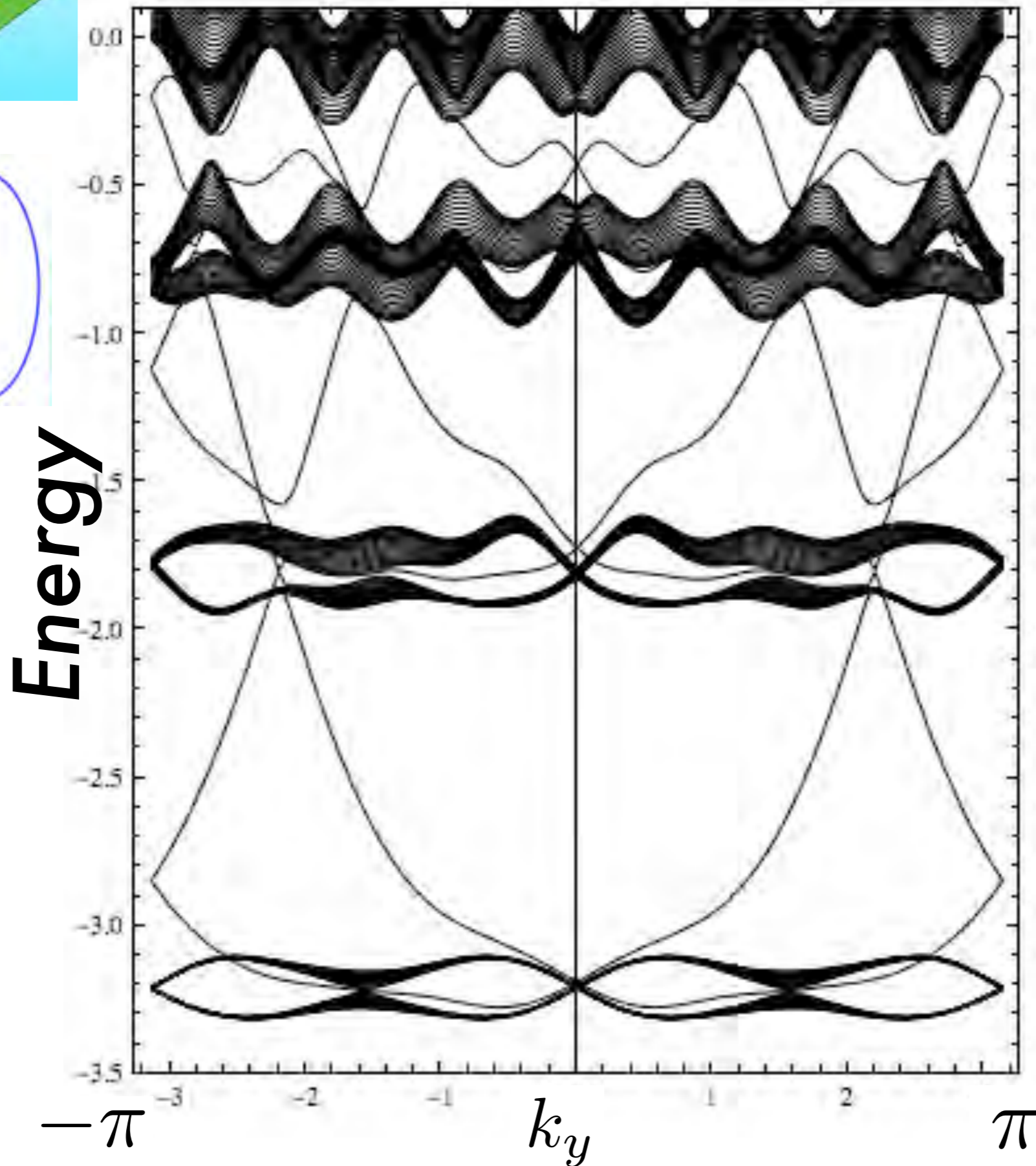
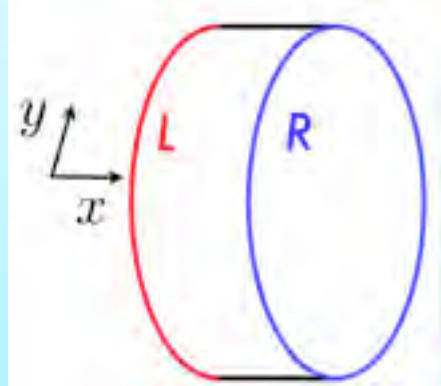
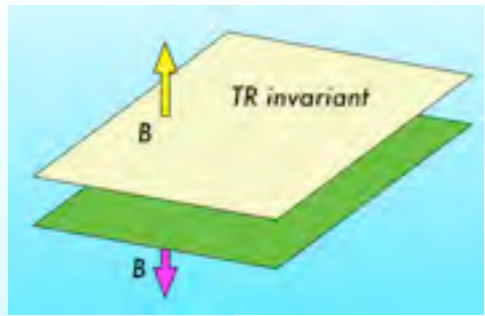


