy}(\mu_F) = \frac{e^2}{\hbar}(2n+1)$$

$$n = 0, 1, 2, 3, \cdots$$

$$\epsilon_{n-1} < \mu_F < \epsilon_n$$



σ_{xy} : $\dfrac{e^2}{h}$ × odd integer

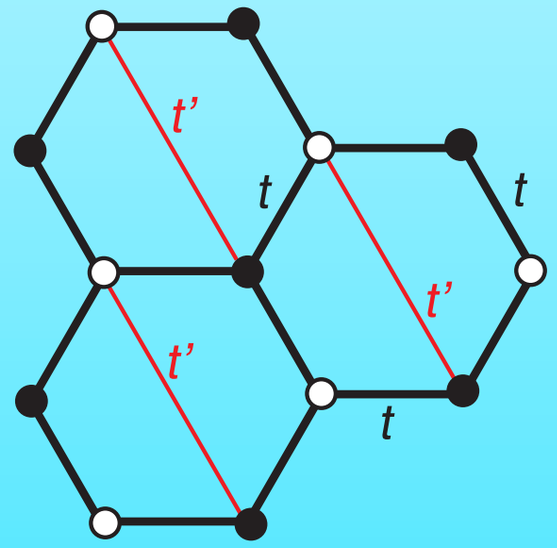
Zheng-Ando 2002
Gusynin-Sharapov, 2005
Peres-Guinea-Neto 2006
...

Graphene under magnetic field

Landau gauge

- In continuum, 2D = $\sum_{k_y}$ (1D harmonic oscillators with parameter k_y)
- Bloch electrons, 2D = $\sum_{k_y}$ (1D Harper problem with parameter k_y)

Hofstadter diagram for the honeycomb Rammal 1985



interpolate graphene (honeycomb) & square lattice

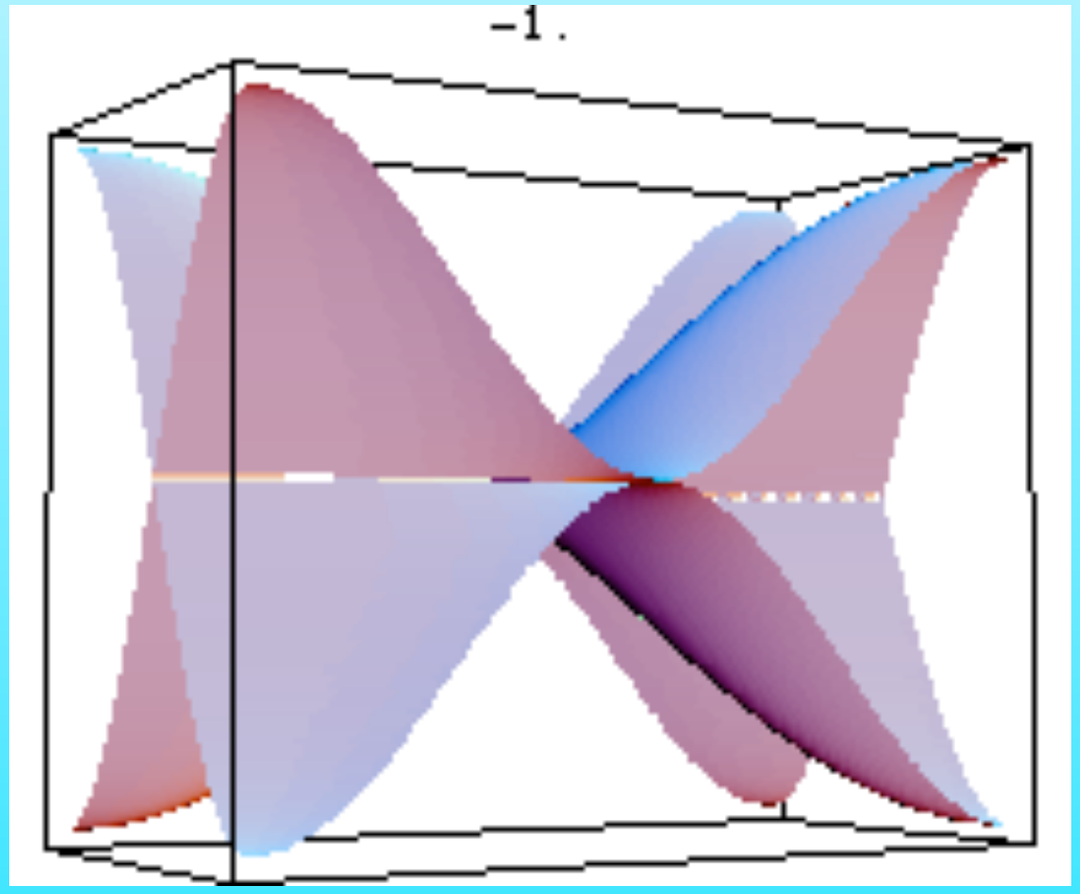
t′/t = 1 : Square Lattice
t′/t =0 :Honeycomb Lattice
t′/t=-1 : π Flux State

YH, Fukui, Aoki, 06

Dirac Cones are Stable!

- ☆ *The Dirac Cornes are not accidental: protected by chiral symmetry*
- ☆ *It has topological stability*

$$-3 < \frac{t'}{t} < 1 \quad \rightarrow \quad \text{Doubled Dirac Cones}$$



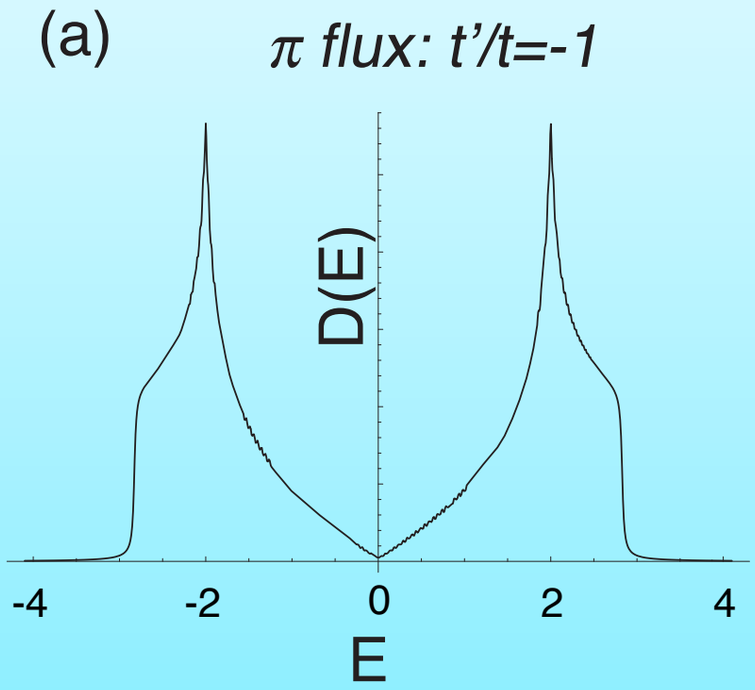
t′/t = 1 : Square Lattice
t′/t =0 :Honeycomb Lattice
t′/t=-1 : π Flux State

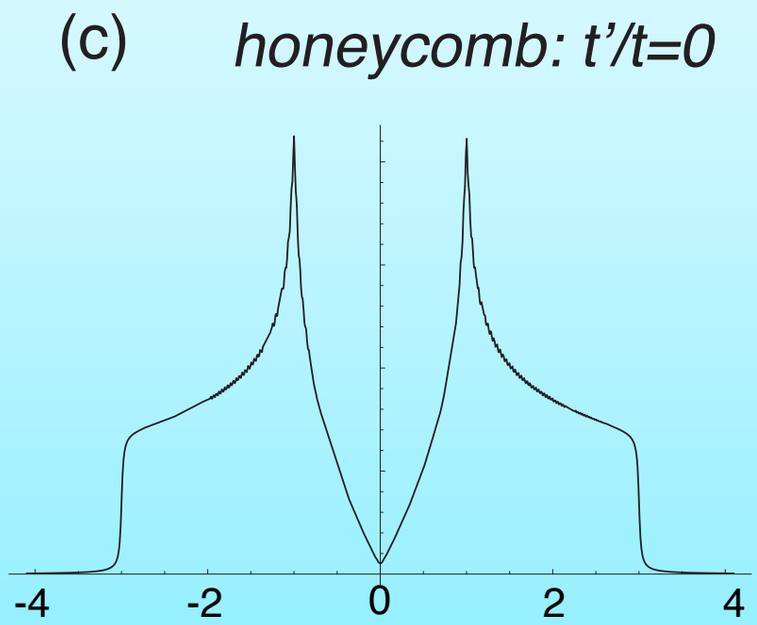
Chiral Symmetry is crucial (ex. bipartite)

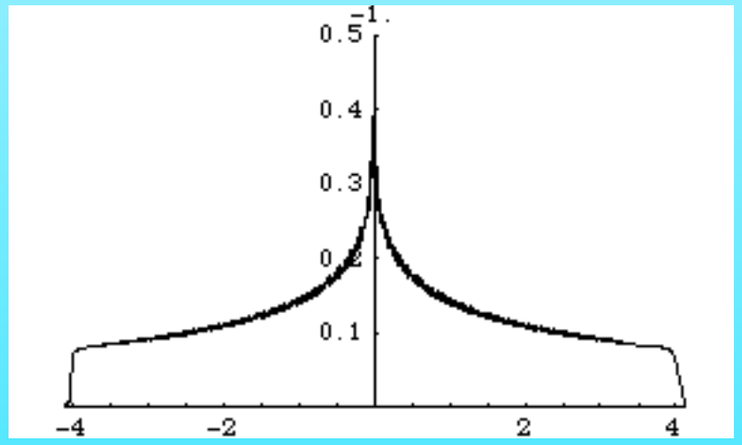
$$\{\Gamma, H\} = \Gamma H + H\Gamma = 0$$

Density of States

- *Vanishing DOS near the zero energy*







t′/t = 1 : Square Lattice
t′/t =0 :Honeycomb Lattice
t′/t=-1 : π Flux State

Stability of the Dirac Cornes!

Topological Stability of the Dirac Cornes

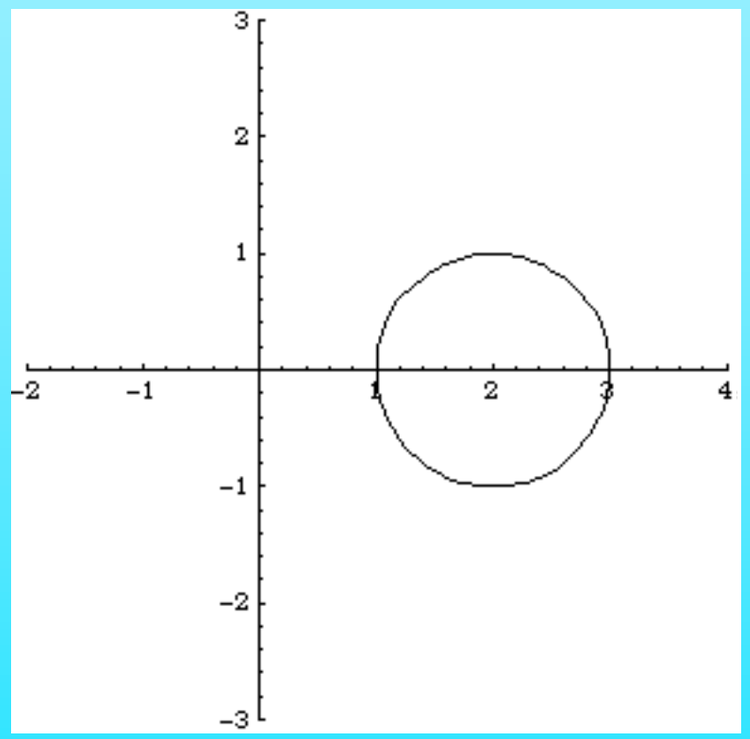
$$H(k_x, k_y) = \begin{pmatrix} 0 & \Delta \\ \Delta^* & 0 \end{pmatrix}$$

Chiral Symmetry
(ex. bipartite)
$$\{\Gamma, H\} = \Gamma H + H\Gamma = 0$$

$$\Delta = -t(1 + e^{ik_y} + e^{ik_x}(1 + re^{-ik_y}), \quad r = t'/t$$

$$E(k_x, k_y) = \pm|\Delta|$$
$$= \pm\sqrt{(1 + \cos k_x + \cos k_y)^2 + (\sin k_x + \sin k_y)^2}$$

☆ General zeros of $\Delta(k_x, k_y)$ ⟶ Dirac Cones



$\Delta(k_x, k_y)$, $k_x : 0 \to 2\pi$: loop $C(k_y)$ in $\mathbb{C}$

loop $C(k_y)$ moves : $k_y : 0 \to 2\pi$

The loop cut the origin⟶ Dirac Cones
Topological Stability
of
the doubled Dirac Cones

Topological meaning of the Hall Conductance

☆ TKNN formula: σ_{xy} as a topological invariant

Kubo formula

$$\sigma_{xy} = \frac{e^2}{h} \sum_{\ell:\epsilon_\ell(k)<E_F} C_\ell$$

Thouless-Kohmoto-Nightingale-den Nijs '82

Sum over the bands below E_F

$$C_\ell = \frac{1}{2\pi i} \int_{T^2:\mathrm{BZ}} F_\ell$$

:First Chern number of the ℓ-th Band

intrinsically integer

unless the energy gap collapses

$$\forall k,\ \epsilon_\ell(k) \neq \epsilon_{\ell\pm1}(k)$$

regularity of the Berry connection

$$F_\ell = dA_\ell = \langle d\psi_\ell | d\psi_\ell \rangle$$

$$A_\ell = \langle \psi_\ell | d\psi_\ell \rangle$$

$$H(k)|\psi_\ell(k)\rangle = \epsilon(k)|\psi_\ell(k)\rangle$$

$$k \in T^2_{\mathrm{BZ}} = \{k = (k_x, k_y)|\ 0 \leq k_x, k_y \leq 2\pi\}$$

$$d = dk_\mu \frac{\partial}{\partial k_\mu}$$

Bulk σ_{xy} by the topological invariant

☆ Hall conductance by Chern number

Counting vortices in the band

$$\sigma_{xy}^{j} = \frac{e^2}{h} \sum_{\substack{\ell=1 \\ \epsilon_\ell(k) < \mu_F,\ \ell = 1, \cdots, j}}^{j} C_\ell, \quad C_\ell = \frac{1}{2\pi i} \int_{BZ} dA_\ell, \quad A_\ell = \langle \psi_\ell | d\psi_\ell \rangle$$

Thouless-Kohmoto-Nightingale-den Nijs 1982

Sum over the filled bands

Need to sum many bands until E=0 (graphene)

Need to cross the van Hove singularity

Numerical difficulty for the weak field (experimental situation)

(continuum theory of Dirac fermion)

Bulk σ_{xy} of the Filled Fermi sea & Dirac Sea

- ☆ *Integration of the NonAbelian Berry Connection of the "Fermi Sea" & "Dirac Sea"*

Numerical advantage for graphene

$$H_j(k)|\psi_j(k)\rangle = \epsilon_j(k)|\psi_j(k)\rangle$$

$\Psi = (\ |\psi_1\rangle, \cdots, |\psi_M\rangle)$ *Collect M states below the Fermi level*

$$A_{FS} \equiv \Psi^\dagger d\Psi = \begin{pmatrix} \langle\psi_1^\dagger|d\psi_1\rangle & \cdots & \langle\psi_1^\dagger|d\psi_M\rangle \\ \vdots & \ddots & \vdots \\ \langle\psi_M^\dagger|d\psi_1\rangle & \cdots & \langle\psi_M^\dagger|d\psi_M\rangle \end{pmatrix}$$

Matrix vector potential of the Fermi (Dirac) Sea
Non Abelian extension for the Chern numbers

$$\sigma_{xy} = \frac{e^2}{h}\frac{1}{2\pi i}\int_{T^2} \mathrm{Tr}_M\, dA_{FS}$$

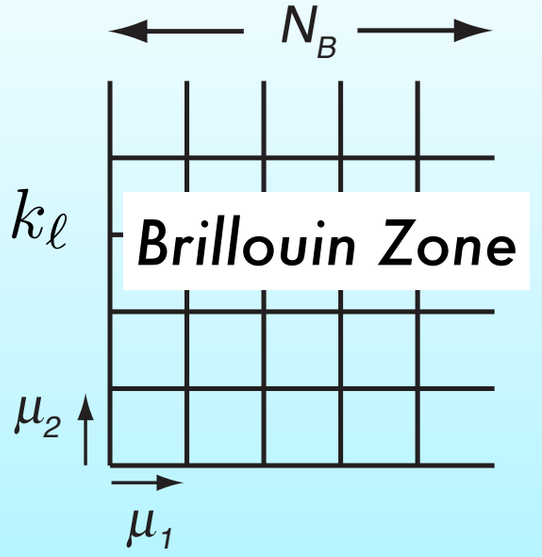
Hatsugai 2004

Numerical Technique from the Lattice gauge theory

☆ Topological Invariant on Discretized Lattice

Lattice in k space (discretization for the integral)

Technical Advantage for large Chern Numbers



$$\sigma_{xy} = \frac{e^2}{h}\frac{1}{2\pi i}\sum F_{1234}$$ gauge invariant

$$F_{1234} = \mathrm{Im}\ \log U_{12}U_{23}U_{34}U_{41}$$

$$U_{mn} = \det \Psi_m^\dagger \Psi_n, \quad \Psi_n = (\psi_1(k_n), \cdots, \psi_j(k_n))$$

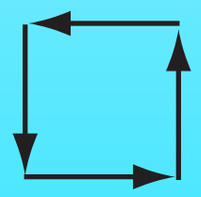
Fermi Sea of j filled bands

$U_\mu(k_\ell)$ →

$$U_\mu(k_\ell) \equiv \langle n(k_\ell)|n(k_\ell+\hat{\mu})\rangle / \mathcal{N}_\mu(k_\ell)$$

$$\mathcal{N}_\mu(k_\ell) = |\langle n(k_\ell)|n(k_\ell+\hat{\mu})\rangle|$$

$\tilde{F}_{12}(k_\ell)$



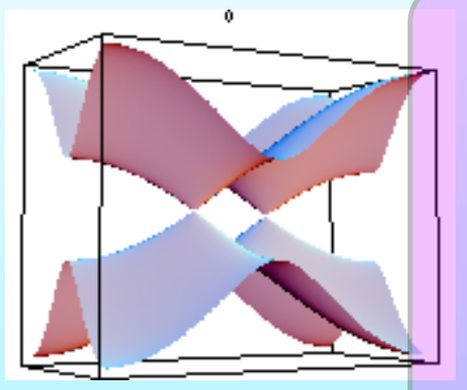
$$\tilde{F}_{12}(k_\ell) \equiv \ln U_1(k_\ell)U_2(k_\ell+\hat{1})U_1(k_\ell+\hat{2})^{-1}U_2(k_\ell)^{-1}$$

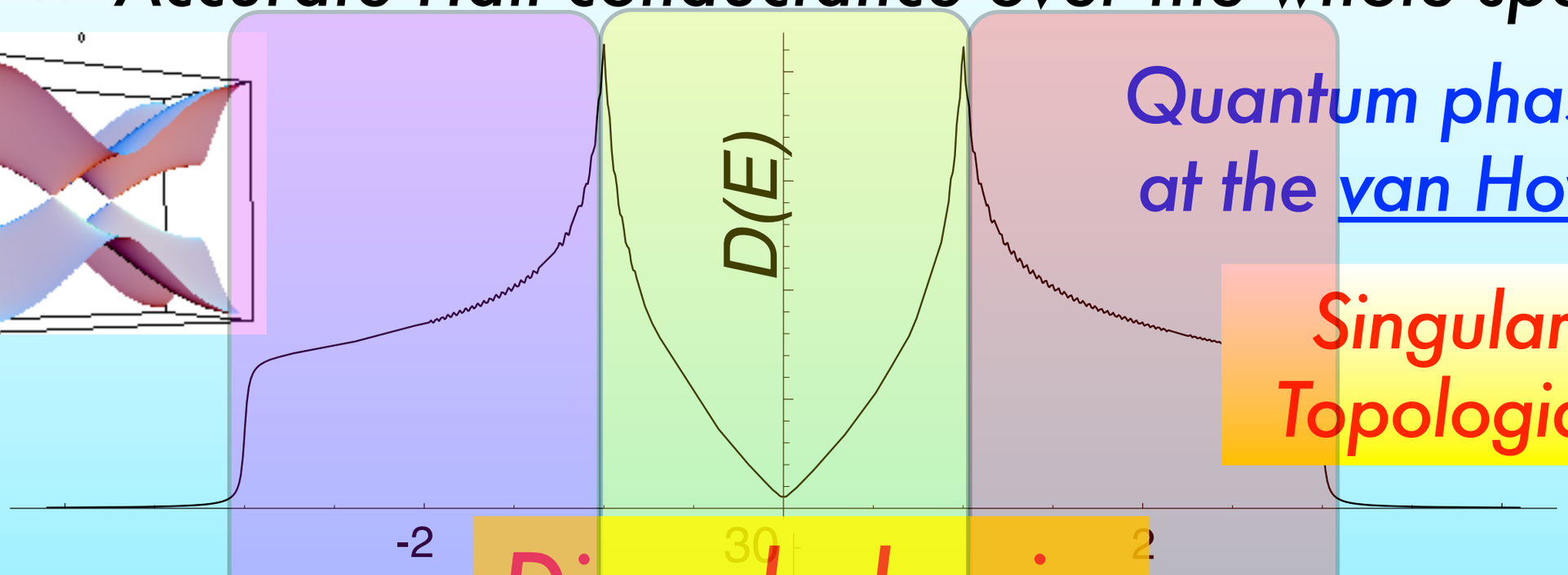
$$-\pi < \tilde{F}_{12}(k_\ell)/i \le \pi$$ (principal value)

Fukui-Hatsugai-Suzuki 2005

Hall Conductace vs chemical potential

☆ Accurate Hall conductance over the whole spectrum





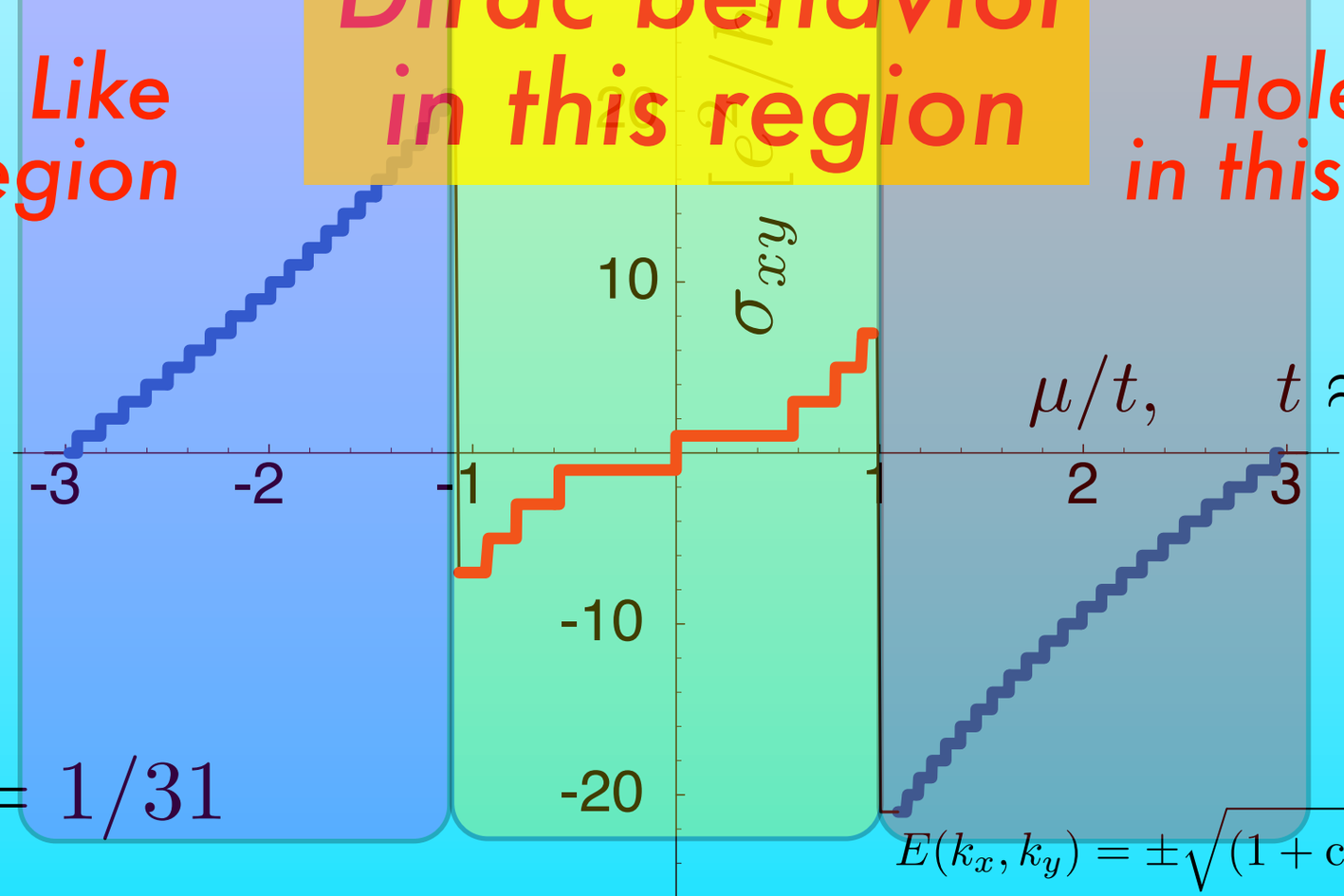
Quantum phase transition
at the van Hove Energies