- ★ *Edge states of  $Z_2$  topological phase*
  - ★ *Spin conserved case*
  - ★ *chiral edge states to helical ones*
  - ★ *Kramers degeneracy*
  - ★  *$Z_2$  characterization by the edge states*

# What's this 2D Quantum Spin Hall state



Edge states are not **chiral**, but **helical**

$$\Theta H(k) \Theta^{-1} \cong H(-k)$$

$$+k \longleftrightarrow -k$$

Not independent  
Correlated

Generically  
TR is broken in  
momentum space

TR is OK  
at special momentum

**$\pi$**  and  **$0$**

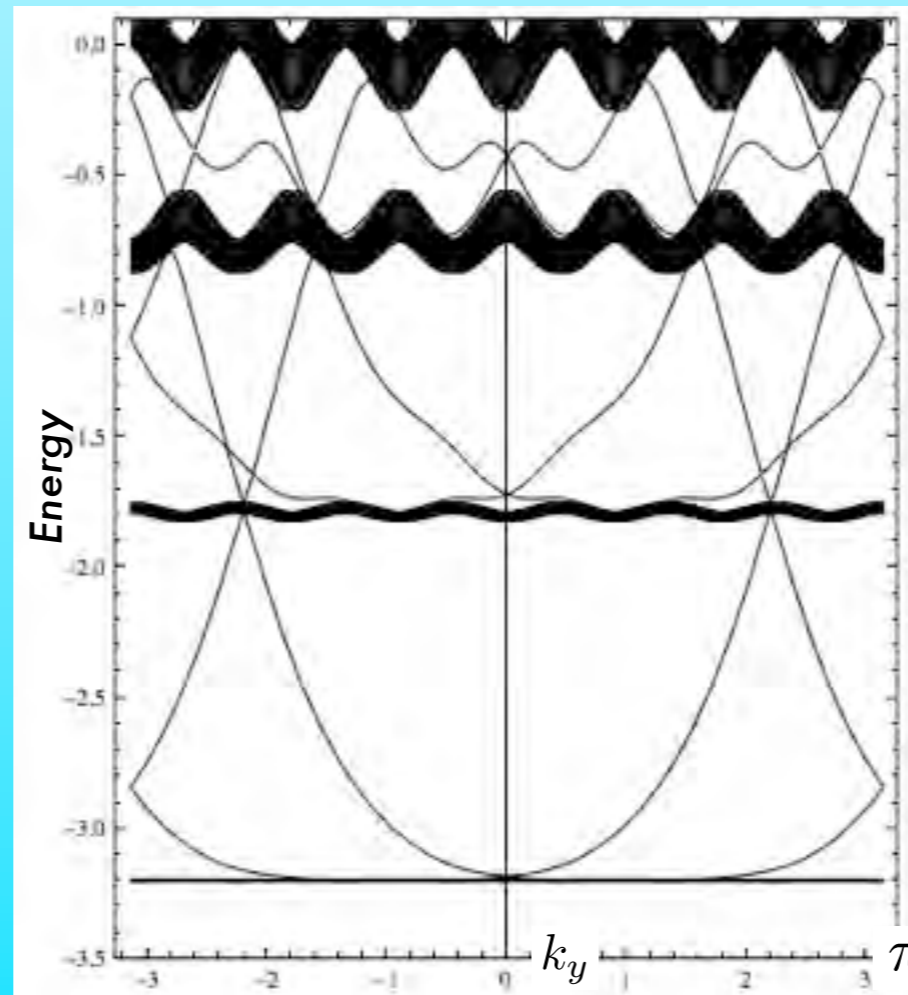
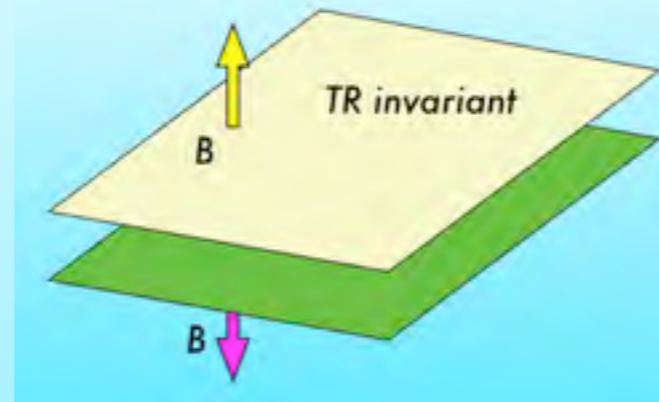
$$H(0) = H(-0)$$

$$H(\pi) = H(-\pi)$$

# Identification of edge states (QSHE)

$Z_2$  edge states

*spin conserved case*

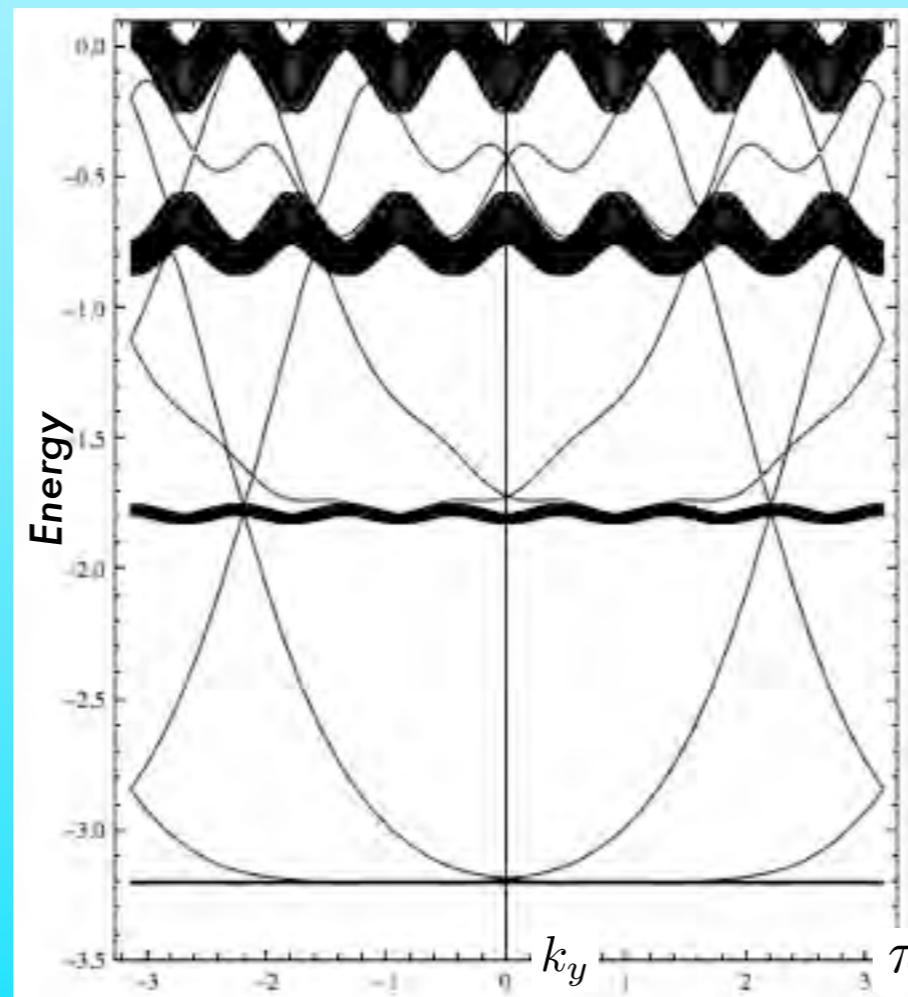
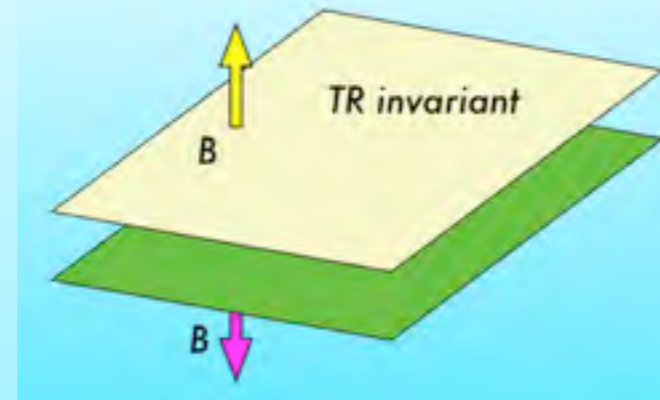