Singularity breaks
Topological Stability

Dirac behavior
in this region

Electron Like
in this region

Hole Like
in this region



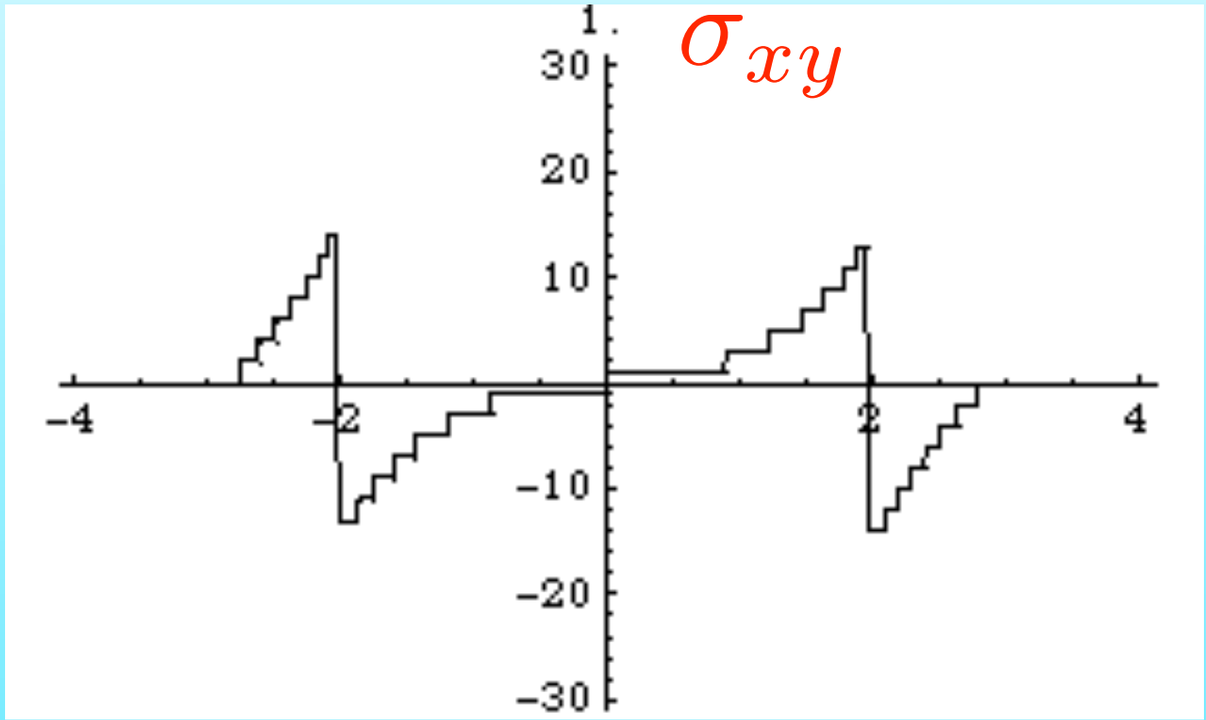
$t \approx 1[\mathrm{eV}]$ for graphene

$\phi = 1/31$

$$E(k_x, k_y) = \pm\sqrt{(1+\cos k_x+\cos k_y)^2+(\sin k_x+\sin k_y)^2}$$

σ_{xy} by Adiabatic Principle

☆ Hofstadter Diagram and σ_{xy} with t′/t



t′/t = 1 : Square Lattice
t′/t =0 :Honeycomb Lattice
t′/t=-1 : π Flux State

Edge states of Graphene

- ***QH edge states in graphene***

  YH, T. Fukui & H. Aoki, '06

  See also, L. Brey, H. A. Fertig, '06

  Edge transport : D. A. Abanin, P. A. Lee, L. S. Levitov '07

- ***Zero modes edge state without magnetic field***

  Fujita et al., '96

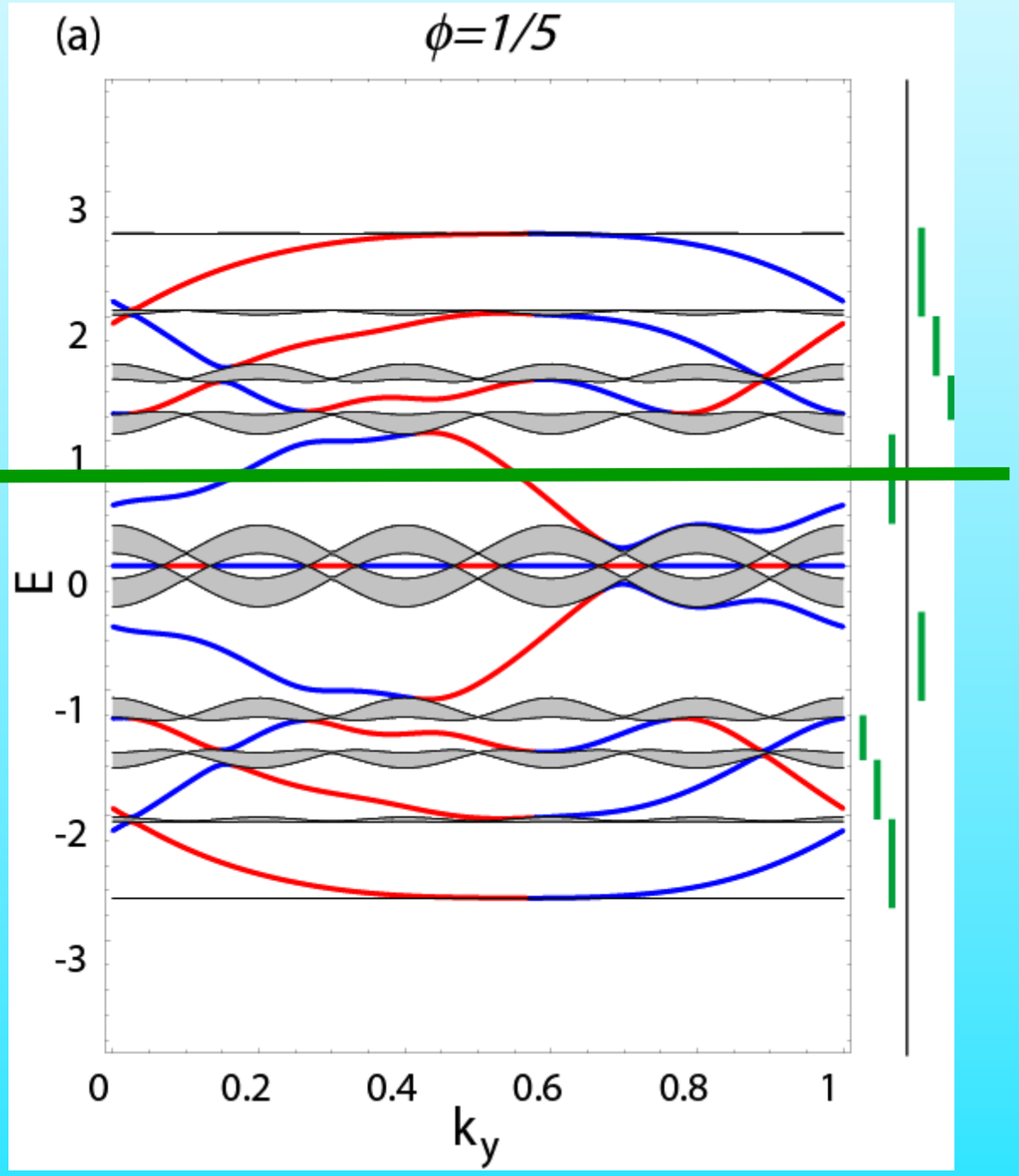
  S. Ryu & YH, '02

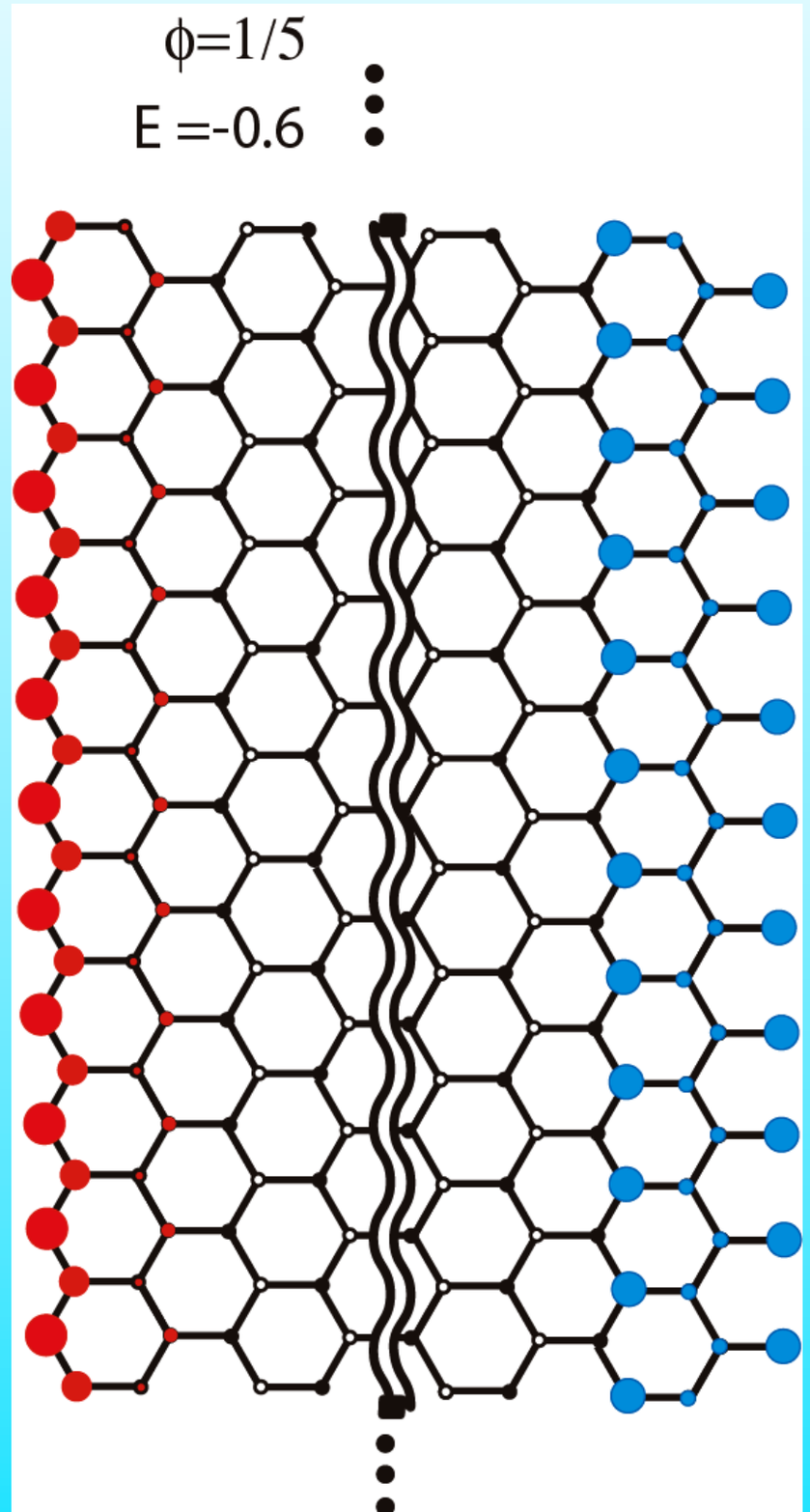
- ***Zero modes with magnetic field***

  M. Arikawa, H. Aoki & YH arXiv:0805.3240 & 0806.2429

Edge States of Graphene with magnetic field

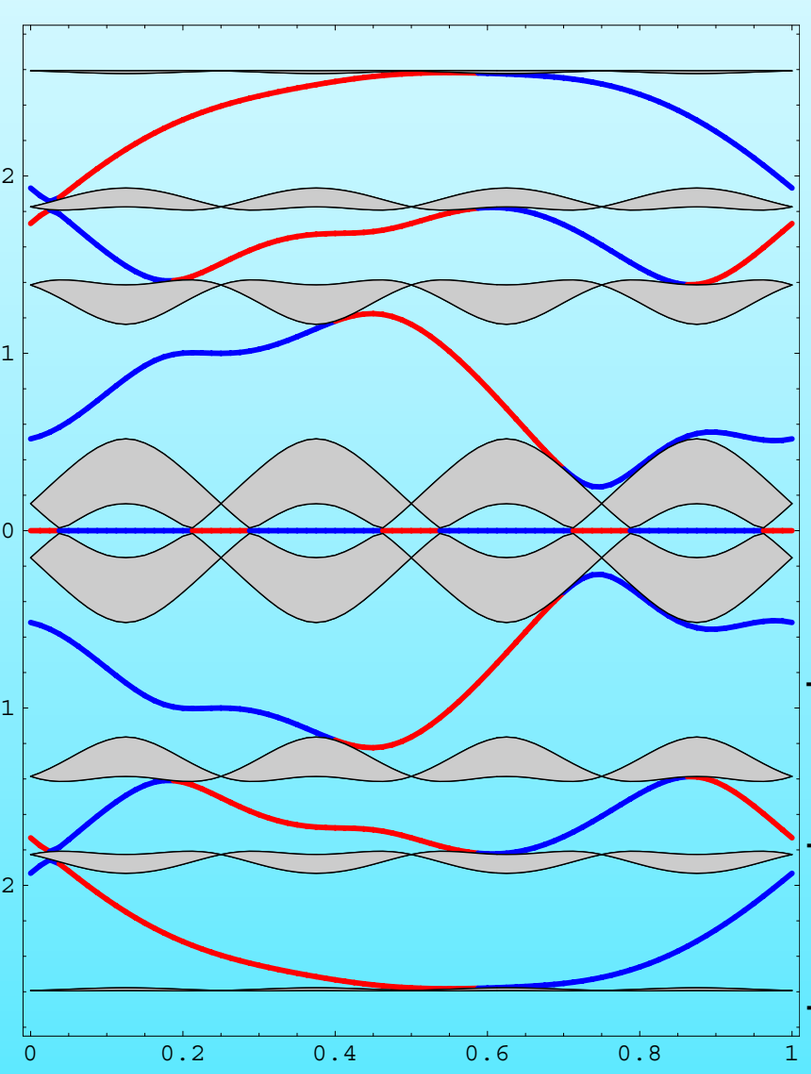
☆ Edge States and their local charges



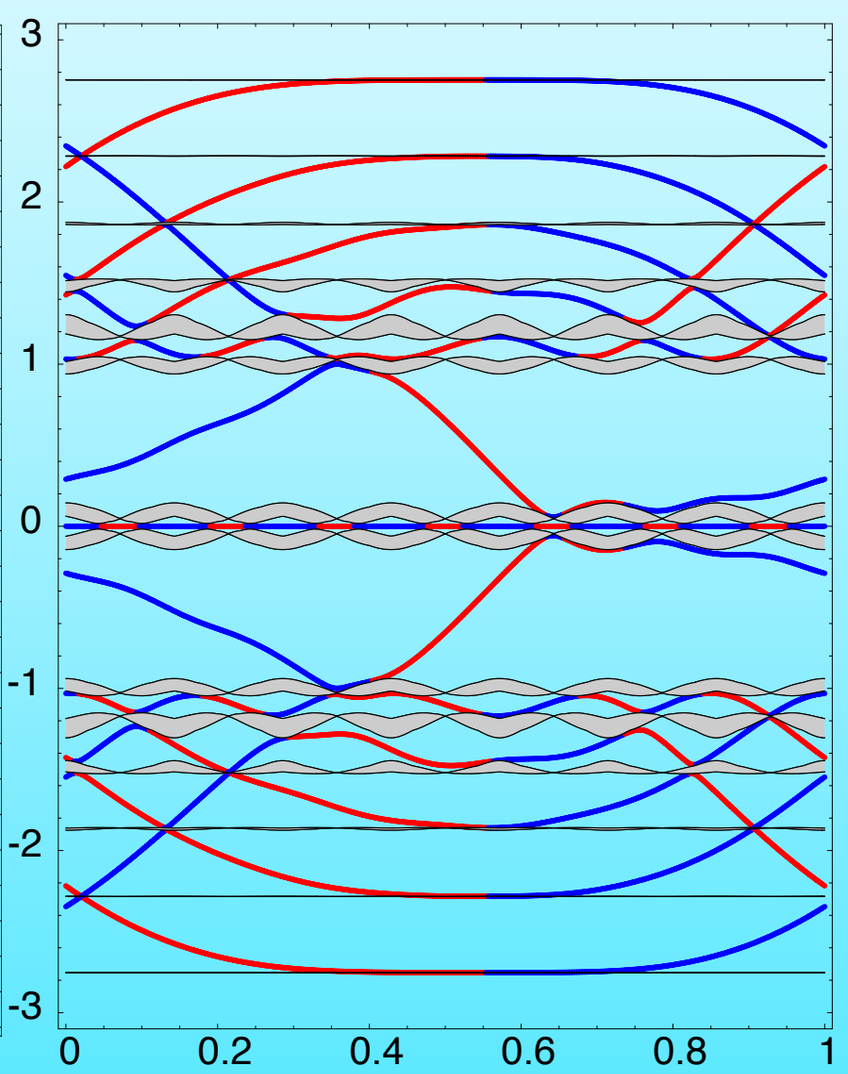


How the Edge states look like ?

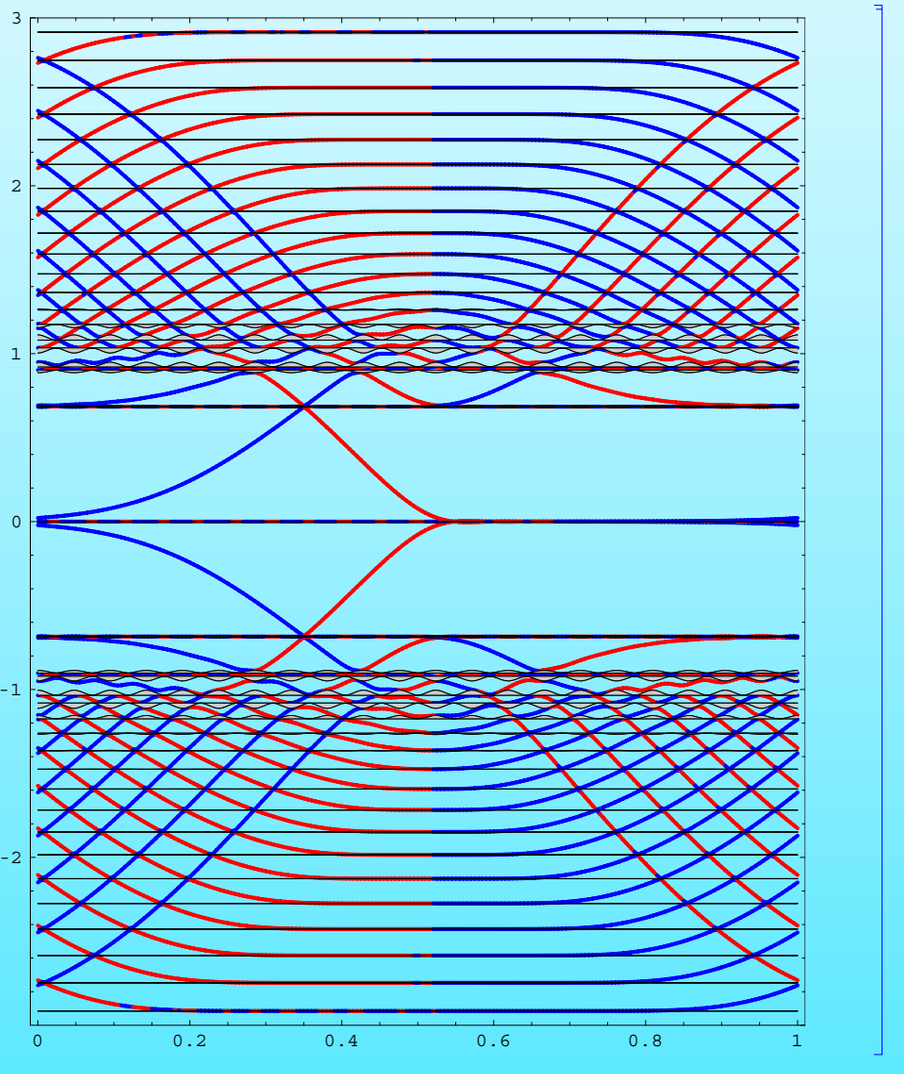
☆ Field dependence



$\phi = 1/4$



$\phi = 1/7$



$\phi = 1/21$

strong magnetic field ⟷ *weak magnetic field*

conitinuum

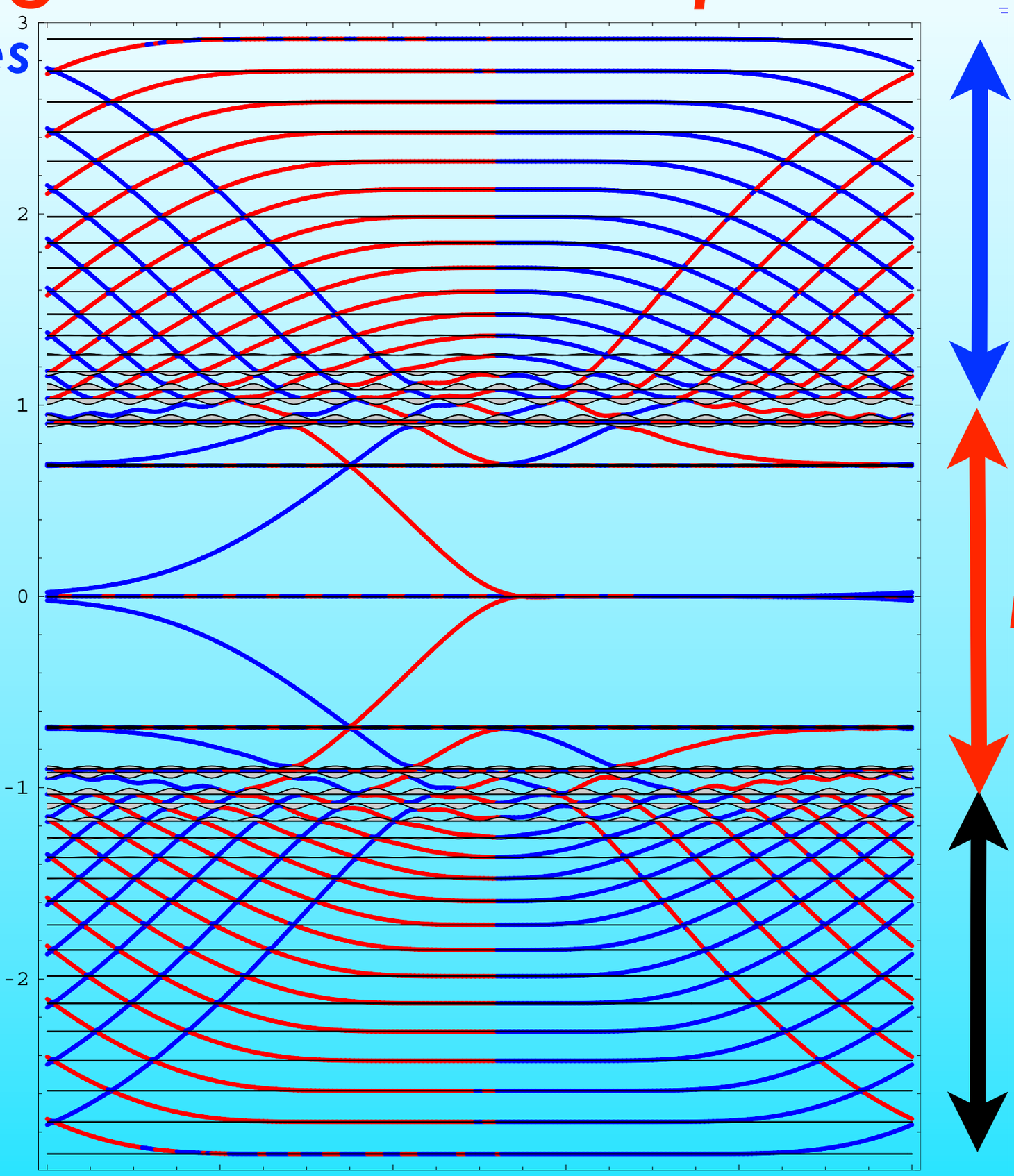
Edge States of Graphene

☆ Zigzag Edges

Full Spectrum

$\phi = 1/21$

Weak Field



Edge State of Holes

Edge State of Dirac Fermions ??

Edge State of Electrons

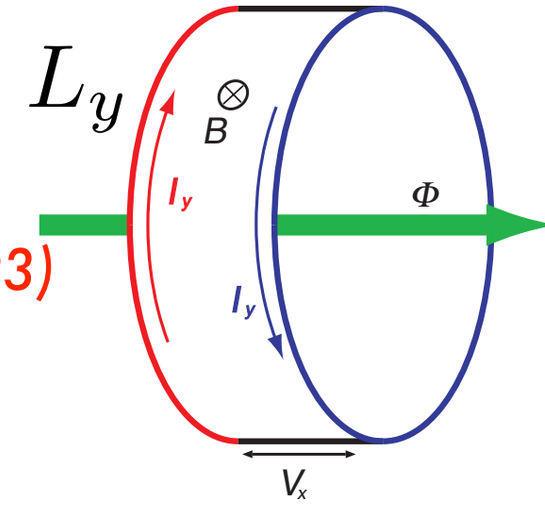
How the edge states determine ?

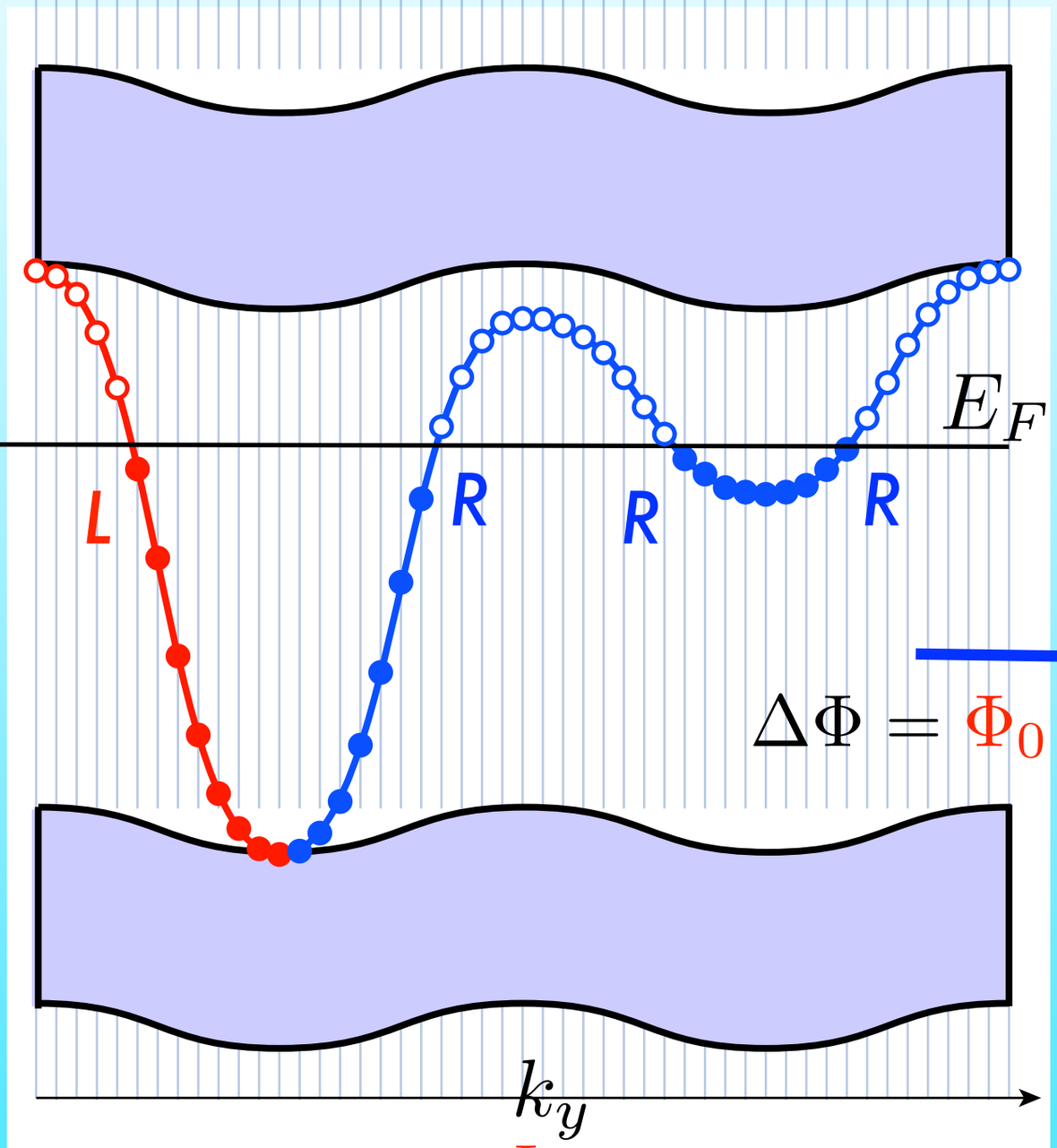
How to calculate σ_{xy} by the edge states?

Laughlin's Argument & Edge States

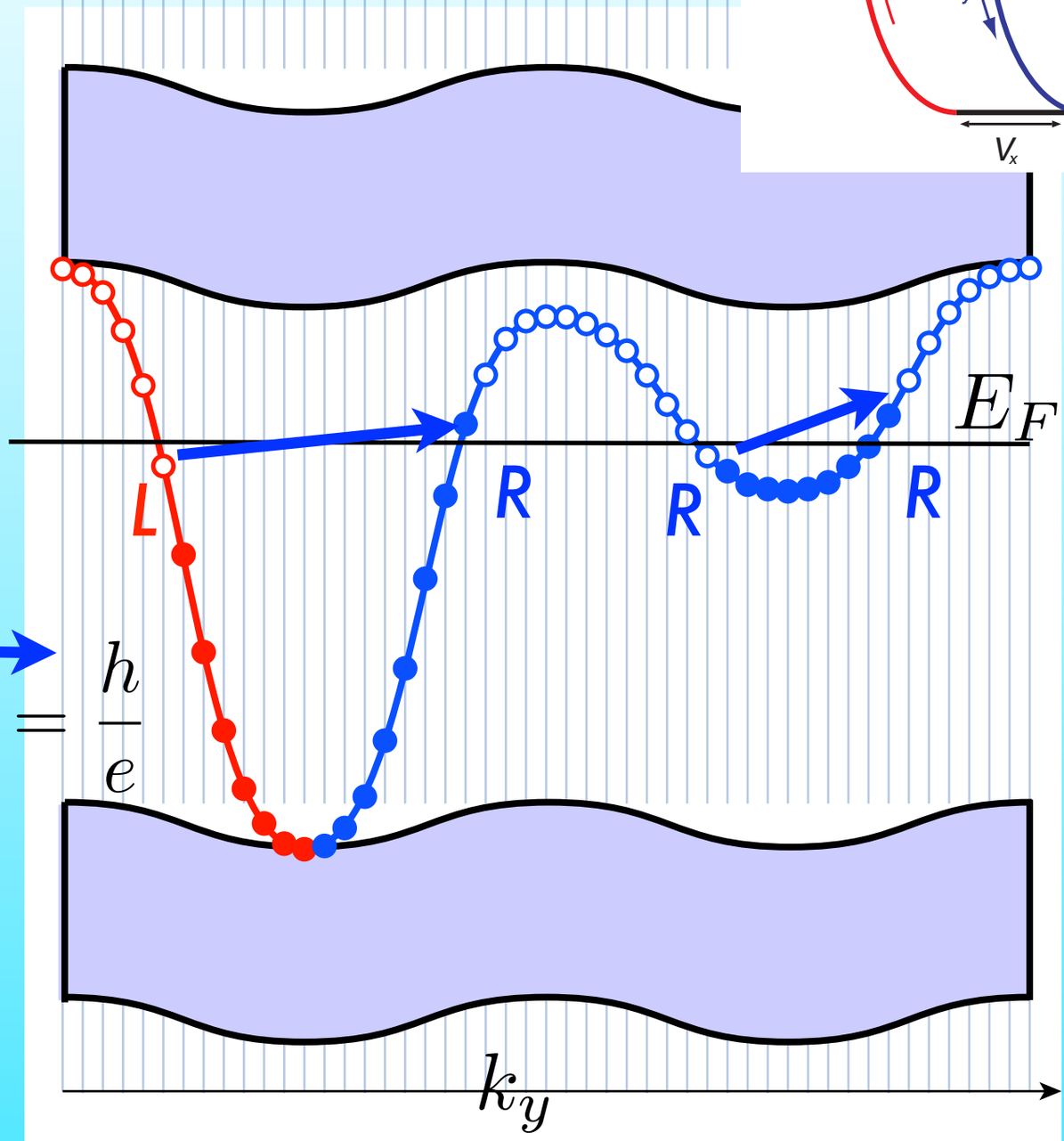
☆ Adiabatic Charge Transfer

Y.H., Phys. Rev. B 48, 11851 (1993)





$$\Delta\Phi = \Phi_0 = \frac{h}{e}$$



$$k_y = 2\pi \frac{n + \frac{\Phi}{\Phi_0}}{L_y}, \; n: \text{ integers}$$

1 Electron is carried from the Left to the right in this case

$$\sigma_{xy} = \frac{e^2}{h} \cdot 1$$

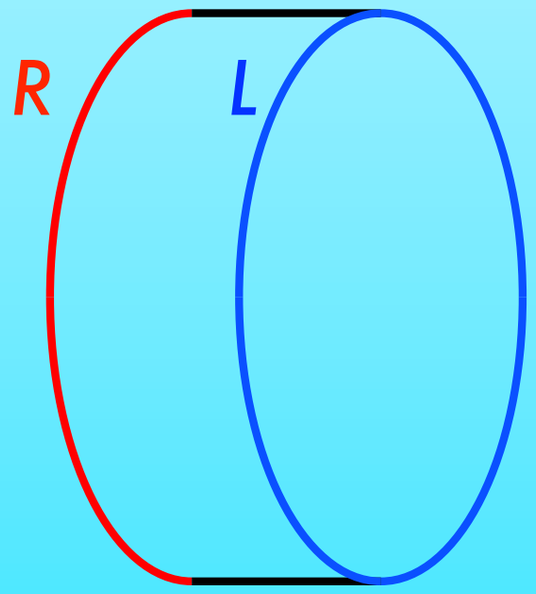
Analytical Consideration of Edge states in Graphene

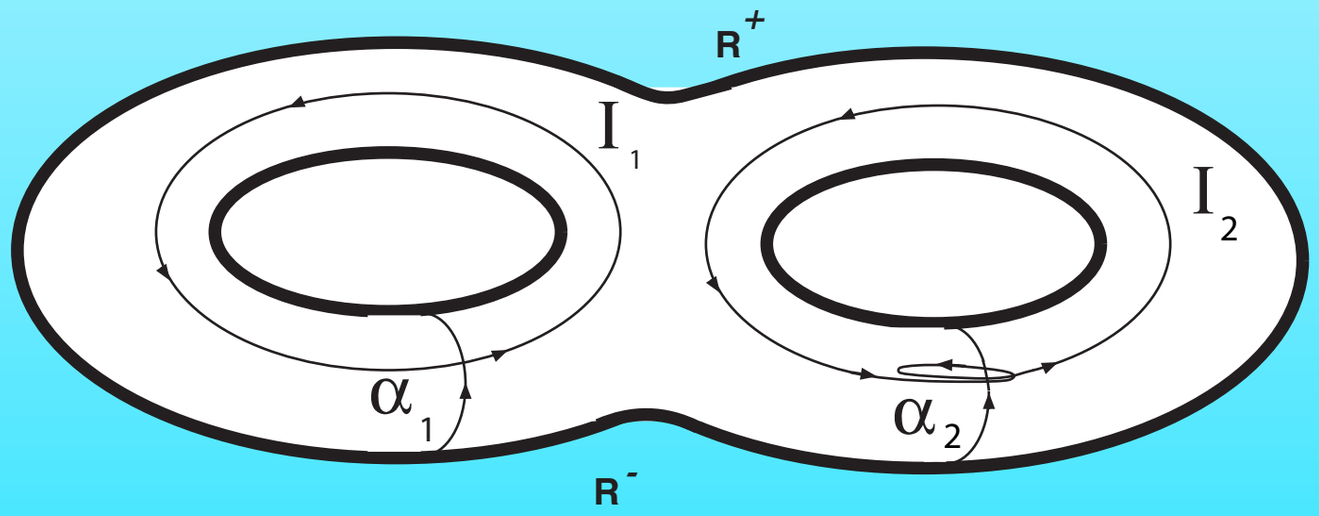
YH, T. Fukui & H. Aoki, Phys. Rev. B74, 205414 (2006)

☆ *Followed by the discussion on a square lattice*

Y.H., Phys. Rev. B 48, 11851 (1993)

Phys. Rev. Lett. 71, 3697 (1993)





Edge State and Bloch State

☆ Bloch electrons, 2D = $\sum$(1D Harper problem with parameter k_y)

As for the 1D Harper equation,

- ☆ Edge state : bound state
- ☆ Bloch state: scattering state

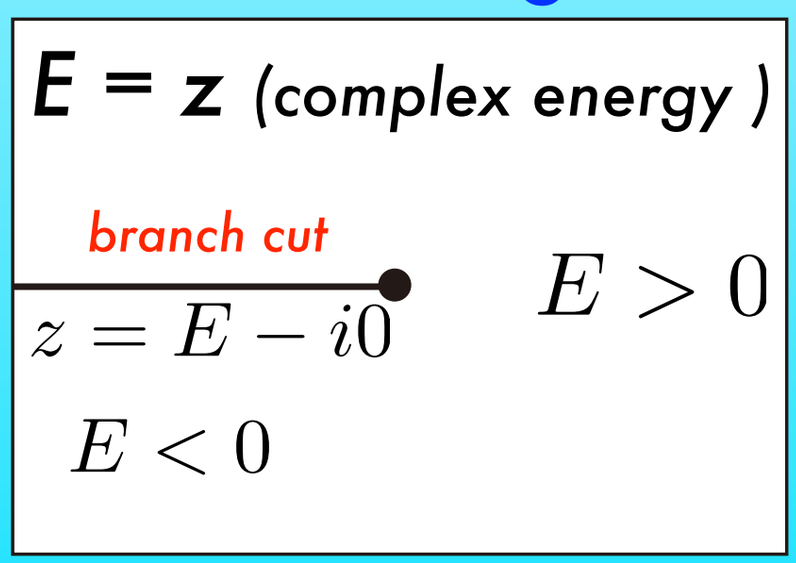
These two can be treated in a unified way by considering complex energy

standard quantum mechanics

- ☆ bound state
- ☆ scattering state

$$E = \frac{\hbar^2 k^2}{2m} \begin{cases} < 0 & k = i\kappa, \quad \psi \sim e^{-\kappa x} \\ > 0 & k \in \mathbb{R}, \quad \psi \sim e^{ikx} \end{cases}$$

E = z (complex energy)



unified description

$$\psi \sim e^{i\sqrt{2mE}\, x/\hbar}$$

energy of the bound state is in the gap region E<0

Analytic Continuation of the Bloch State

- The Edge State is obtained from the Bloch State by Analytical continuation

  Y.H., Phys. Rev. B 48, 11851 (1993)
  Phys. Rev. Lett. 71, 3697 (1993)

  - Energy of the Bloch state ψ_B is in the band
  - Energy of the edge state ψ_E is in the gap
- Complex energy surface : genus Q Riemann surface

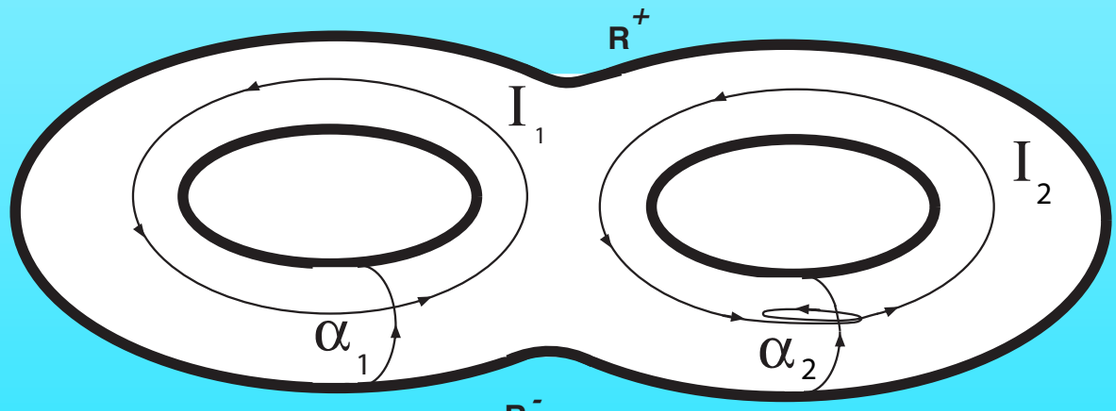
  ψ_B & ψ_E :Unified on Complex Energy surface

  - Energy bands : branch cuts, 2 Riemann sheets required
  - Q branch cuts
  - genus (number of holes) g=Q-1 Riemann surface
  - g : number of the energy gaps

$$\phi = P/Q$$

Flux per hexagon

Complex Energy surface of Harper eq.



$$\sqrt{(z-\lambda_1)(z-\lambda_2)\cdots(z-\lambda_{2Q-1})(z-\lambda_{2Q})}$$

Wave function & Riemann Surface Σ_g

As for fixed k_y of the 1D-Harper systems

☆ *Zeros of the Bloch fn. defines the Edge State Energies*

Energy bands ⟷ *Branch cuts*

Energy gaps ⟷ *Holes*

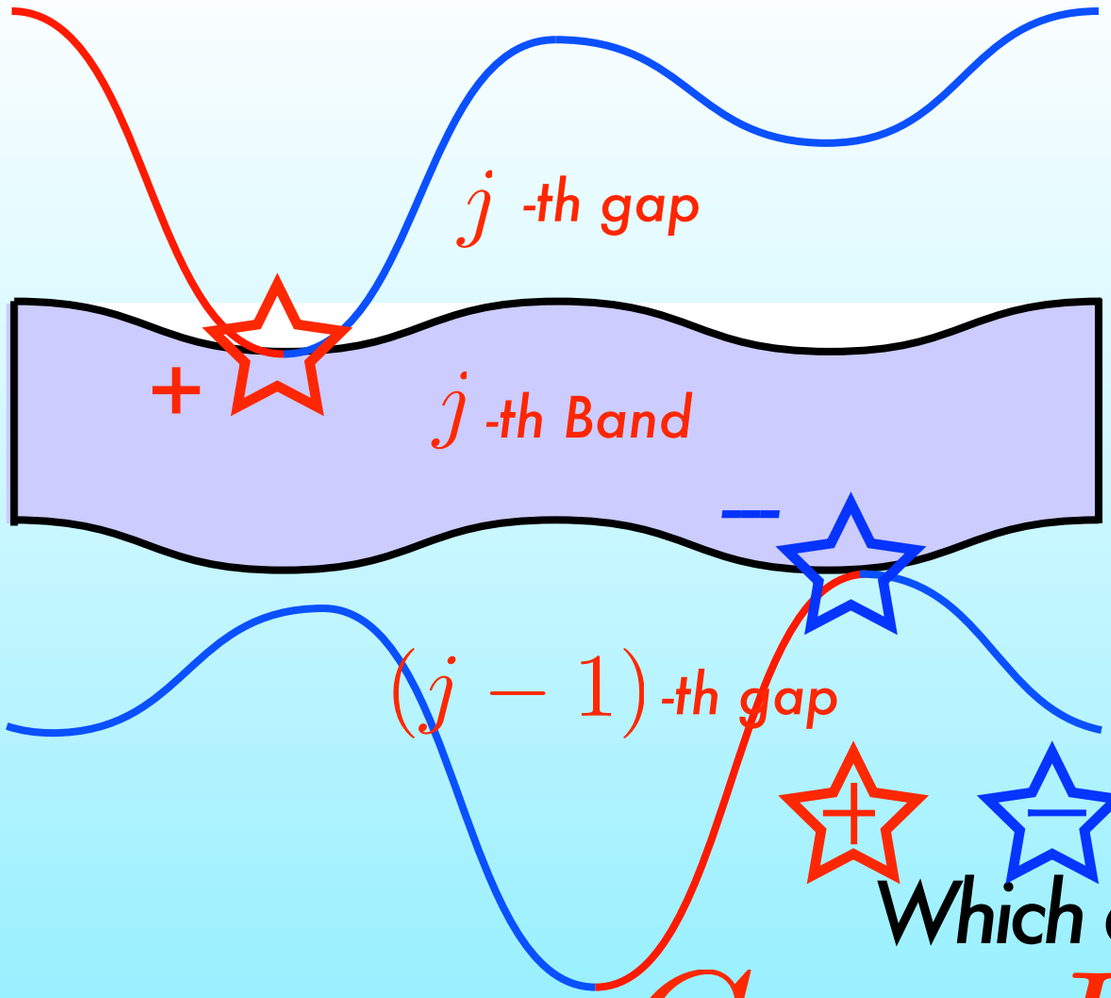
$$\phi = P/Q$$
$$g = Q - 1$$

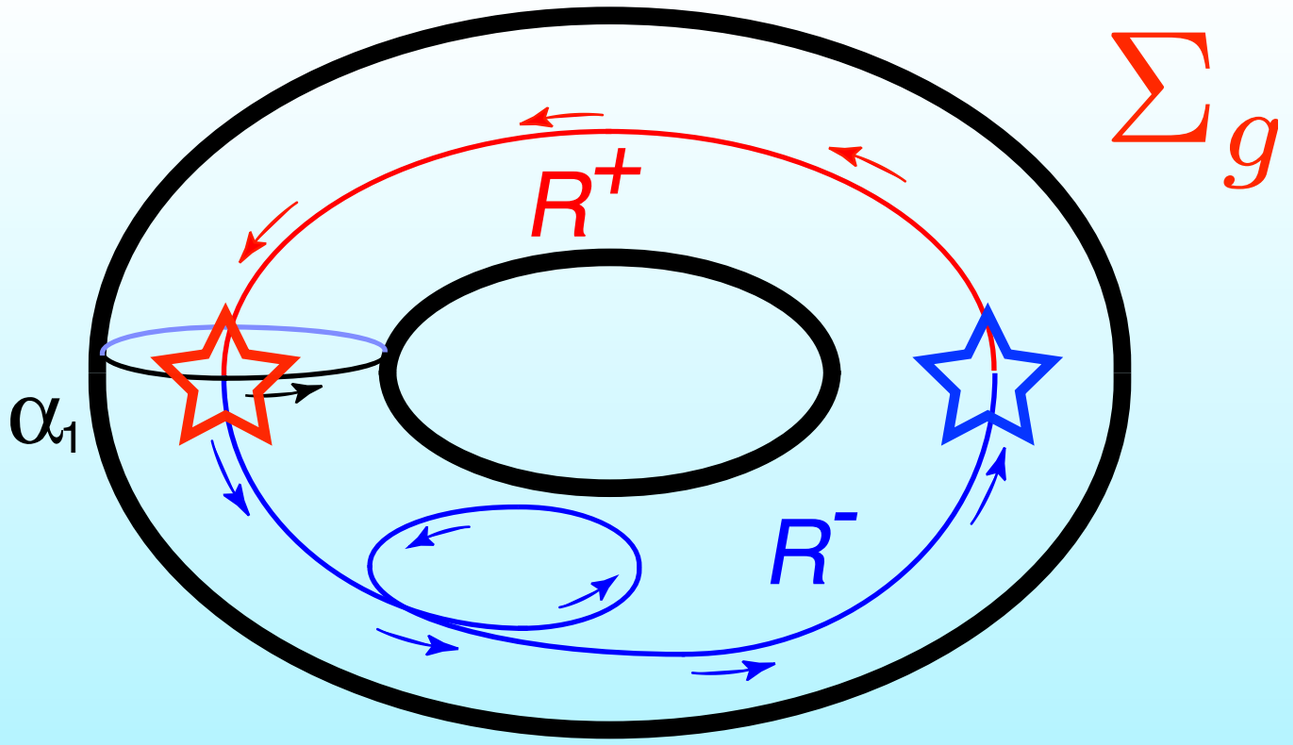
W. fn. is localized at — *The zero of the Bloch fn. is on*

the left edge ⟷ *the upper Riemann Surface* R^+

the right edge ⟷ *the upper Riemann Surface* R^-

☆ *Changing* $k_y \in [0, 2\pi]$ *, the zero of the Bloch function in the j-th gap makes a closed loop on* Σ_g





Y.H., Phys. Rev. Lett. 71, 3697 (1993)