# Identification of edge states (QSHE)

$Z_2$  edge states

spin conserved case

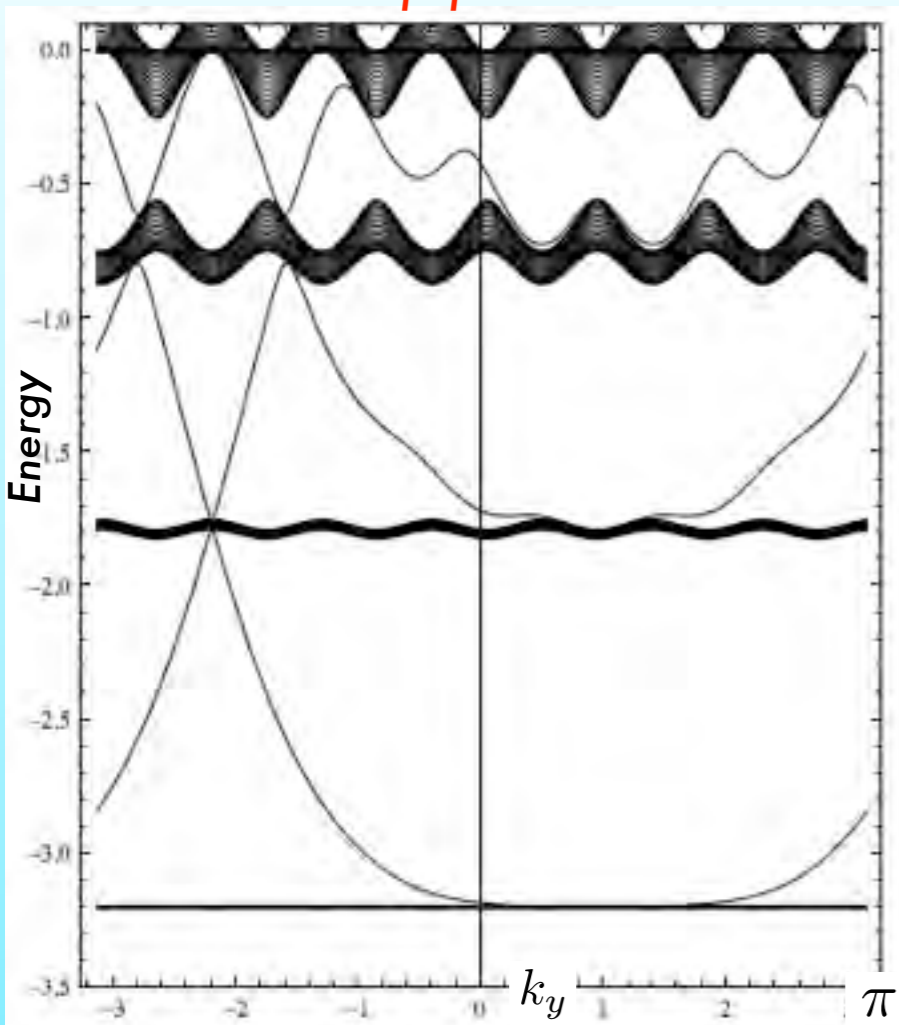
decompose into up & down