Which contribure to the Chern number of the Bulk

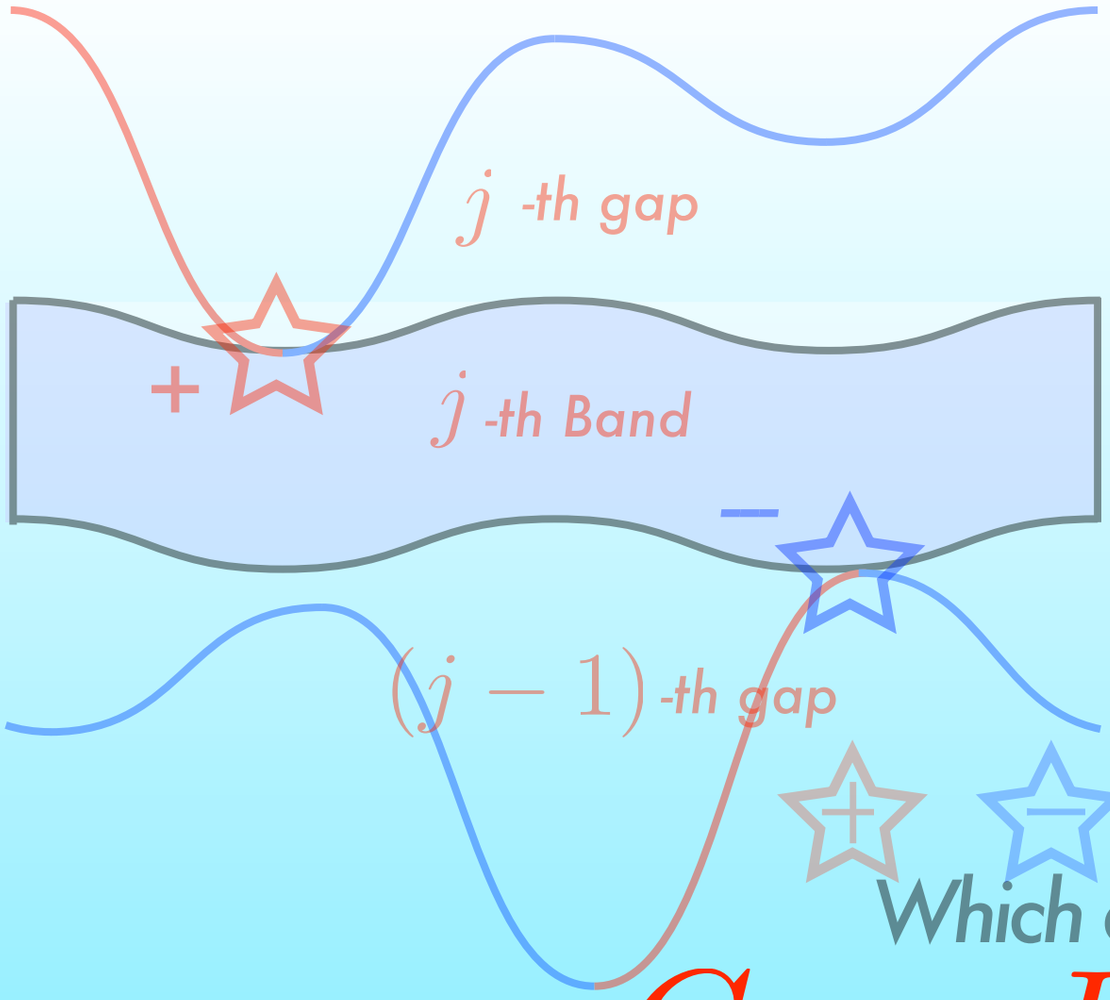
$$C_j = I_j - I_{j-1}$$

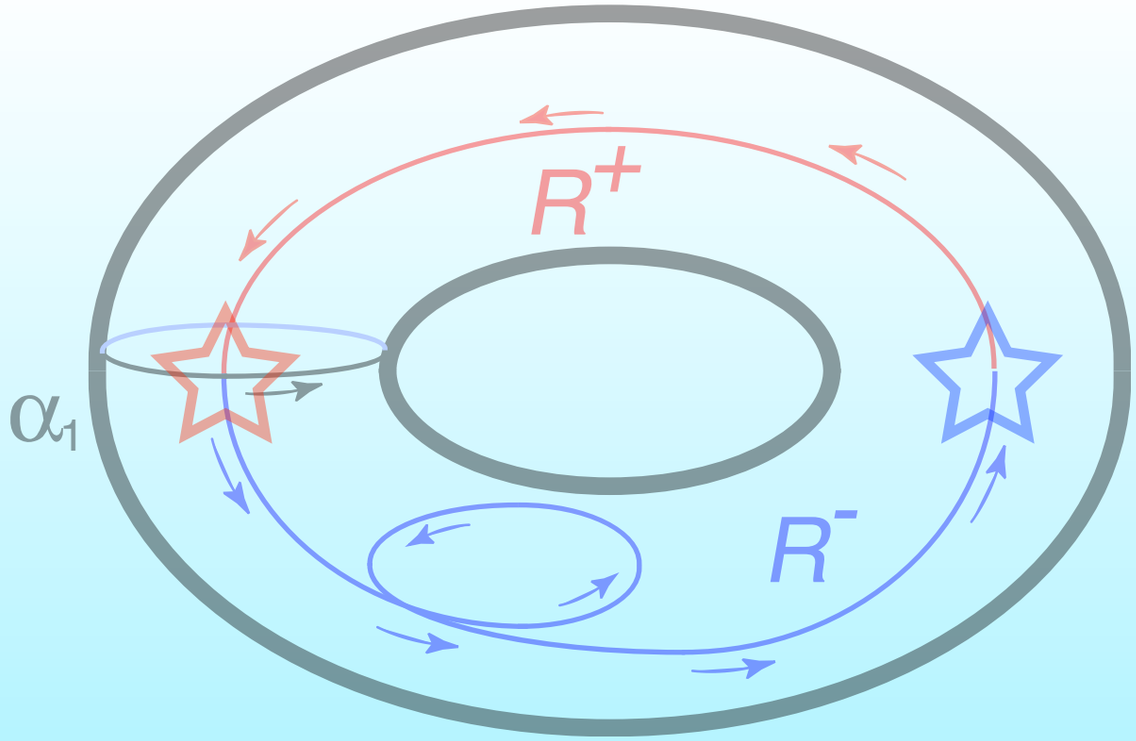
Chern # = winding # Difference between the neighboring gaps

$$\sum_{j=1,\cdots \ell} C_j = I_\ell, \ (\because I_0 = 0)$$

$$\sigma_{xy}^{\text{bulk}} = \sigma_{xy}^{\text{edge}}$$

Bulk-Edge Correspondence in their topological numbers





Y.H., Phys. Rev. Lett. 71, 3697 (1993)

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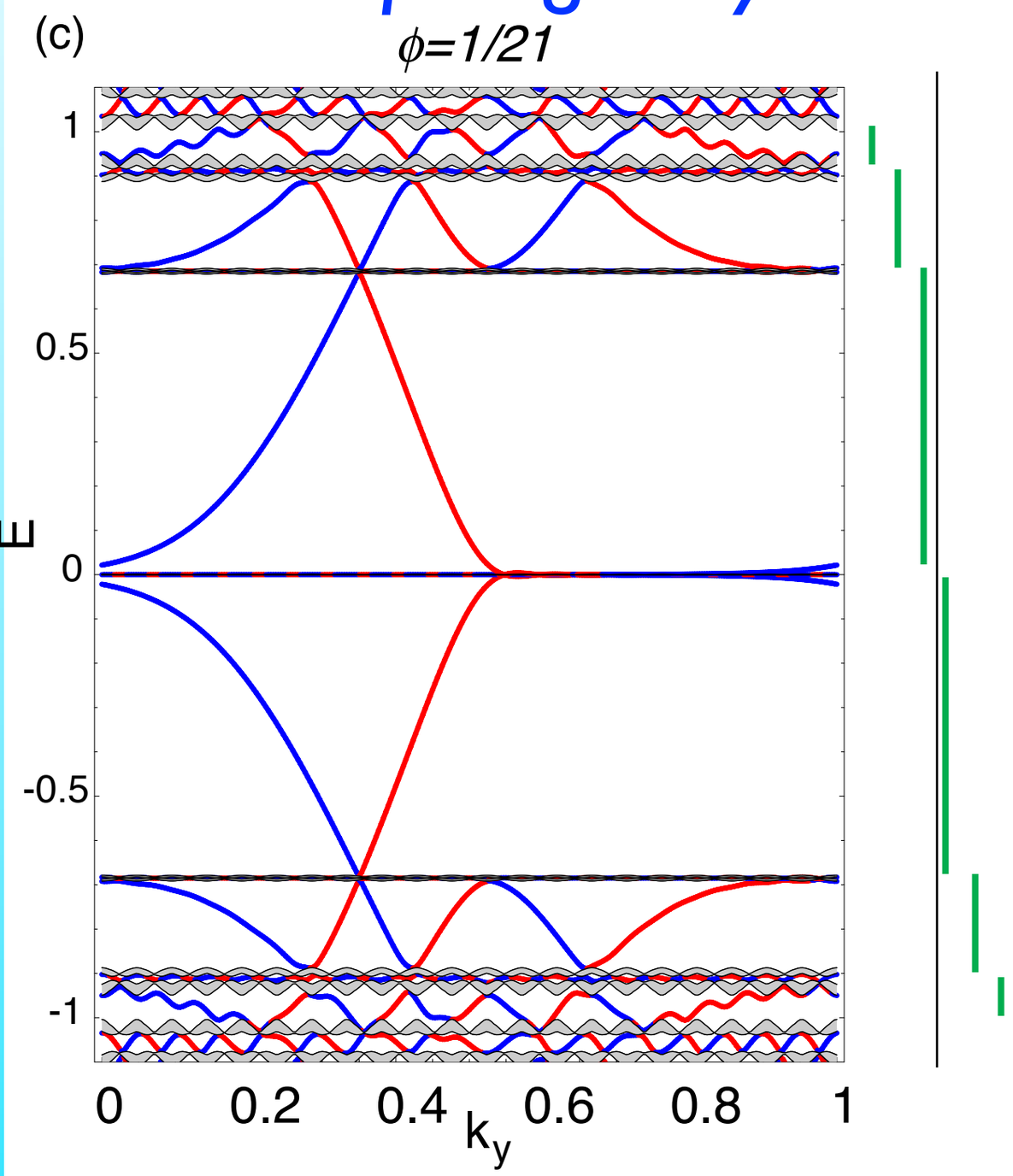
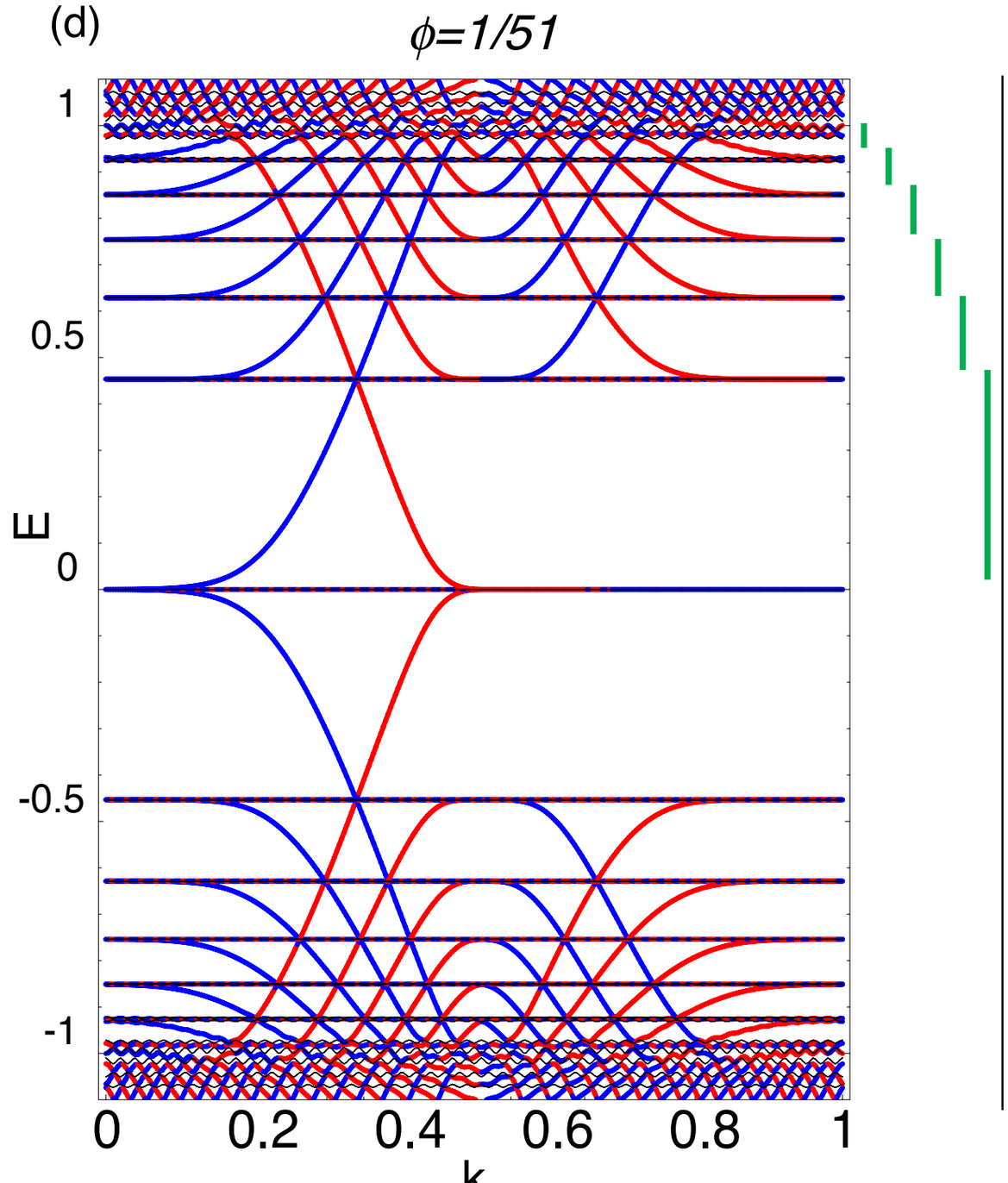
$$\sigma_{xy}^{\text{bulk}} = \sigma_{xy}^{\text{edge}}$$

Bulk-Edge Correspondence in their topological numbers

Bulk – Edge Correspondence

- ☆ Analytically $\sigma_{xy}^{\text{bulk}} = \sigma_{xy}^{\text{edge}}$
- ☆ Topologically Near Zero





k_y

Another type of Edge states in Graphene

- ☆ *Quantum Hall edge states*
  - ☆ *Topologically protected edge states*
- ☆ *Symmetry protected edge states*
  (with topological origin & stability)

Dirac Dispersion : Chiral Symmetry!

without magnetic field

S. Ryu (now KITP) & YH

with magnetic field

M. Arikawa, H. Aoki & YH arXiv:0805.3240 & 0806.2429

Ref.[1] Phys. Rev. B65, 212510 (2002)
[2] Phys. Rev.Lett. 89, 077002(2002)
[3] Physics C 388-389, 78 (2003), ibid 90 (2003)
[4] Phys. Rev. B67, 165410 (2003)
[5] Physica E 22, 679 (2004)

New feature : topological compensation

can be observed by STM experiments

Without magnetic field

Localized Boundary State in Carbon Sheet (1)

now called as Graphene

Tight-binding Model Calculation

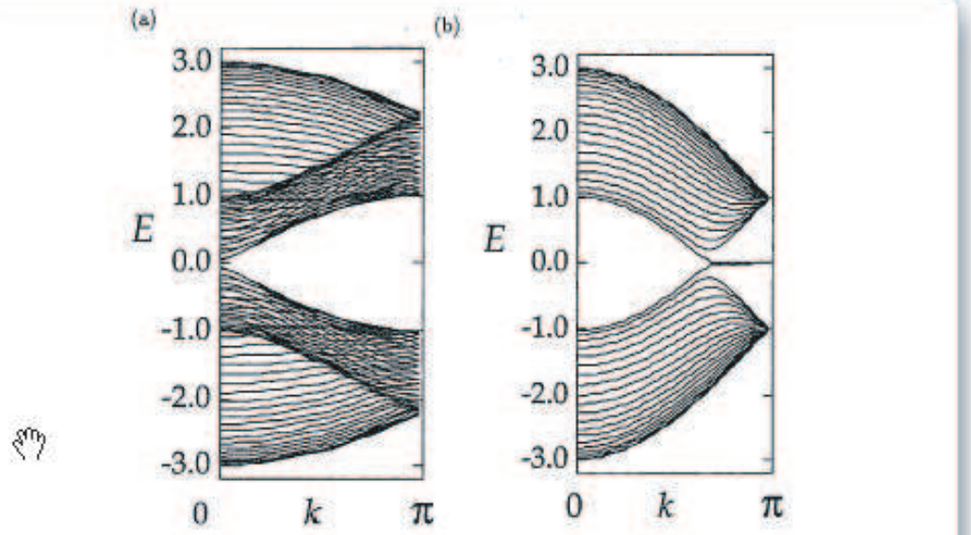


Fig. 2. Band structure of (a) armchair and (b) zigzag ribbons with width $N=20$.

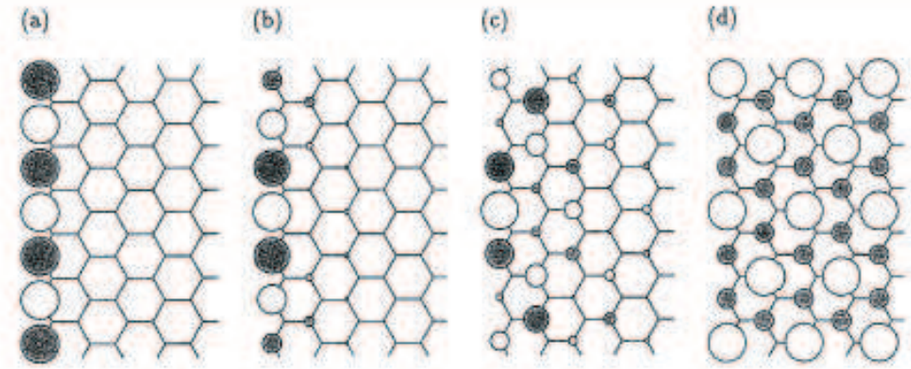


Fig. 3. Schematic figures of the real part of analytic solutions for the edge state in semi-infinite graphite, when (a) $k=\pi$, (b) $8\pi/9$, (c) $7\pi/9$ and (d) $2\pi/3$.

" Peculiar Localized State at Zigzag Graphite Edge " M. Fujita, K. Wakabayashi, K. Nakada and K. Kusakabe, JPSJ 65, 1920 (1996)

Localized Boundary State in Carbon Sheet (2)

Local Spin Density Functional Appr. Calculation

VOLUME 87, NUMBER 14 PHYSICAL REVIEW LETTERS 1 OCTOBER 2001

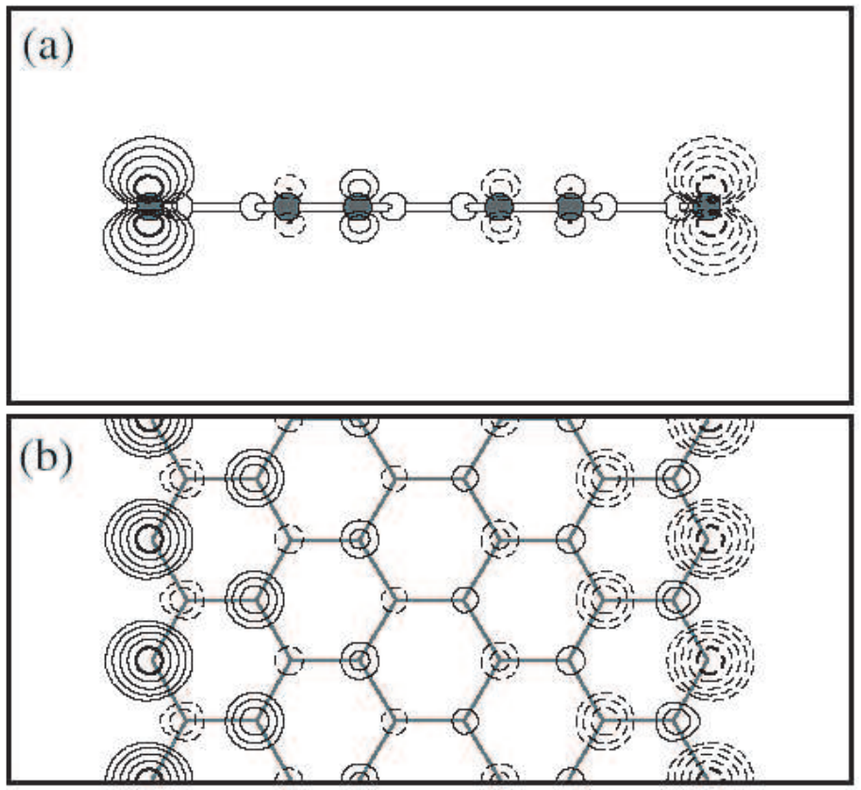


FIG. 1. Contour plots of spin density $n_\uparrow(\boldsymbol{r}) - n_\downarrow(\boldsymbol{r})$ (a) on a plane perpendicular to a graphite flake with zigzag edges and (b) on a plane including the graphite flake. In (a) the edges are perpendicular to the plane and C atoms on the plane are depicted by shaded circles. Positive and negative values of the spin density are shown by solid and dashed lines, respectively. Each contour represents twice (or half) the density of the adjacent contour lines.

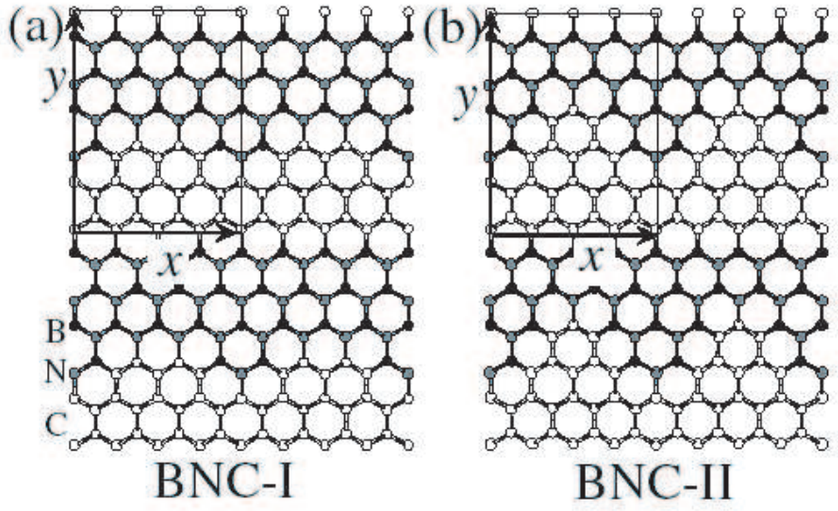


FIG. 2. Top views of fully optimized BNC heterosheets, (a) BNC-I and (b) BNC-II. White, shaded, and black circles denote C, B, and N atoms, respectively. The rectangle in each figure denotes the unit cell.

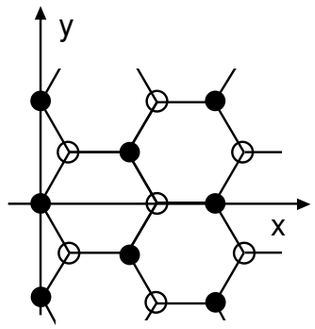
B, N, and C atoms have been observed indeed [8–10]. Second, the phase separation of graphite and BN regions leading to the striped structures above is energetically favorable. In fact, we have performed the total-energy calculations for graphite, BN, BC, and NC heterosheets by DFT. The calculated bond energies of B-C and N-C are smaller than that of graphite by 1.52 and 0.81 eV, respectively. On the other hand, the bond energy of B-N is smaller than that of graphite only by 0.31 eV. Third, undulation

"Magnetic Ordering in Hexagonally Bonded Sheets with First-Row Elements", Okada, Oshiyama, Phys. Rev. Lett. 87, 146803 (2001)

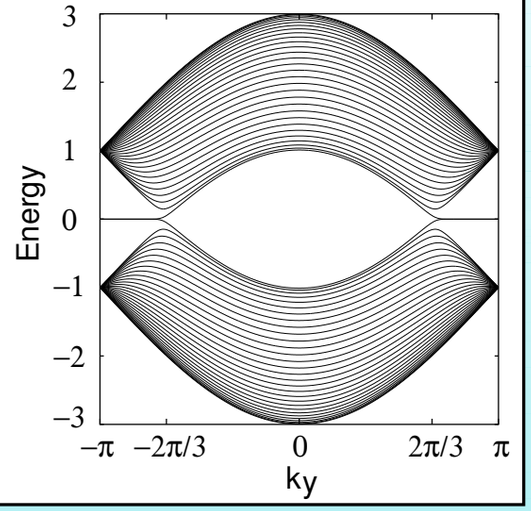
Zero Energy Edge States : cont.

- Graphite Ribbons (zigzag, bearded, and armchair edges)

(a)

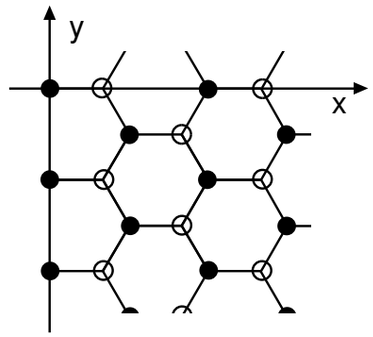


Zigzag

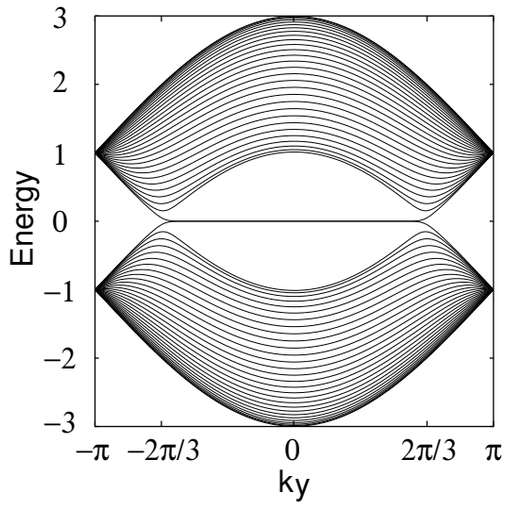


Zero Energy Edge States !

(b)

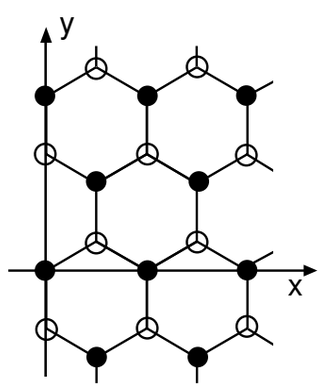


Bearded

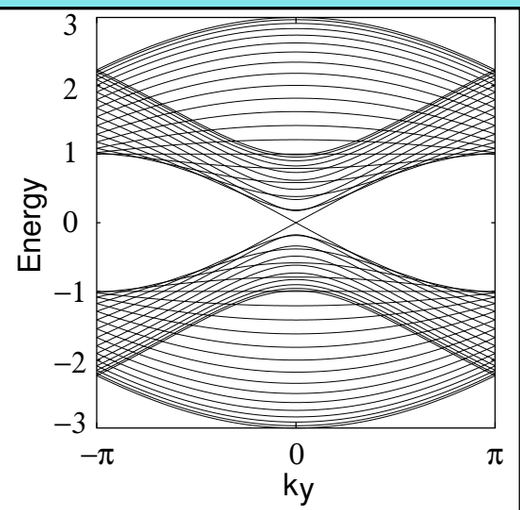


Zero Energy Edge States !

(c)



Armchair



No Edge States !

When the zero modes exist?

S.Ryu & Y.Hatsugai, Phys. Rev. Lett. 89, 077002 (2002)
Y.Hatsugai., J. Phys. Soc. Jpn. 75 123601 (2006)
Kuge, Maruyama, Y. Hatsugai, arXiv:0802.2425

1D two site problem <u>with</u> boundaries

Consider Berry phase of the bulk (<u>without</u> boundaries) Zak

$$\gamma = \int A = \int d\vec{k} \cdot \vec{\mathcal{A}} \qquad \vec{\mathcal{A}} = \langle \psi(k) | \vec{\nabla}_k \psi(k) \rangle$$

Require Local Chiral Symmetry
(ex. bipartite)

$$\{\Gamma, H\} = \Gamma H + H\Gamma = 0$$

⇒ Quantized

$$\gamma = \int A = \begin{cases} \pi \\ 0 \end{cases}$$

$\gamma = \pi$ ⇒ Zero energy localized states EXIST

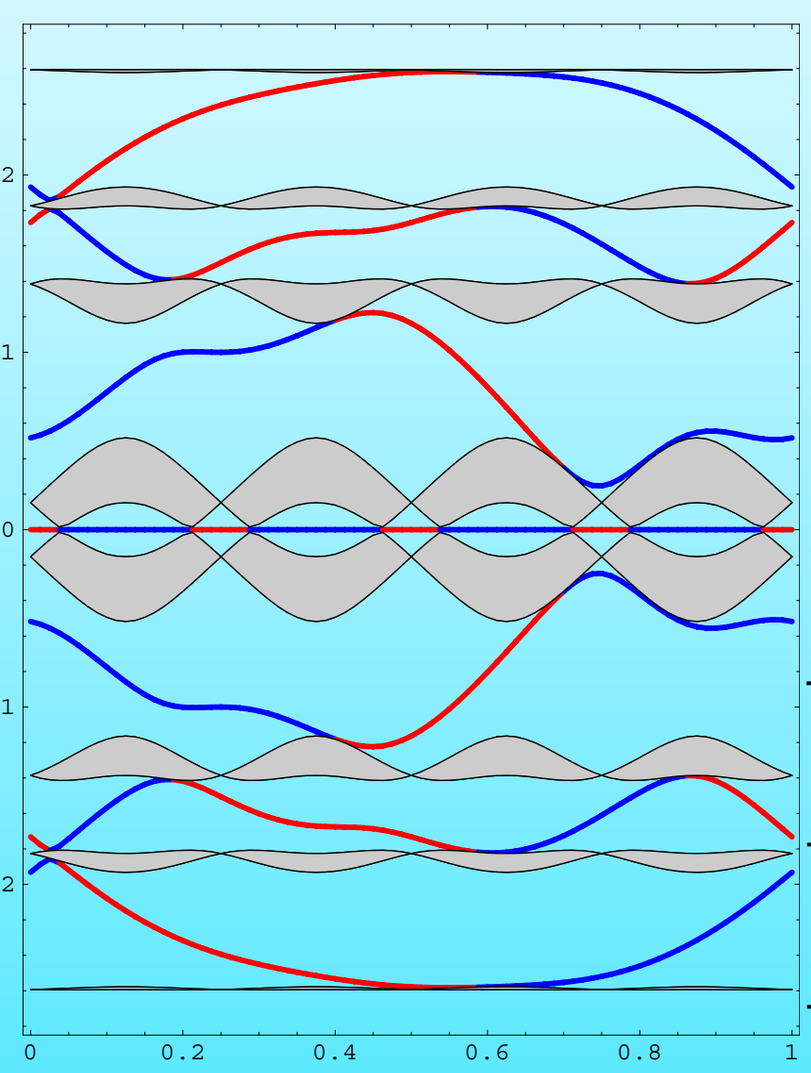
: There exists odd number of zero modes

Symmetry protected zero modes: bulk-edge correspondence

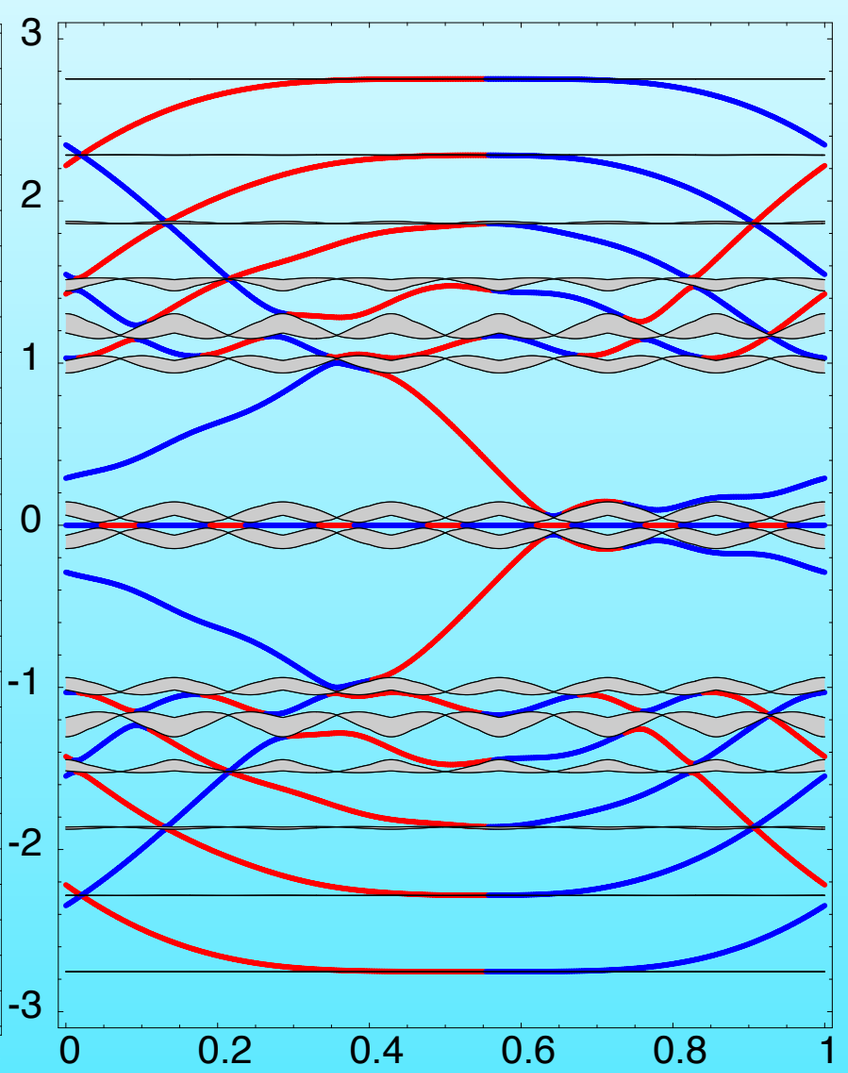
With magnetic field

Close look at E=0

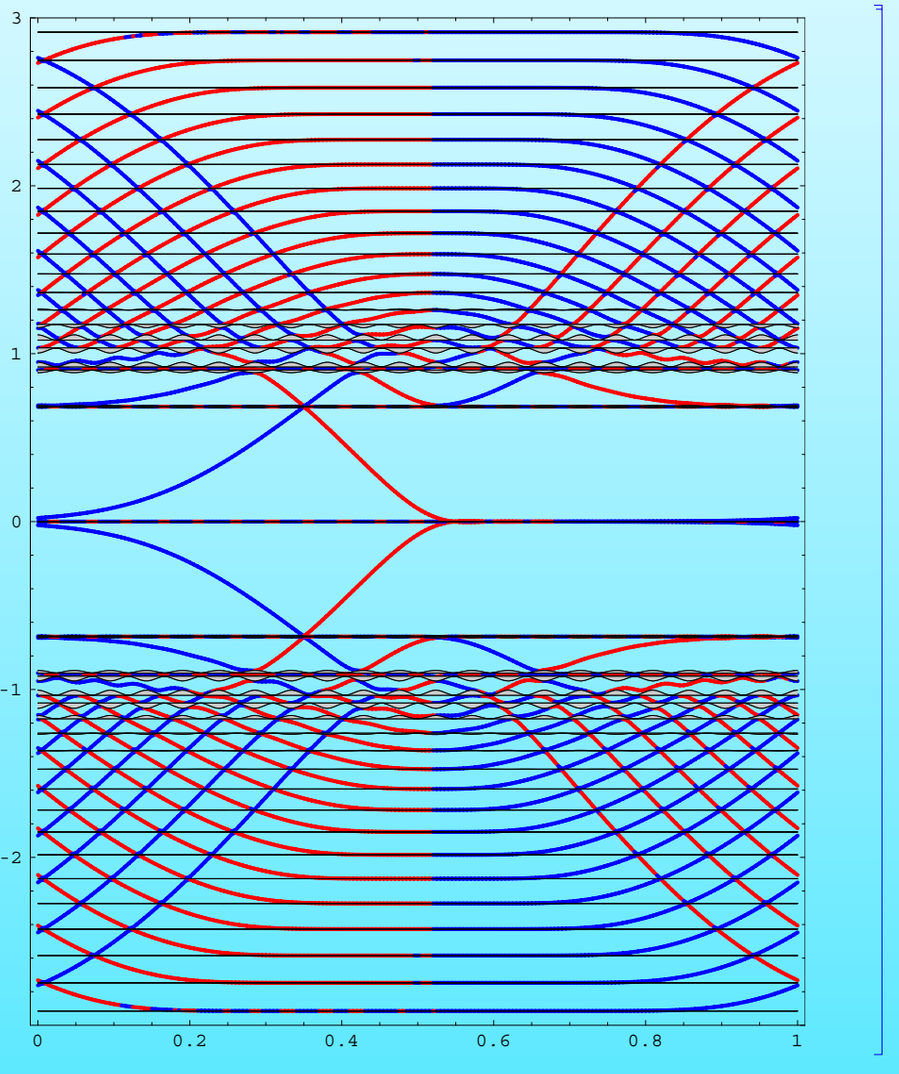
- *n=0 Landau Level*



$\phi = 1/4$



$\phi = 1/7$

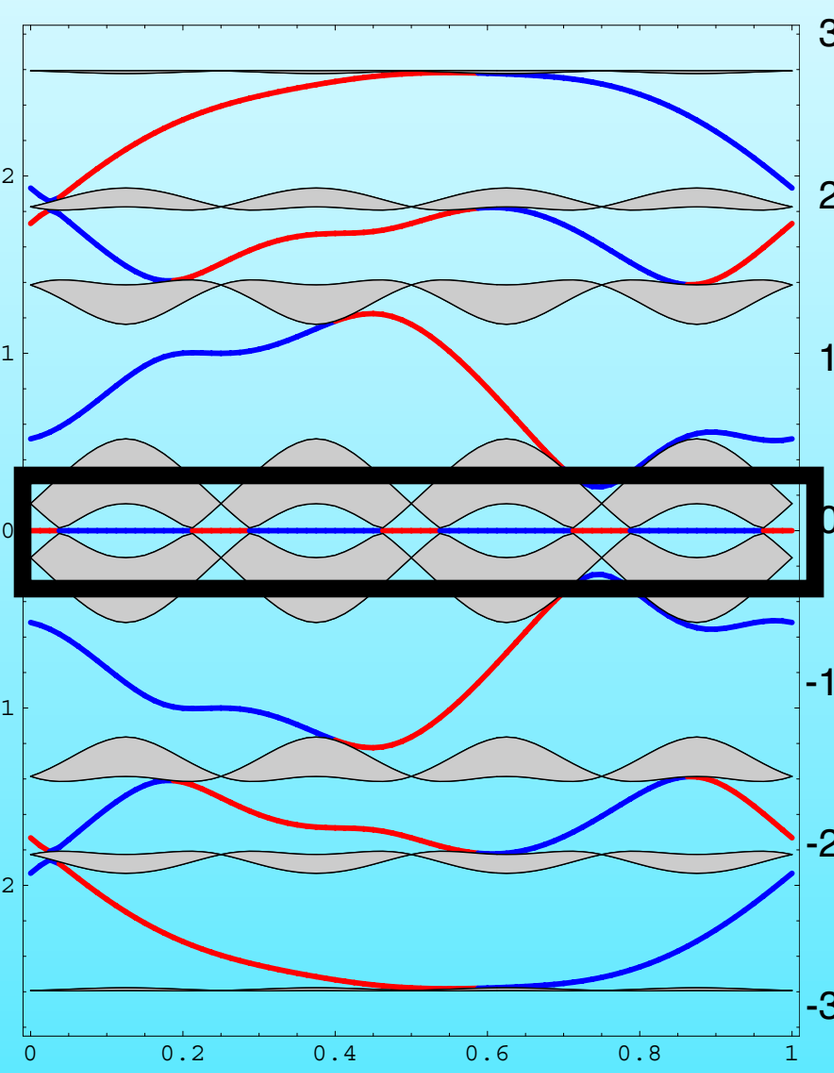


$\phi = 1/21$

strong ⟷ *weak*

Close look at E=0

- *n=0 Landau Level*



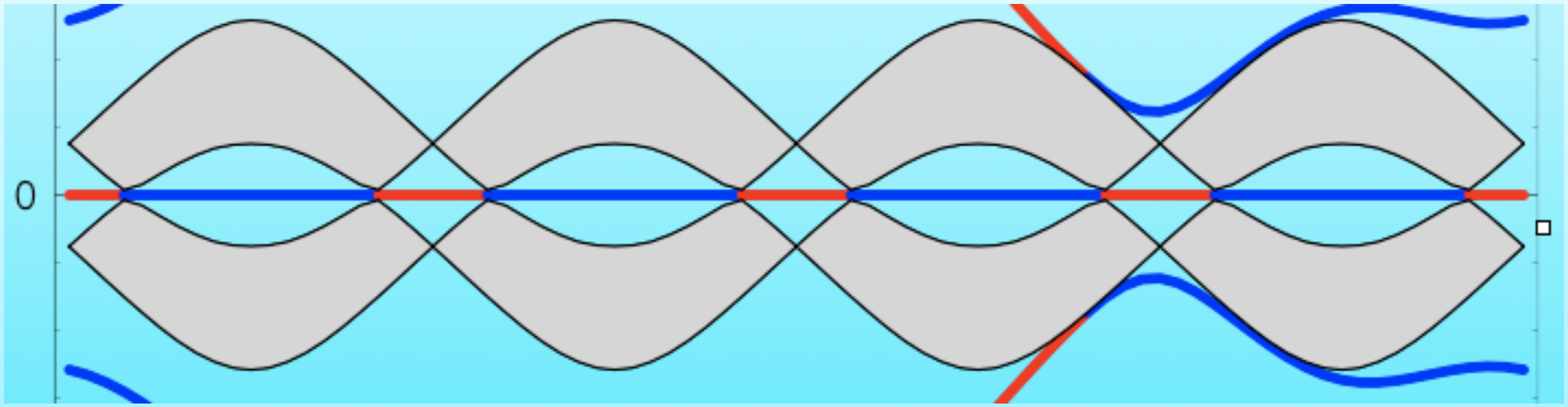
$\phi = 1/4$ $\phi = 1/7$ $\phi = 1/21$

strong ⟷ *weak*

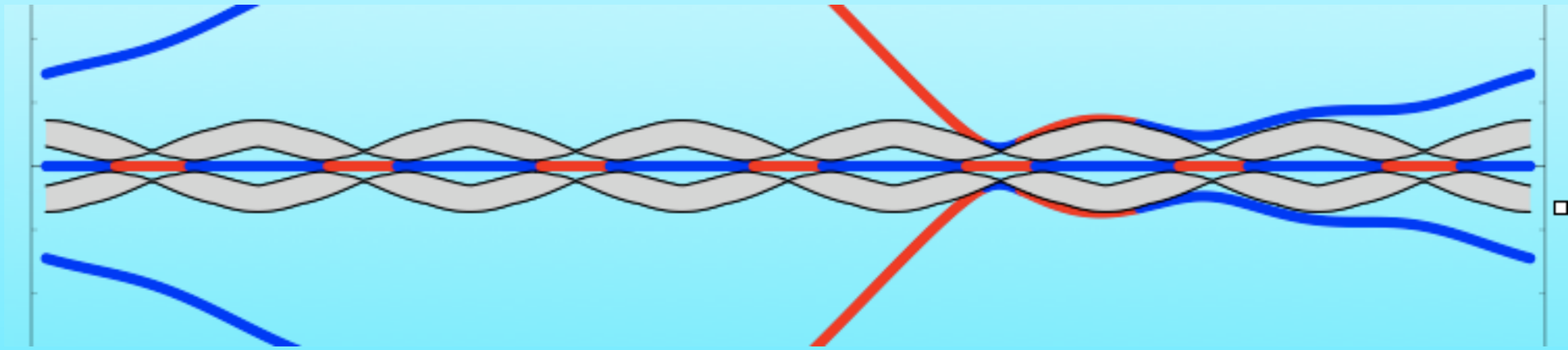
Close look at E=0

☆ *n=0 Landau Level*

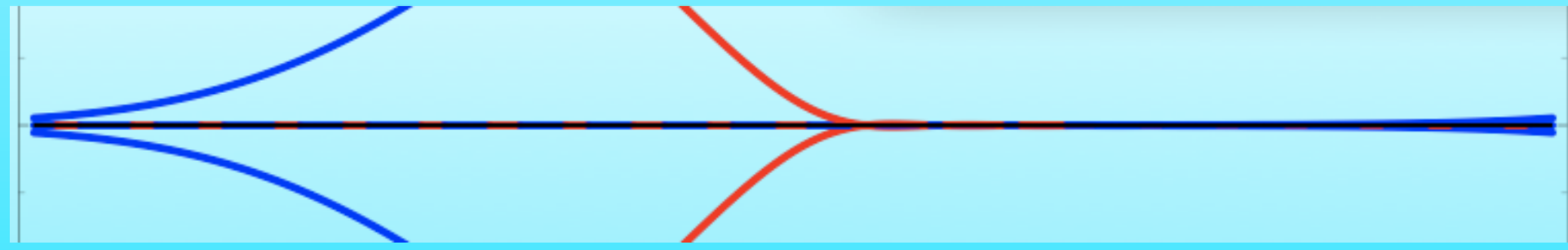
$\phi = 1/4$



$\phi = 1/7$



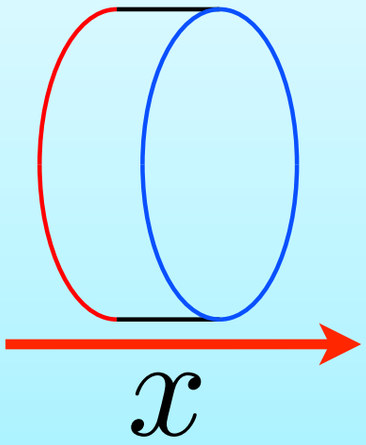
$\phi = 1/21$



Bulk Landau Level and the zero mode edge states coexist

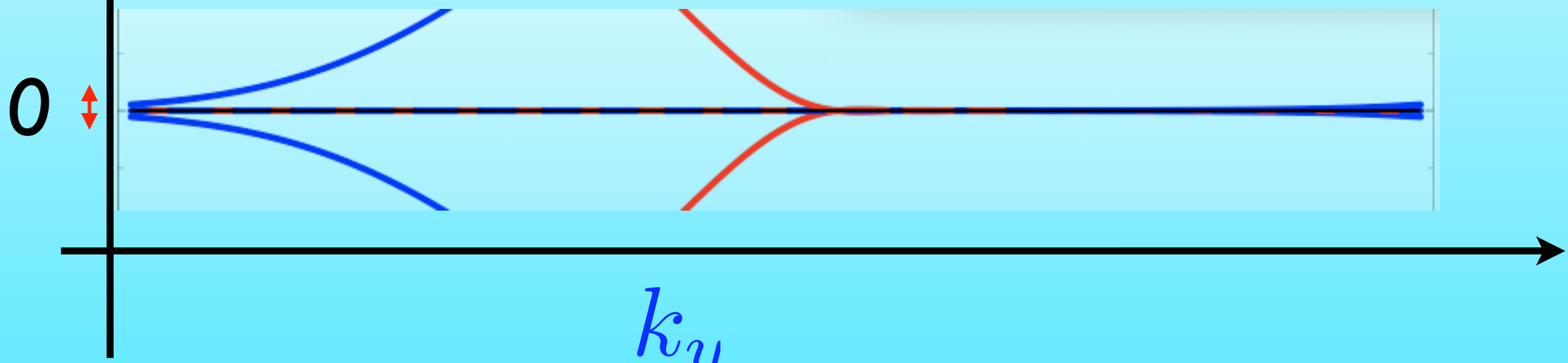
Charge density around E=0

Integrated local density of state (LDOS)



$$I(x) = \frac{1}{2\pi} \int_{-E_C}^{E_C} dE \int_{-\pi}^{\pi} dk_y \, |\Psi(x, k_y, E)|^2$$