# Identification of edge states (QSHE)

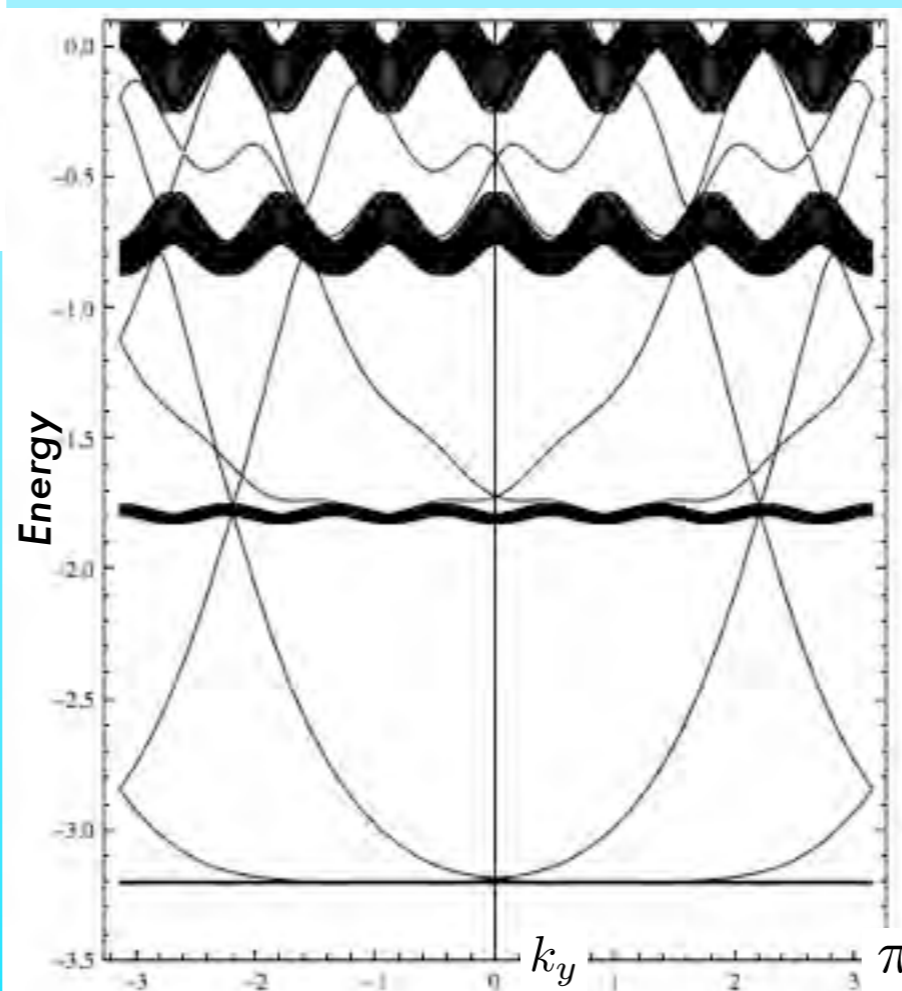
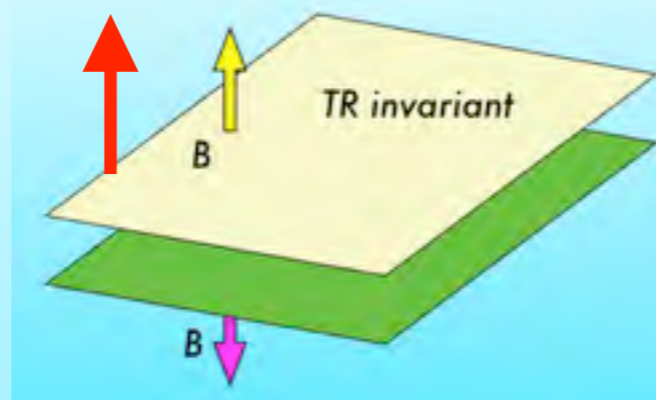
$Z_2$  edge states