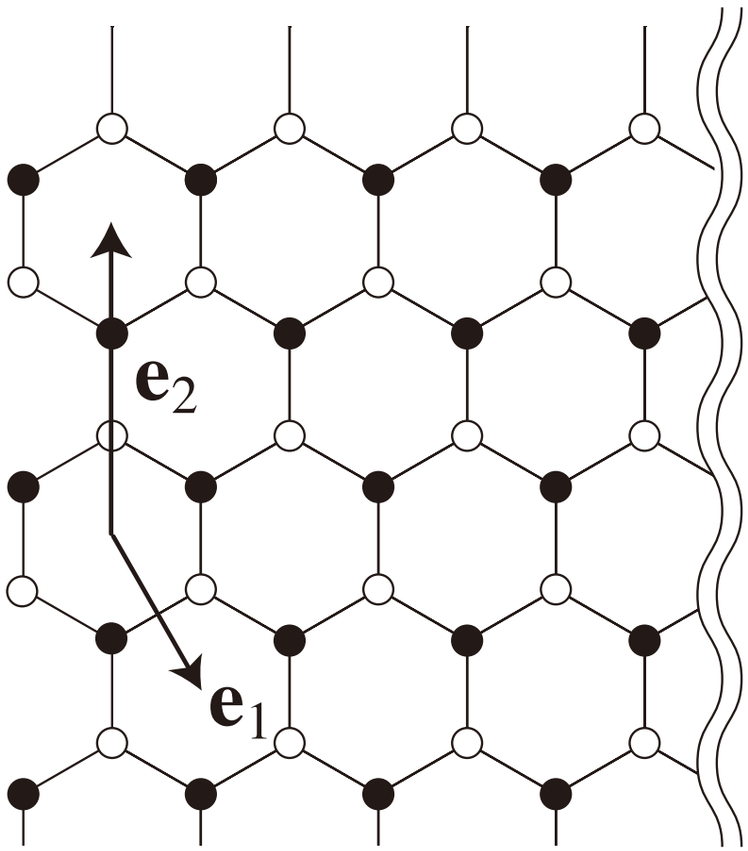
$$E_C \sim 0.05\, t$$



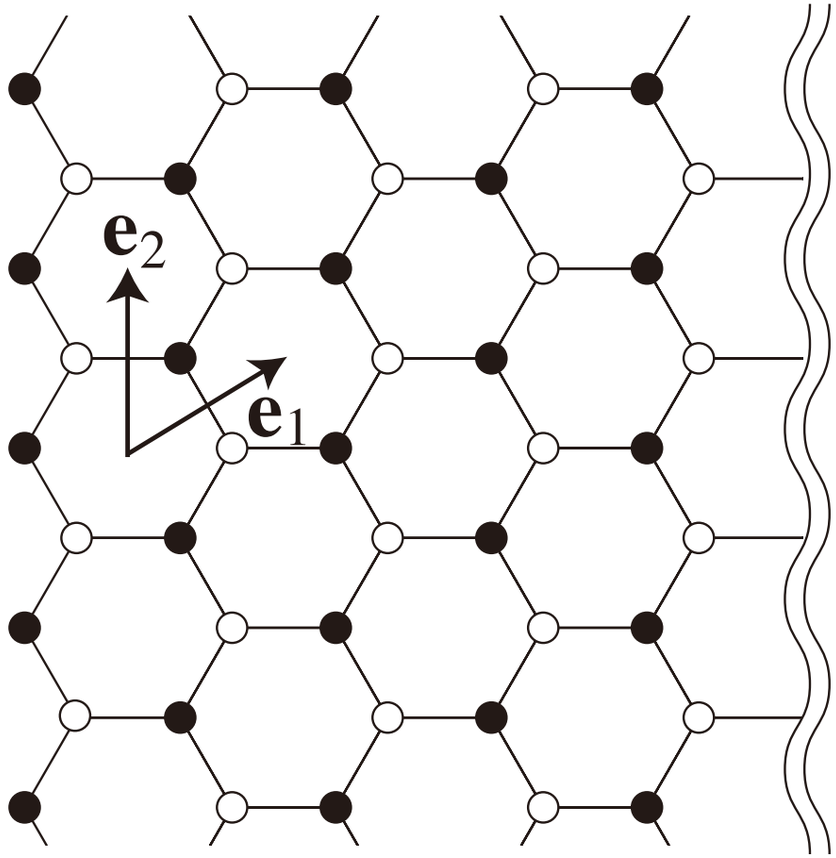
This can be observed by the STM image

Charge density around E=0

2 types of edges : Zigzag & Armchair



(a) Armchair edge



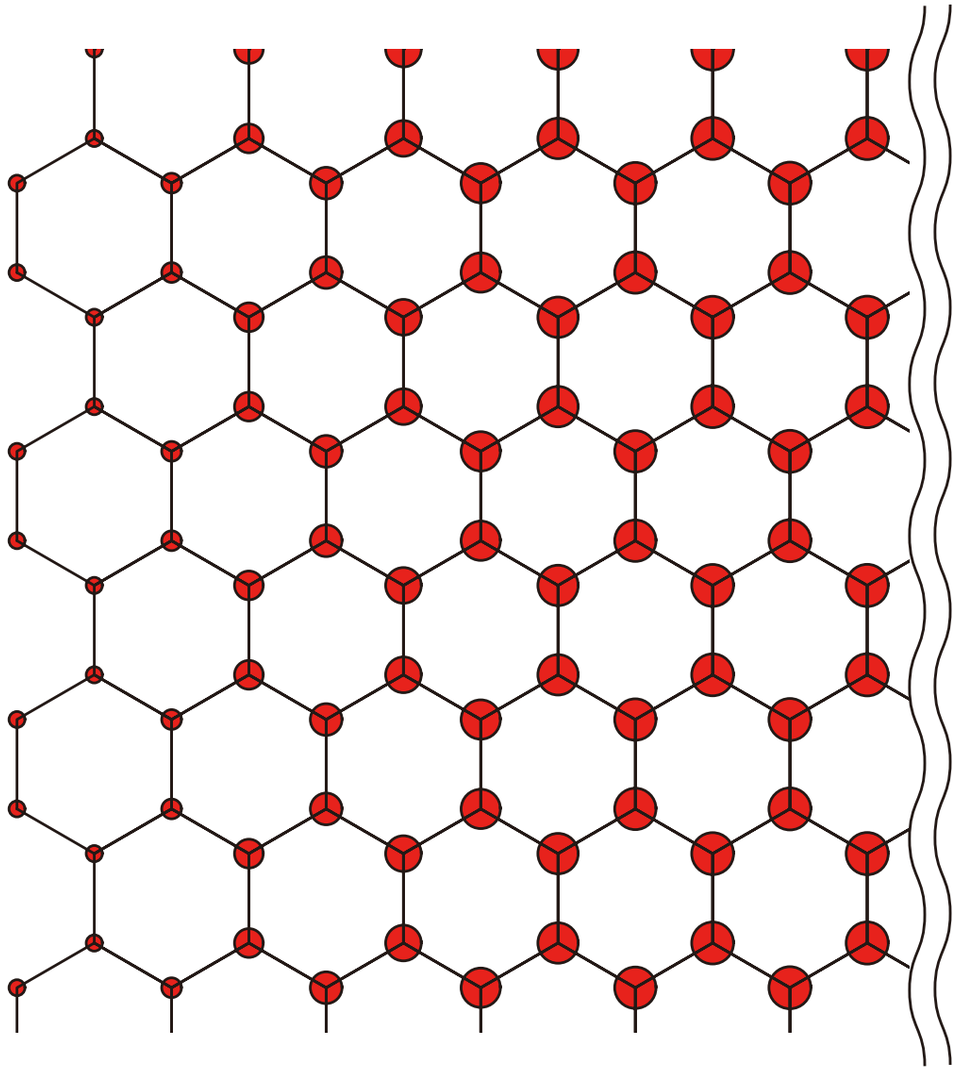
(b) Zigzag edge

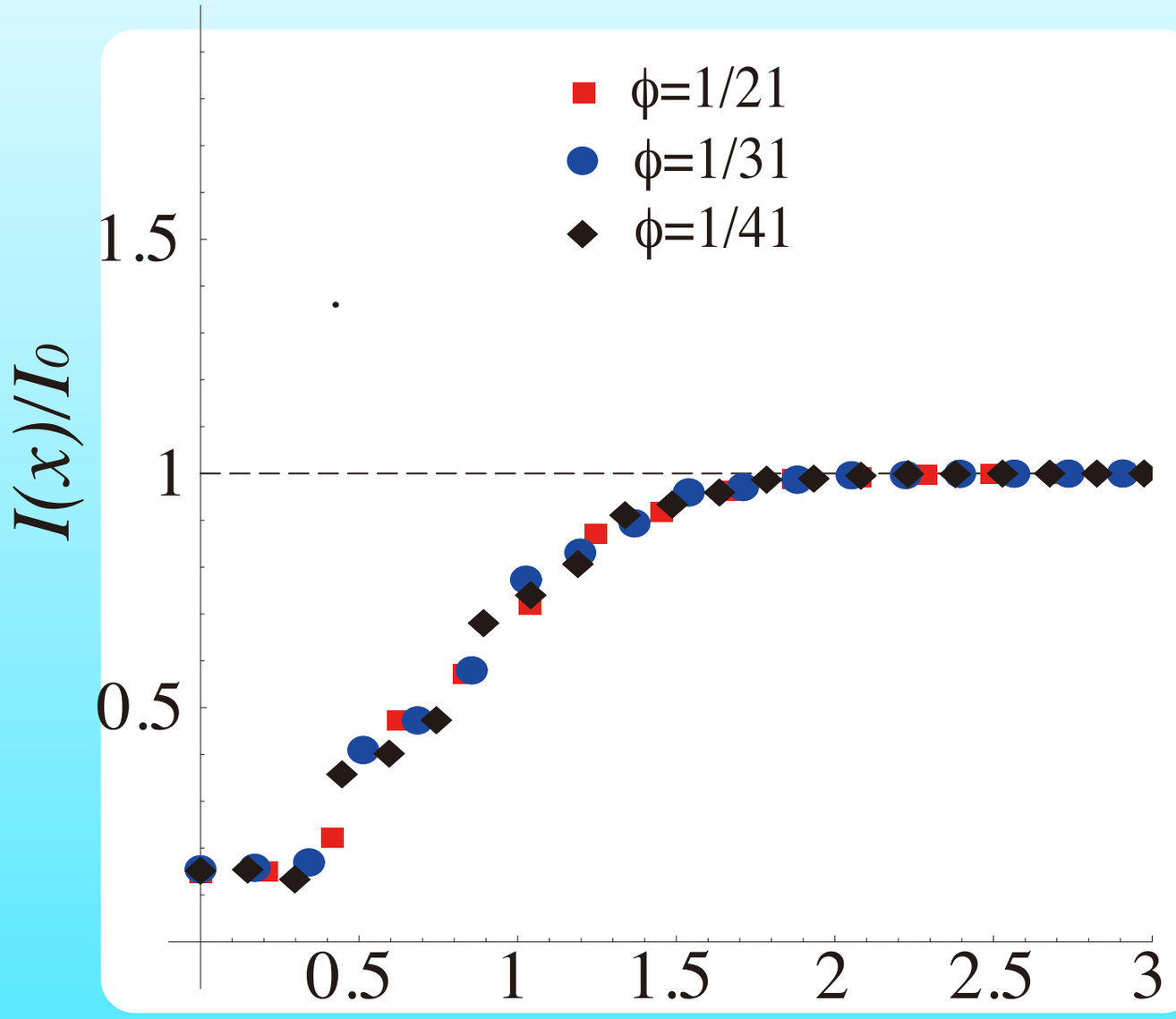
LDOS around E=0

$$I(x) = \frac{1}{2\pi} \int_{-E_C}^{E_C} dE \int_{-\pi}^{\pi} dk_y \, |\Psi(x, k_y, E)|^2$$

Armchair

x





Suppression near the edge

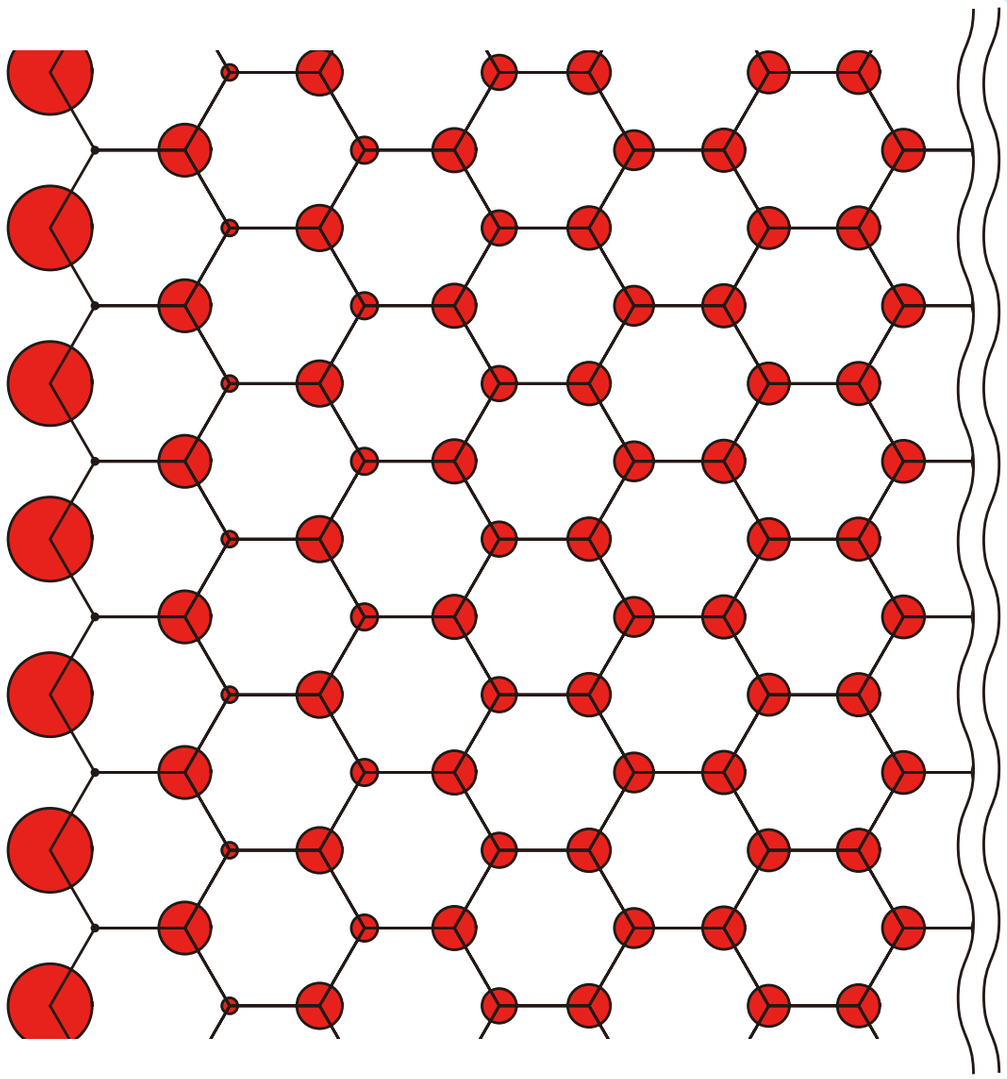
Standard behavior due to edge potential

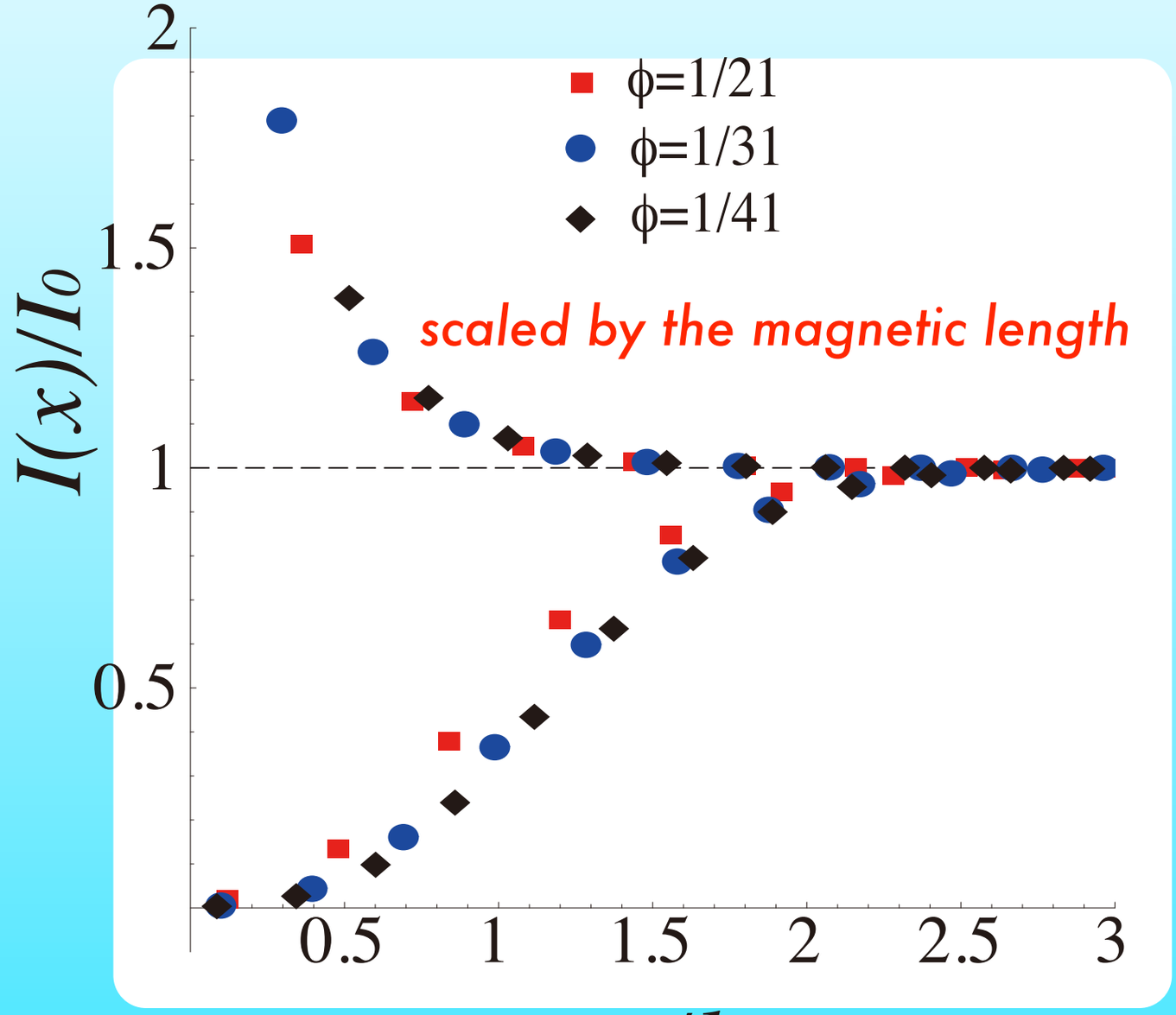
LDOS around E=0

$$I(x) = \frac{1}{2\pi} \int_{-E_C}^{E_C} dE \int_{-\pi}^{\pi} dk_y \, |\Psi(x, k_y, E)|^2$$

Zigzag

x





STM observable

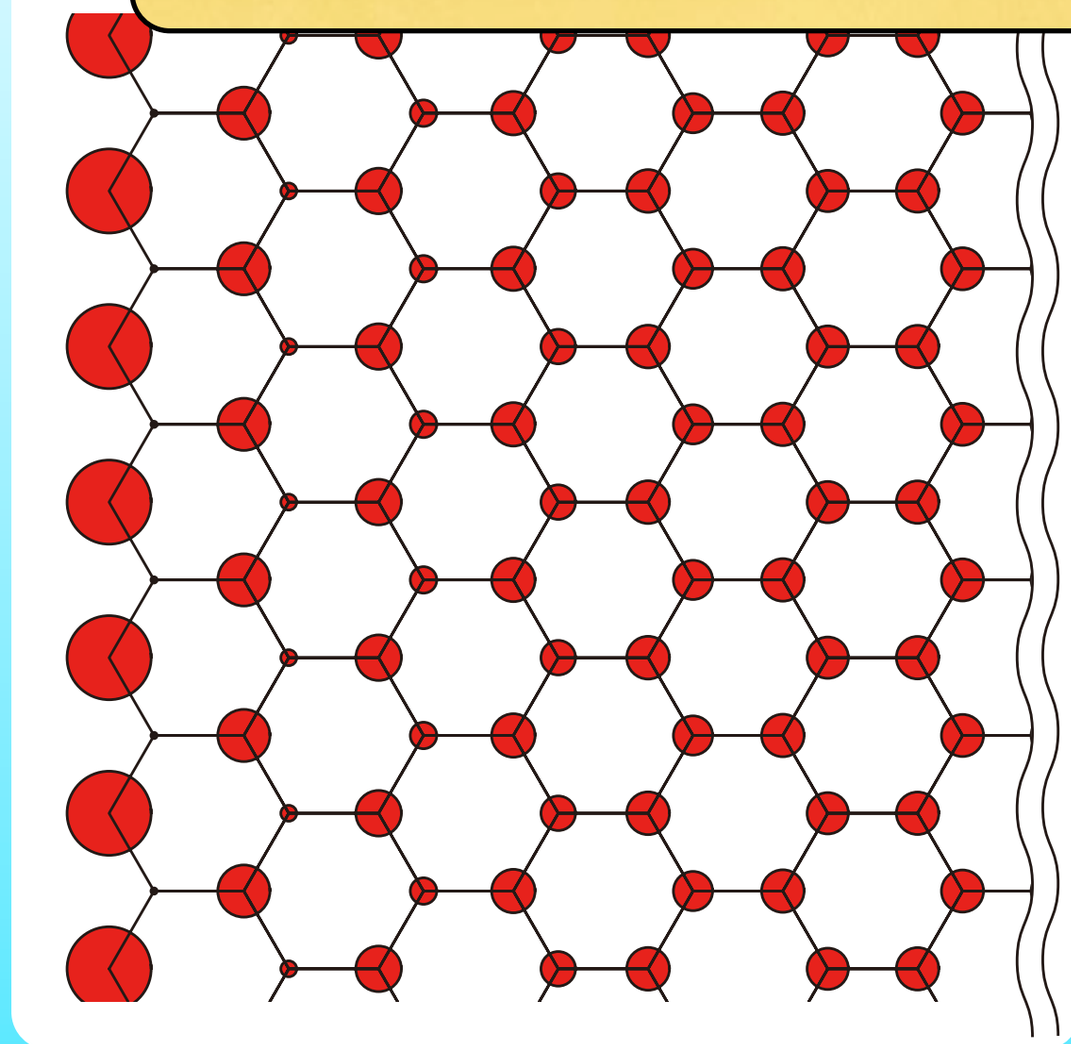
Strong enhancement near the edge

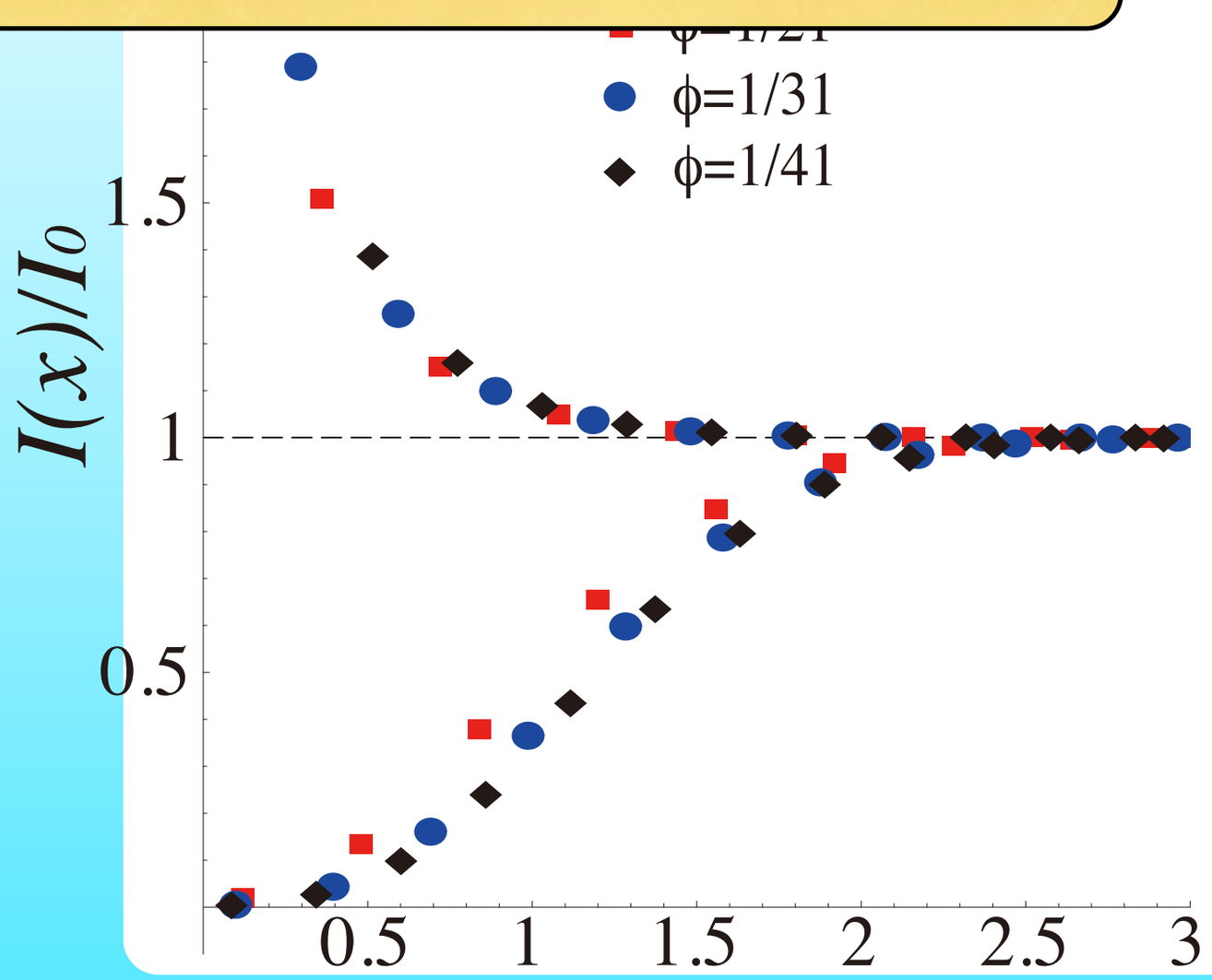
Characteristic feature of the Graphene Zigzag edges!

Zero mode contribution ?

Not exactly true !

Topological Compensation by the Bulk !





Strong enhancement near the edge

Characteristic feature of the Graphene Zigzag edges!

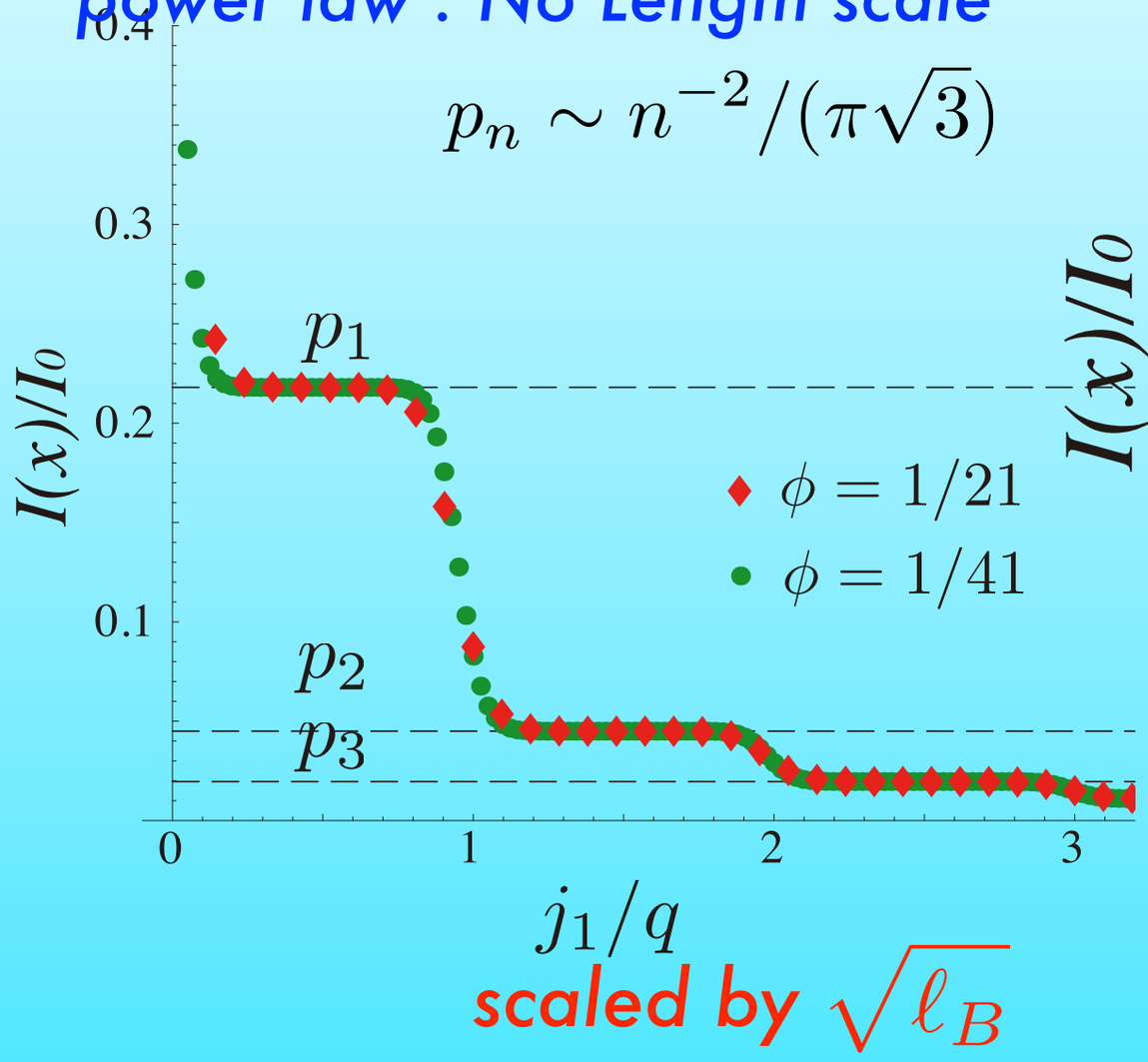
Topological Compensation by Bulk

$$I(x) = \frac{1}{2\pi} \int_{-E_C}^{E_C} dE \int_{-\pi}^{\pi} dk_y \, |\Psi(x, k_y, E)|^2$$

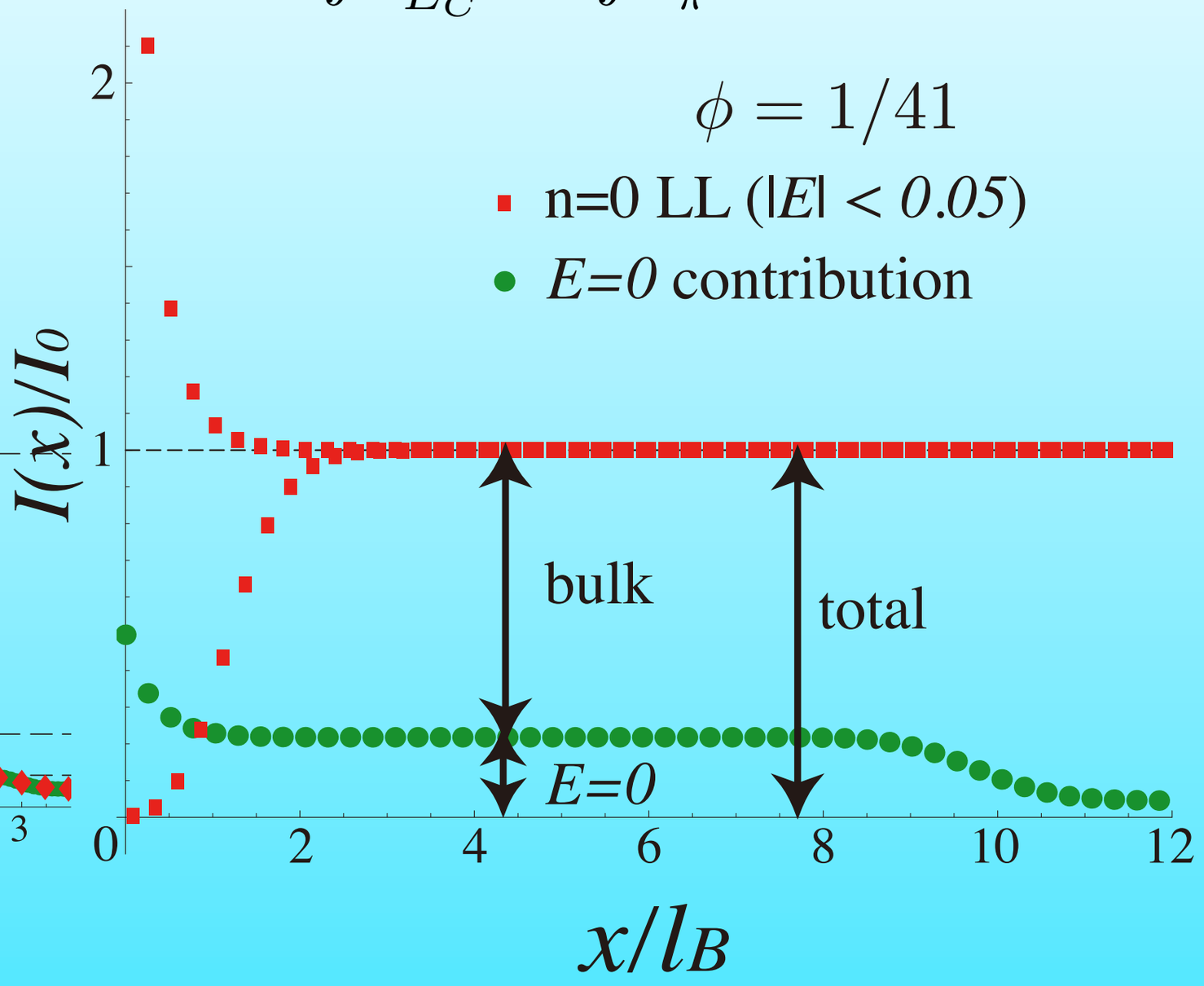
Only the Zero mode

power law : No Length scale

$$p_n \sim n^{-2}/(\pi\sqrt{3})$$



scaled by $\sqrt{\ell_B}$



Bulk does give substantial contribution

Characteristic feature of the Graphene Zigzag edges!

Possible quantum liquid of Graphene

many body effects

edge states as basic objects to condense

edge states as 2D analogue of the solitons in 1D

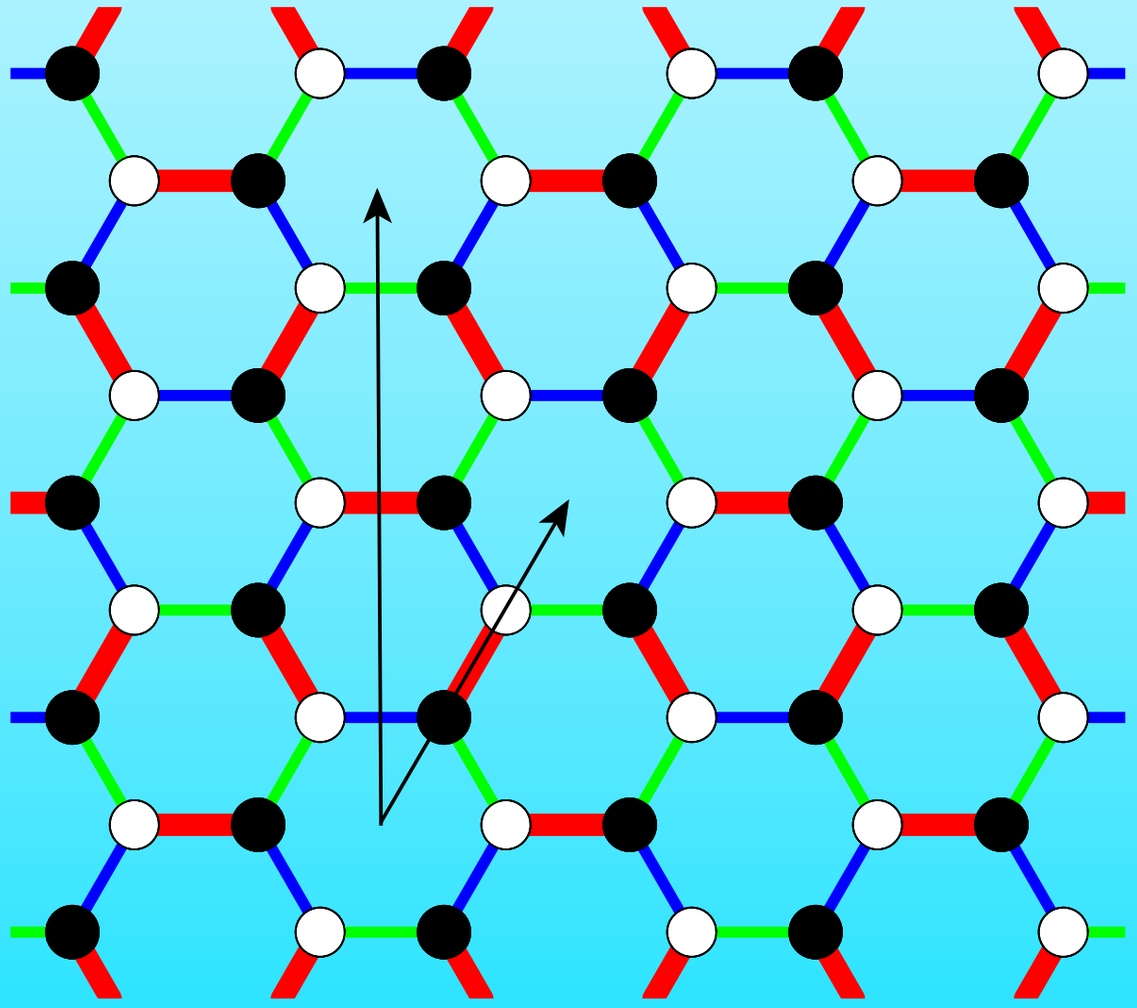
Y. Hatsugai, T. Fukui, H. Aoki,
"Topological low-energy modes in N=,0 Landau levels of graphene: a possibility of a quantum-liquid ground state", arXiv:0804.4762, Physica E 40, 1530 (2008).

Can the E=0 Landau level be split in graphene??

A possible candidate = Bond-ordering or dimerization

bond-order parameter! $\langle c_a^\dagger c_b \rangle$

Affleck-Marston '88
Voit '92
Nakamura '99



- Origin ? (with Magnetic field)
  - Electron-Electron interaction
  - (Jahn-Teller?)

Breaking the equivalence among 3 kinds of bonds in a Kekule pattern

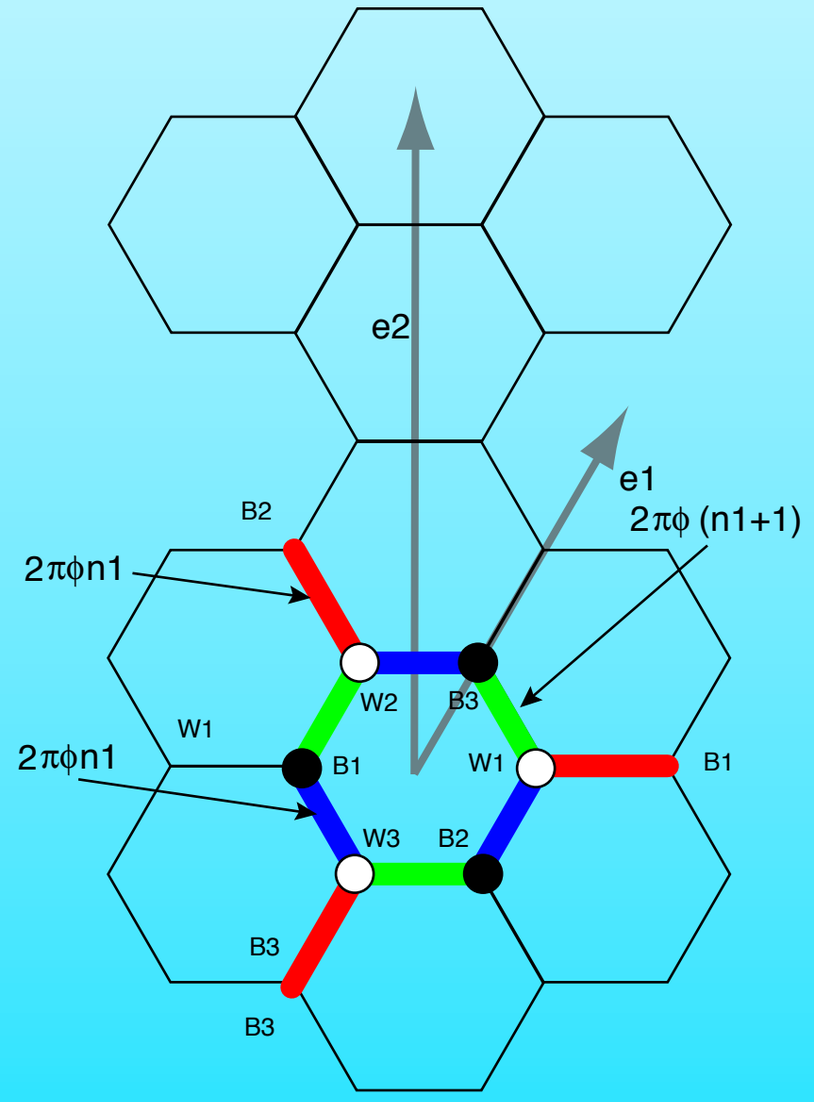
- Preserve Chiral Symmetry
- Three Fold Degeneracy
- Local Kekule pattern
- Enlarged unit cell (3times)

Static Bond-Order and the Enlarged Unit cell

☆ *Spinless Fermion model on Graphene with Nearest-neighbor Coulomb interaction*

$$H_{int} = \sum_{\langle ij \rangle} t_{ij} c_i^\dagger c_j + h.c. + V n_i n_j$$

Simplest case:



Affleck-Marston type mean Hamiltonian

$$V n_a n_b \approx -\Delta_{ab}^* c_a^\dagger c_b + h.c. + \text{const.}$$

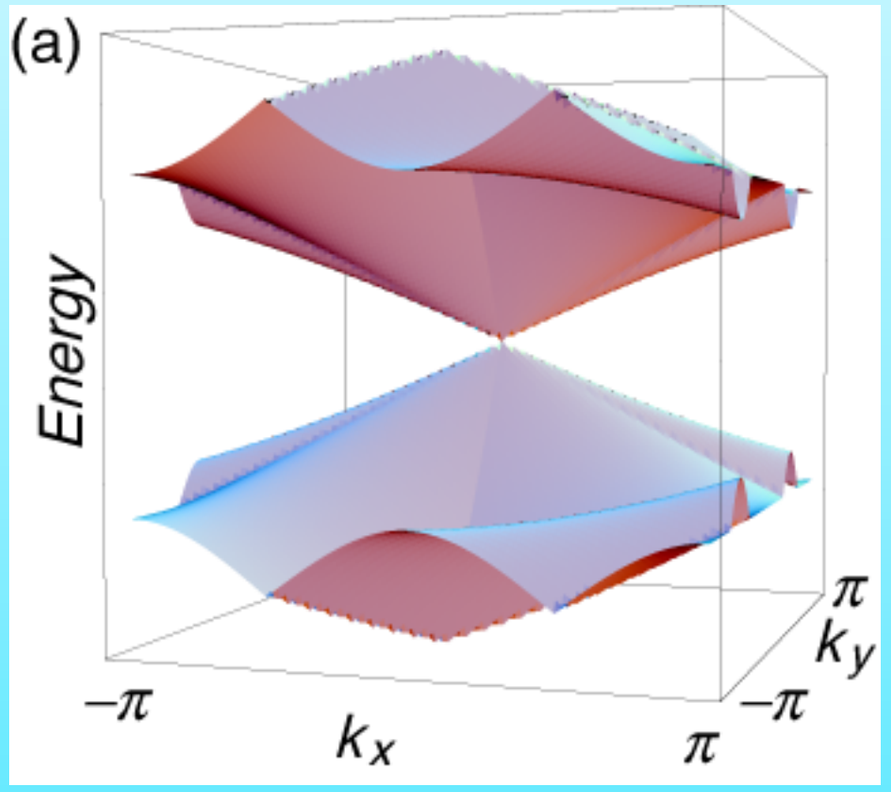
$$\Delta_{ab} = V \langle c_a^\dagger c_b \rangle = |\Delta_{ab}| e^{i\theta_{ab}}$$

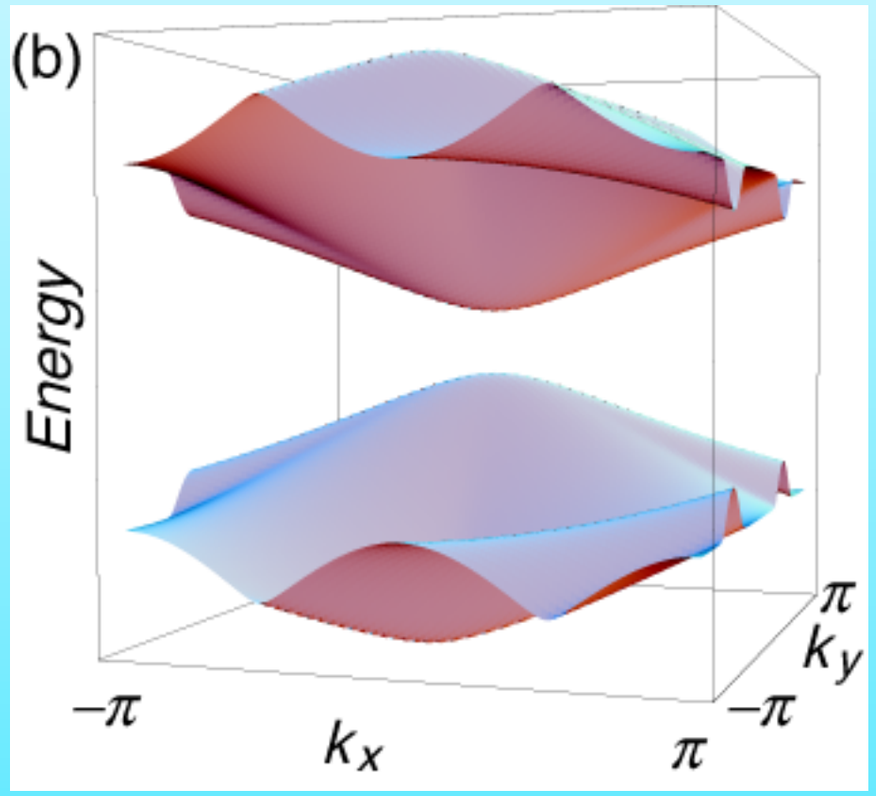
Self consistent solution

$$\begin{aligned} H_{MF} = \sum & \; t_G \left[e^{i2\pi\phi(n_1+1)} c_{\bullet 3}^\dagger(n) c_{\circ 1}(n) \right. \\ & \left. + c_{\bullet 1}^\dagger(n) c_{\circ 2}(n) + c_{\bullet 2}^\dagger(n) c_{\circ 3}(n) \right] \\ & + t_B \left[c_{\bullet 3}^\dagger(n) c_{\circ 2}(n) \right. \\ & \left. + e^{i2\pi\phi n_1} c_{\bullet 1}^\dagger(n) c_{\circ 3}(n) + c_{\bullet 2}^\dagger(n) c_{\circ 1}(n) \right] \\ & + t_R \left[c_{\bullet 1}^\dagger(n + 2e_1 - e_2) c_{\circ 1}(n) \right. \\ & + e^{i2\pi\phi n_1} c_{\bullet 2}^\dagger(n - e_1 + e_2) c_{\circ 2}(n) \\ & \left. + c_{\bullet 3}^\dagger(n - e_1) c_{\circ 3}(n) \right] + h.c. \end{aligned}$$

Energy Dispersion (without Magnetic Field)

- *Enlarged Unit cell* — *We <u>assume</u> a static bond-ordering*
- *Chiral Symmetry is preserved*
- *Dirac Fermions get masses*
  - *The energy gap opens up*





Particle-Hole Symmetric

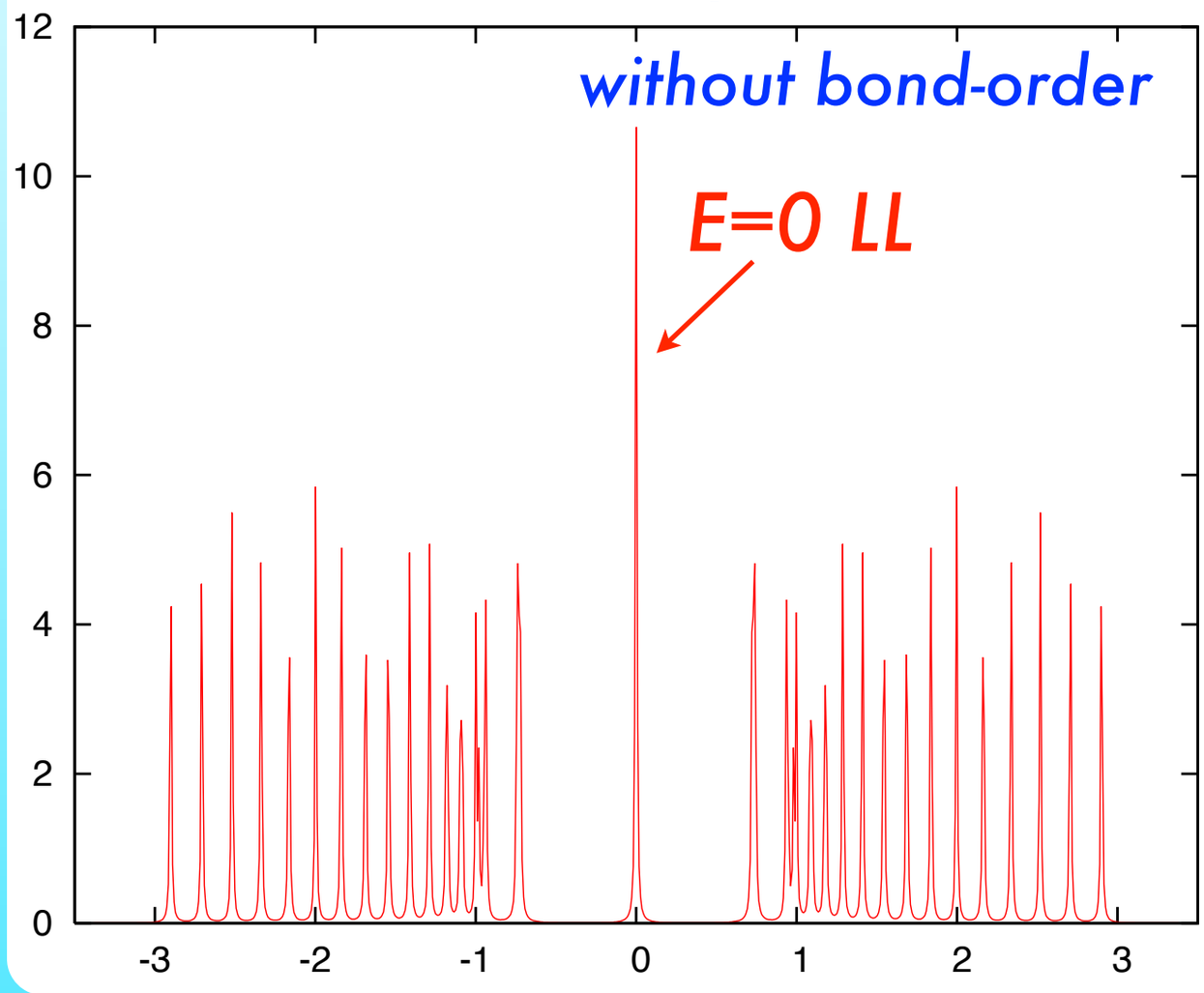
$$E(\boldsymbol{k}) \approx \pm|\det \boldsymbol{D}(\boldsymbol{k})|$$ ***(small gap & near E=0)***

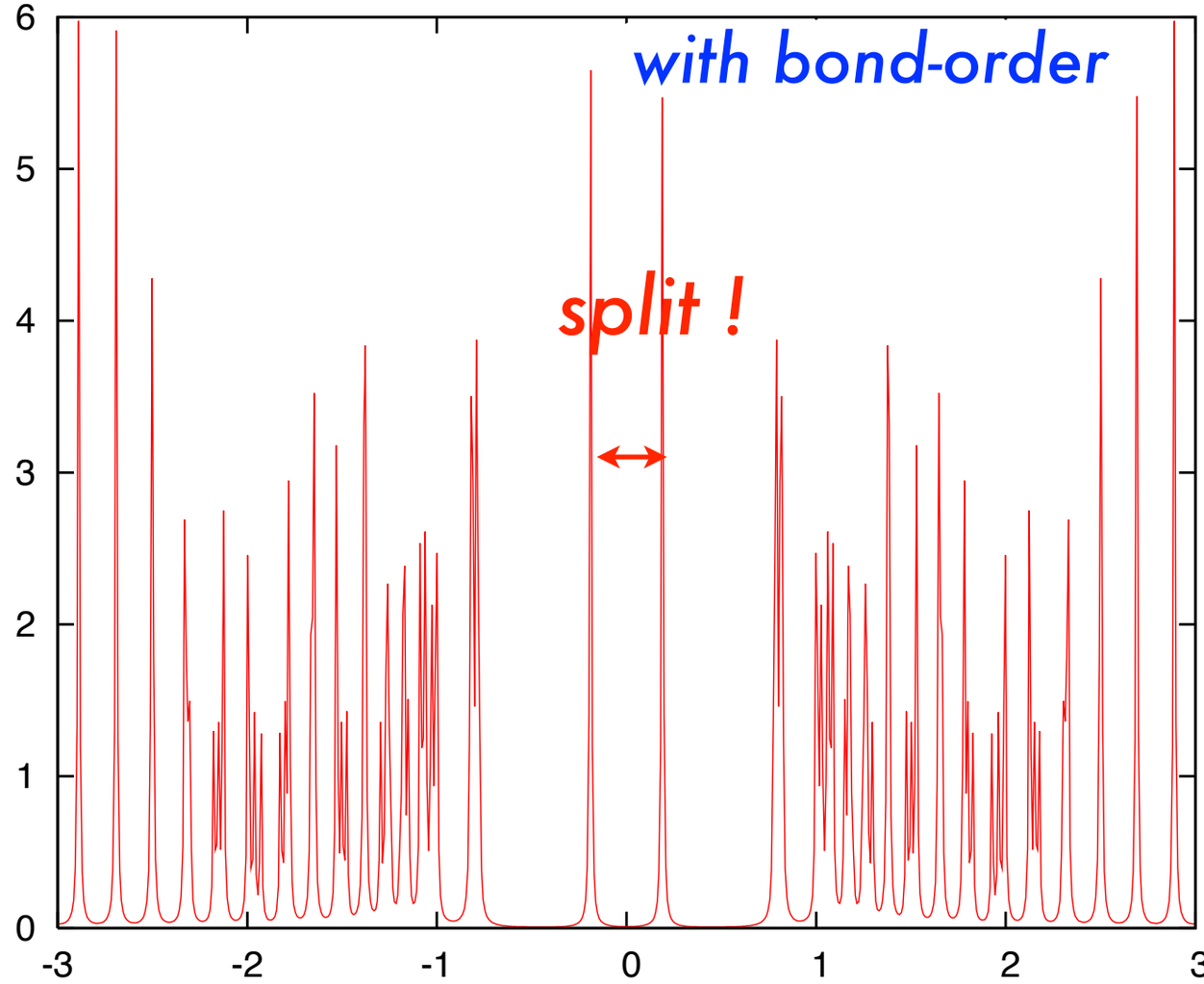
$$\boldsymbol{D} = \begin{pmatrix} t_R e^{-i(2k_1-k_2)} & t_G & t_B \\ t_B & t_R e^{-i(-k_1+k_2)} & t_G \\ t_G & t_B & t_R e^{ik_1} \end{pmatrix}$$

Landau level Structures with bond-orders

☆ Characteristic *E=0 Landau Level* of the Dirac Fermion *splits* !