upspin



spin conserved case

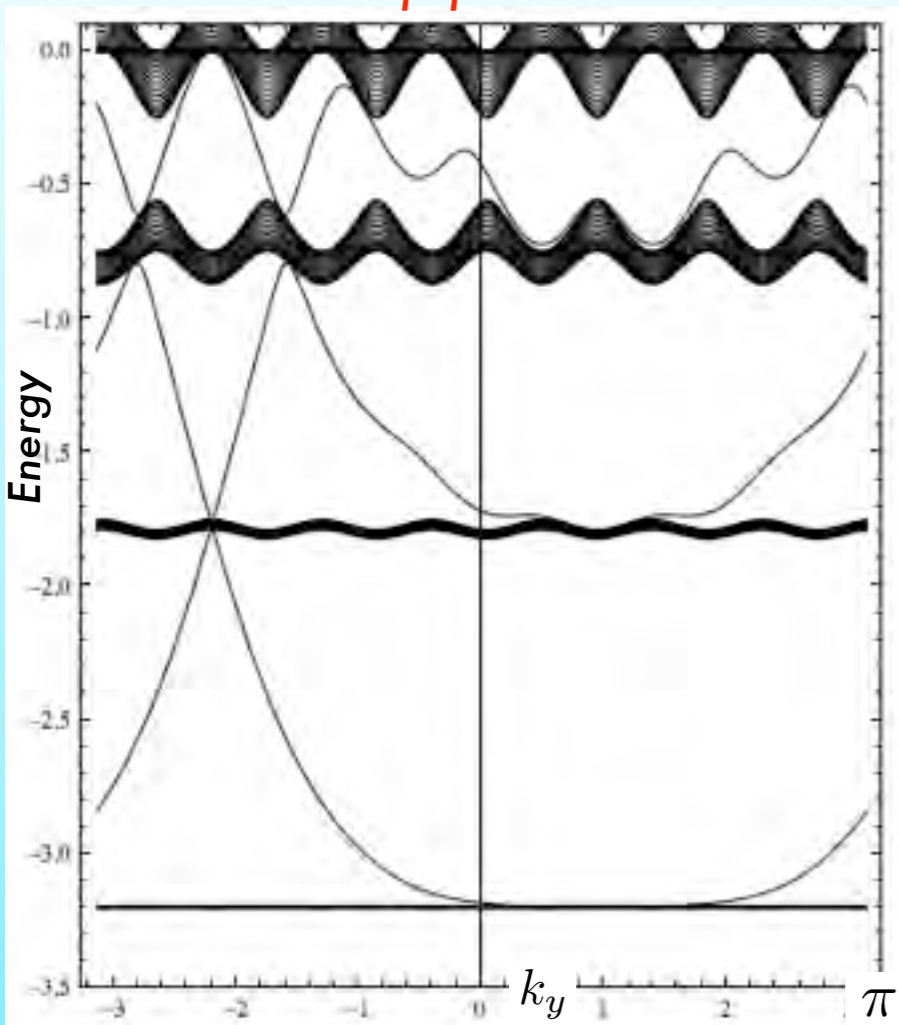
decompose into up & down