One particle Density of states





Bond-Order Formaion (*in a mean field level*)

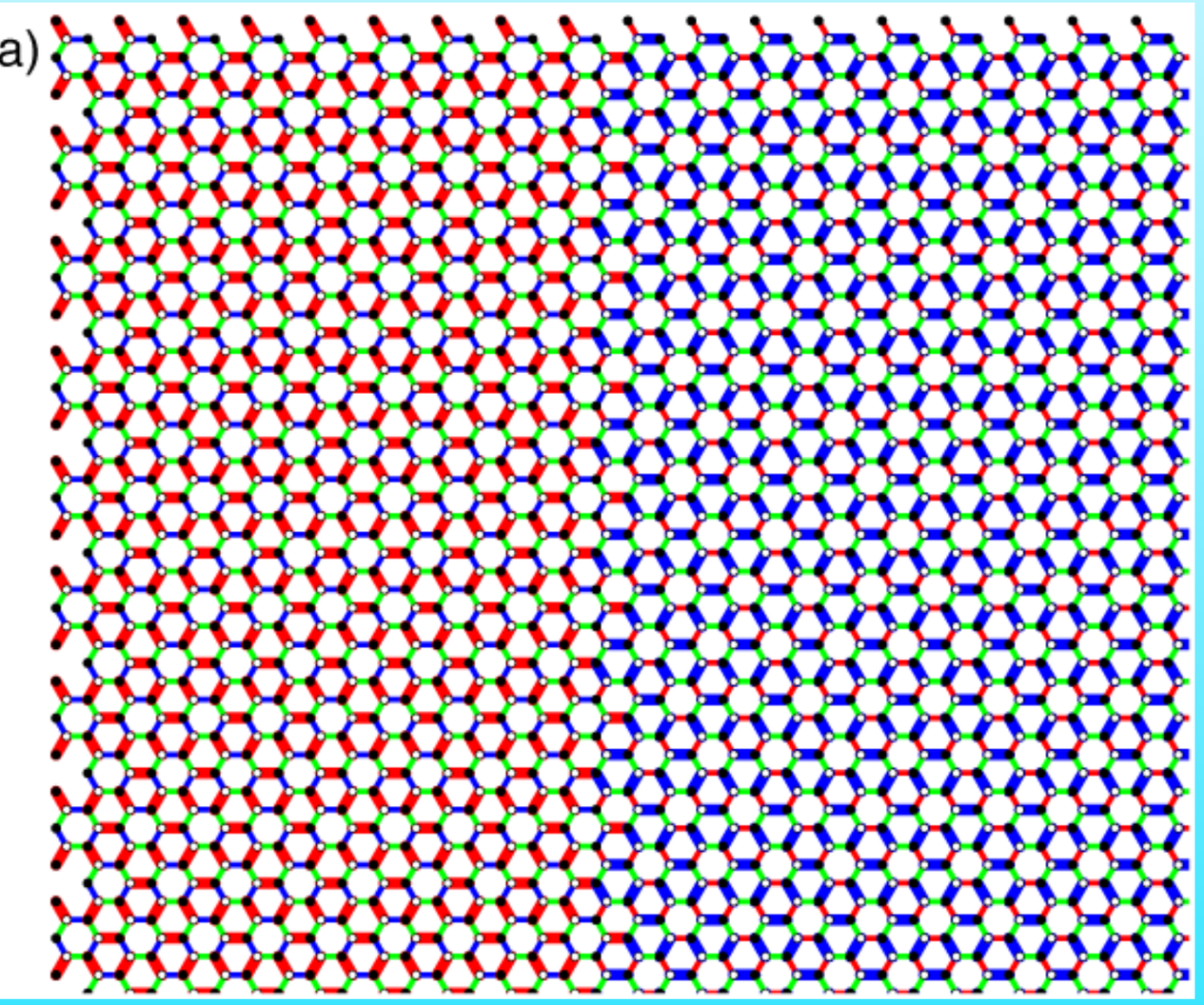
Peierls Instability of the Flat band as of the E=0 Landau Level

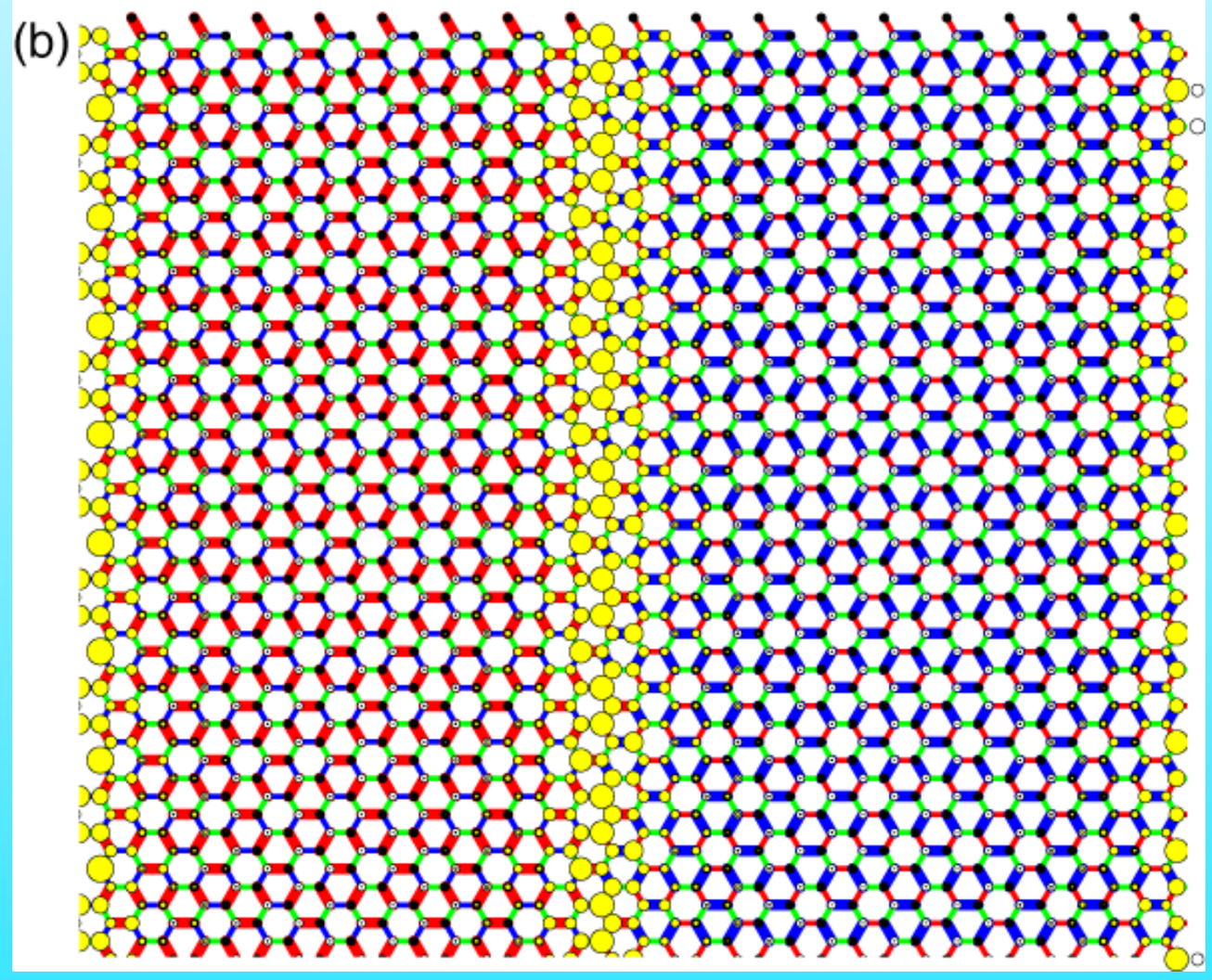
Bond-Ordering with Domains (Strip domain)

- ☆ Straight line domain boundaries
- ☆ A possible snap shot configuration of the dynamical bond-order

Domain shape (strip)

Charge Density of the in-gap states



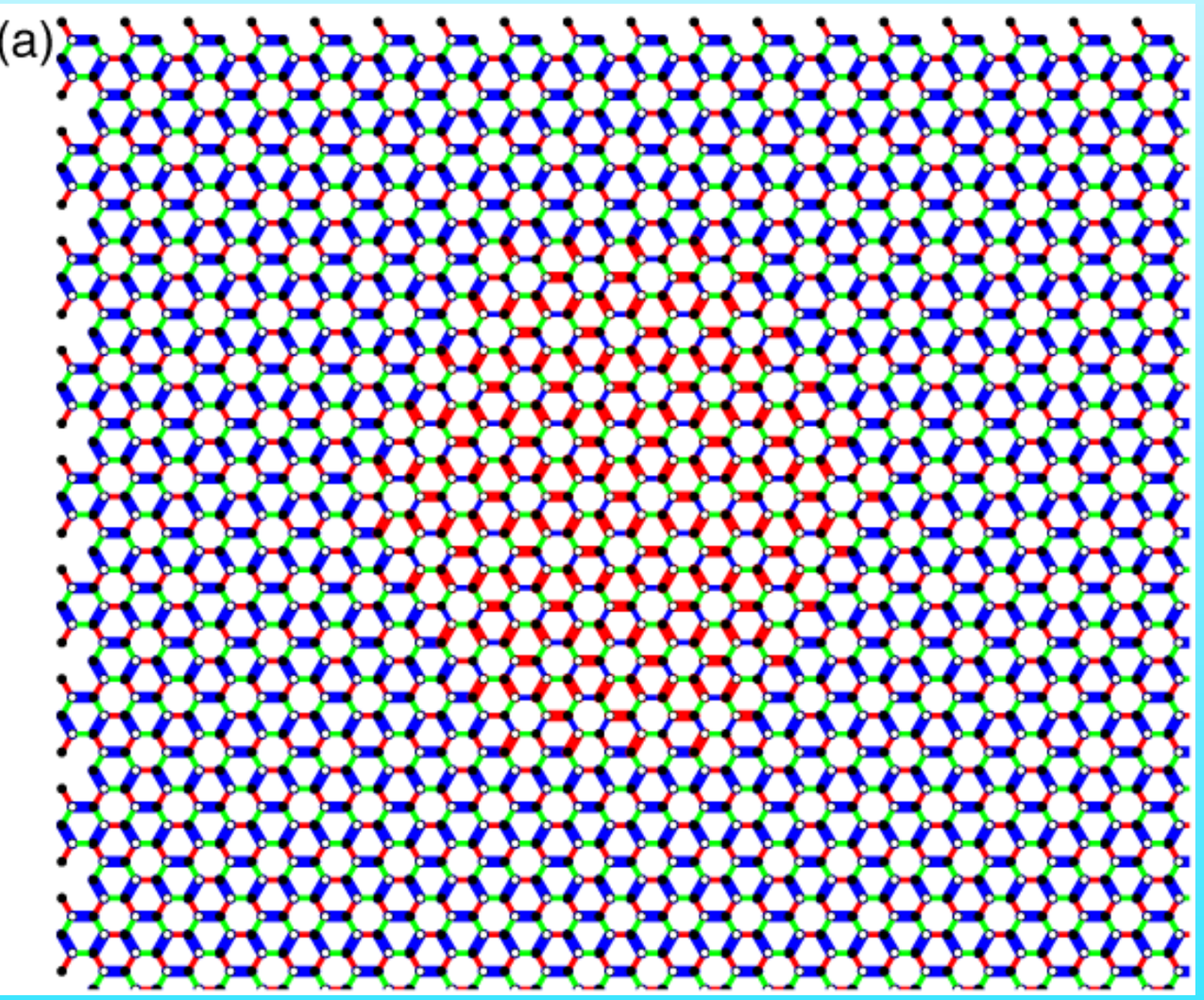


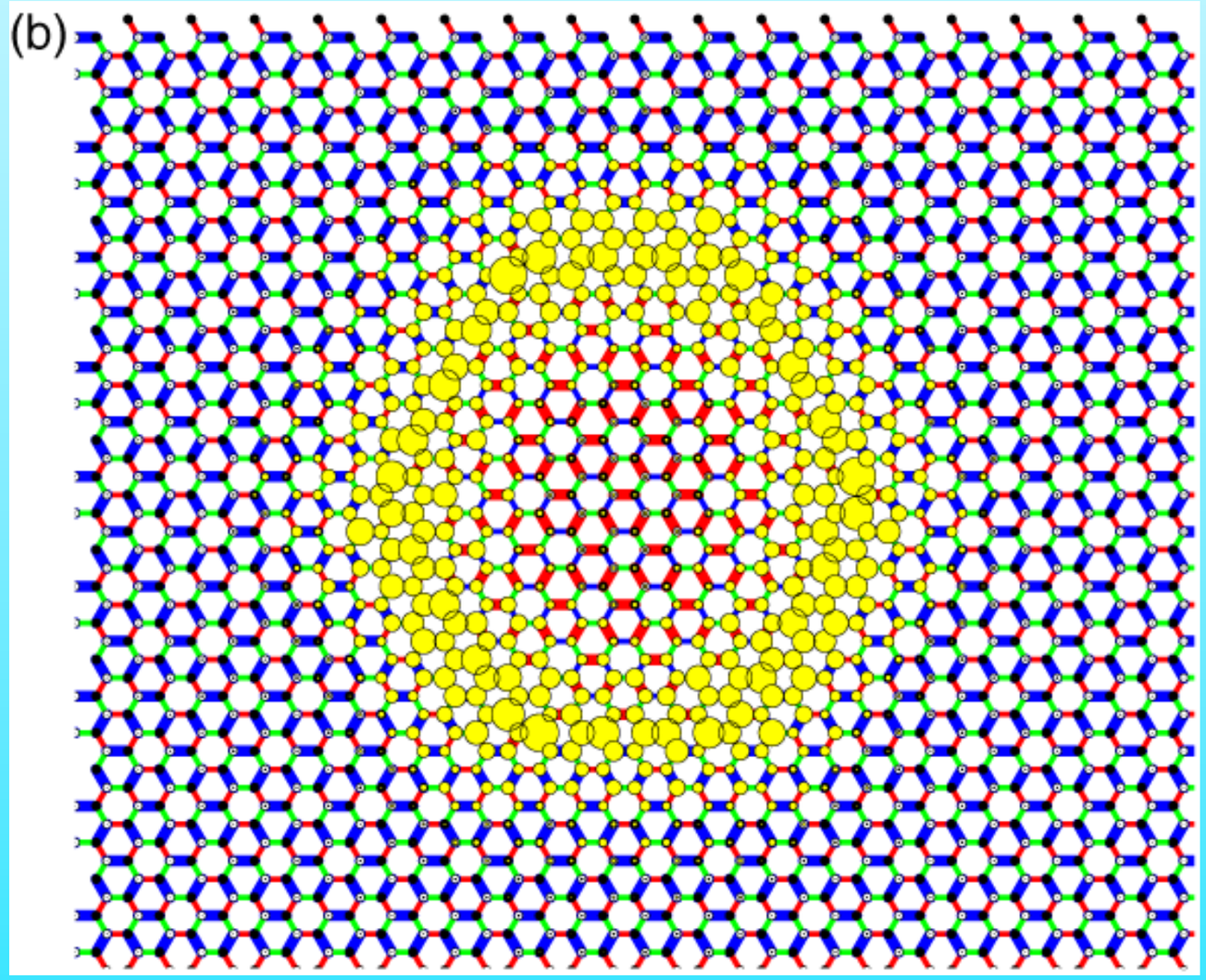
Bond-Ordering with Domains (circular domain)

- ☆Closed loop domain boundaries
- ☆A possible snap shot configuration of the dynamical bond-order

Domain shape (closed loop)

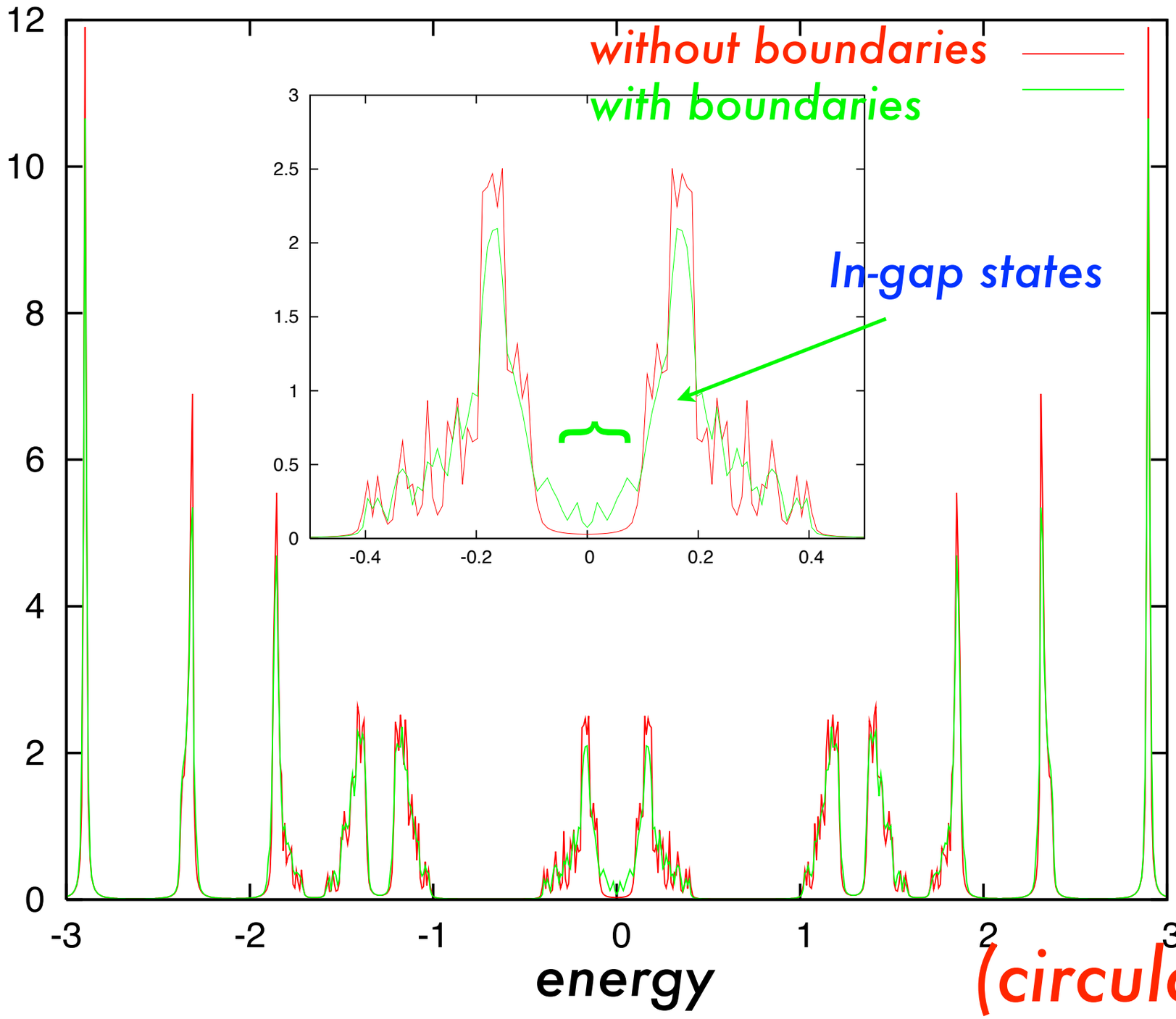
Charge Density of the in-gap states





In-gap states between the split E=0 Landau Level

One particle Density of states



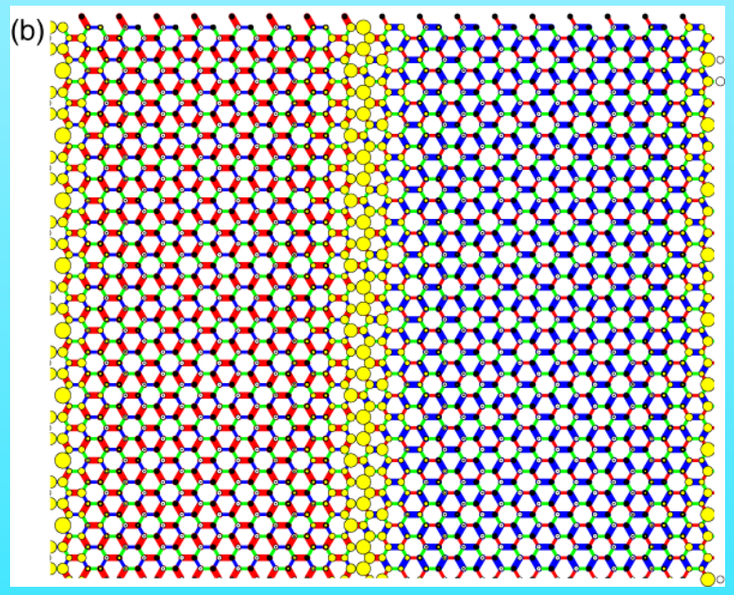
(circular domain)

True ground state of graphene
with interaction

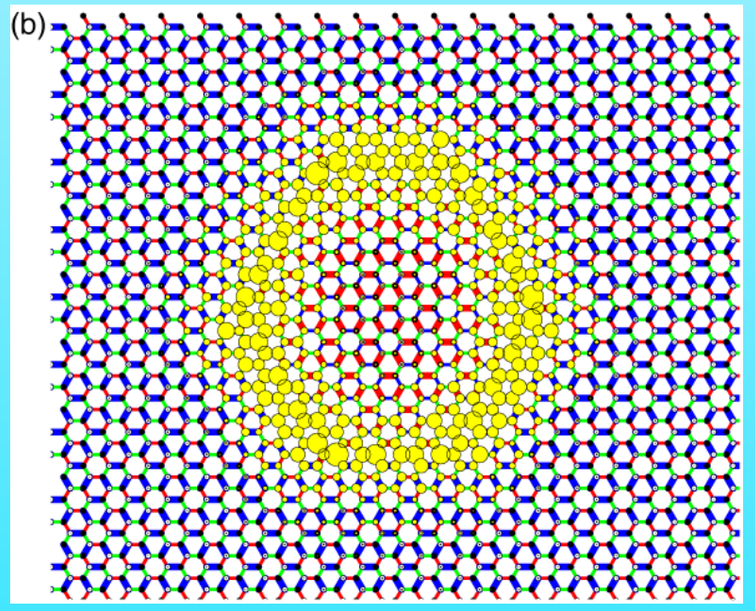
edge states as 2D analogue of the solitons in 1D

edge states as basic objects to condense

Quantum liquid
by the condensation of the edge states?



Snapshots ?



Y. Hatsugai, T. Fukui, H. Aoki, arXiv:0804.4762, Physica E 40, 1530 (2008).

Summary

- Topological Aspects of Graphene
  - QHE by the Bulk
  - QHE by the edge
  - Bulk – Edge Correspondence
- Another Edge states of Graphene
  - without magnetic field
  - with magnetic field :
    Coexistence of the Bulk and edge states at E=0
    (STM observable)
- Possible quantum liquids with bond order
  - As a condensate of the loops by the edge states