# Identification of edge states (QSHE)

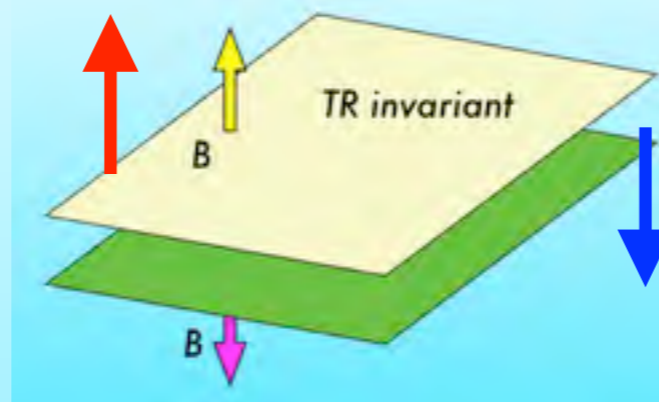
$Z_2$  edge states

upspin

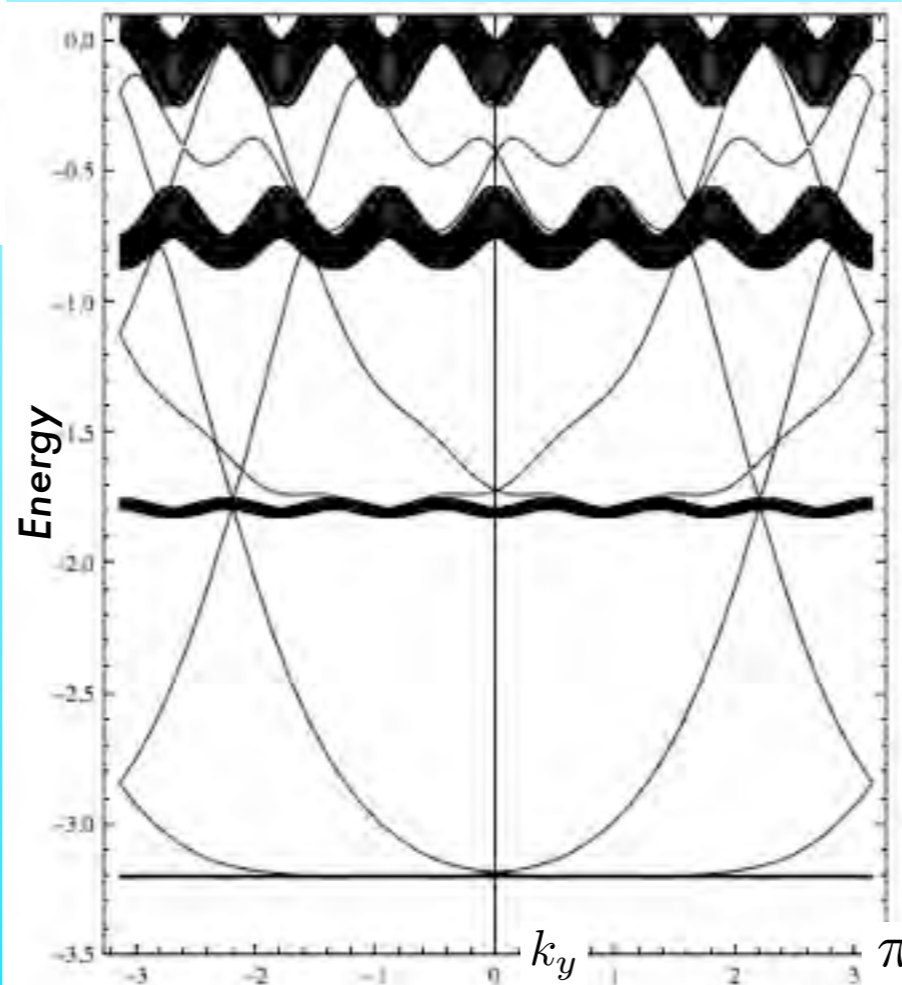
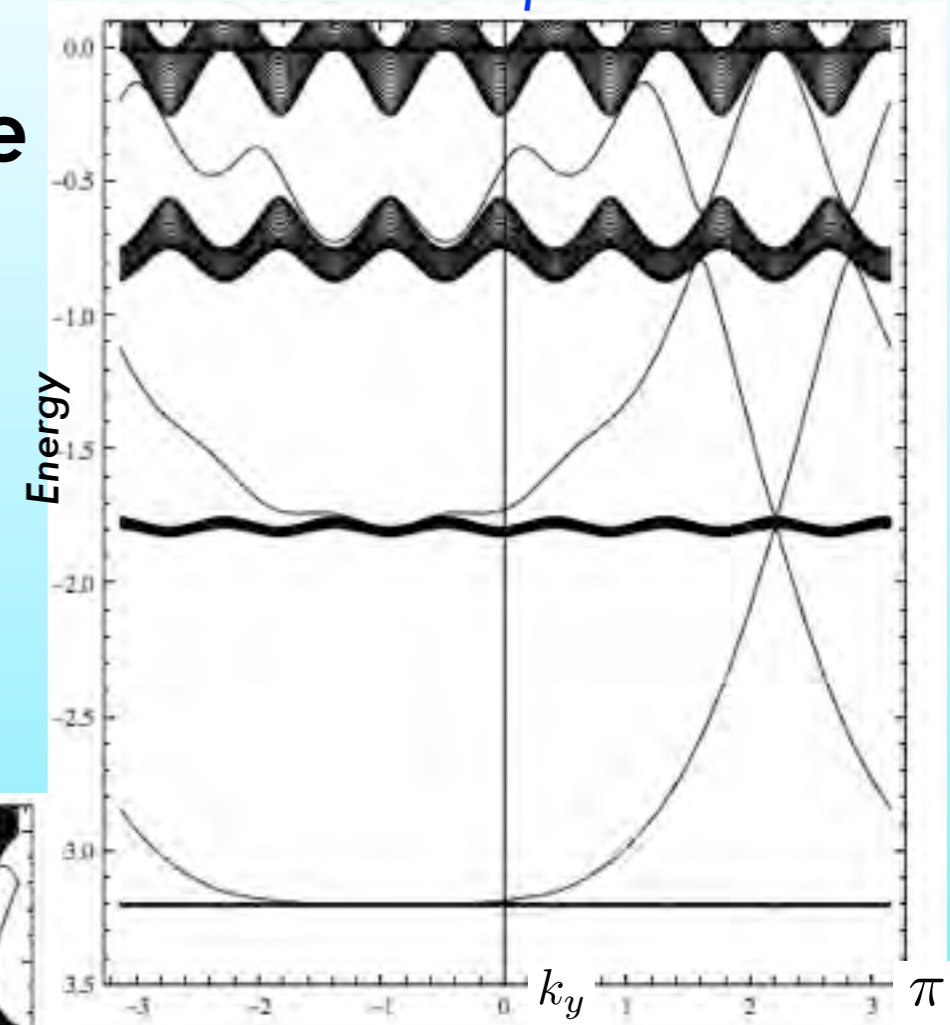


spin conserved case

decompose into up & down



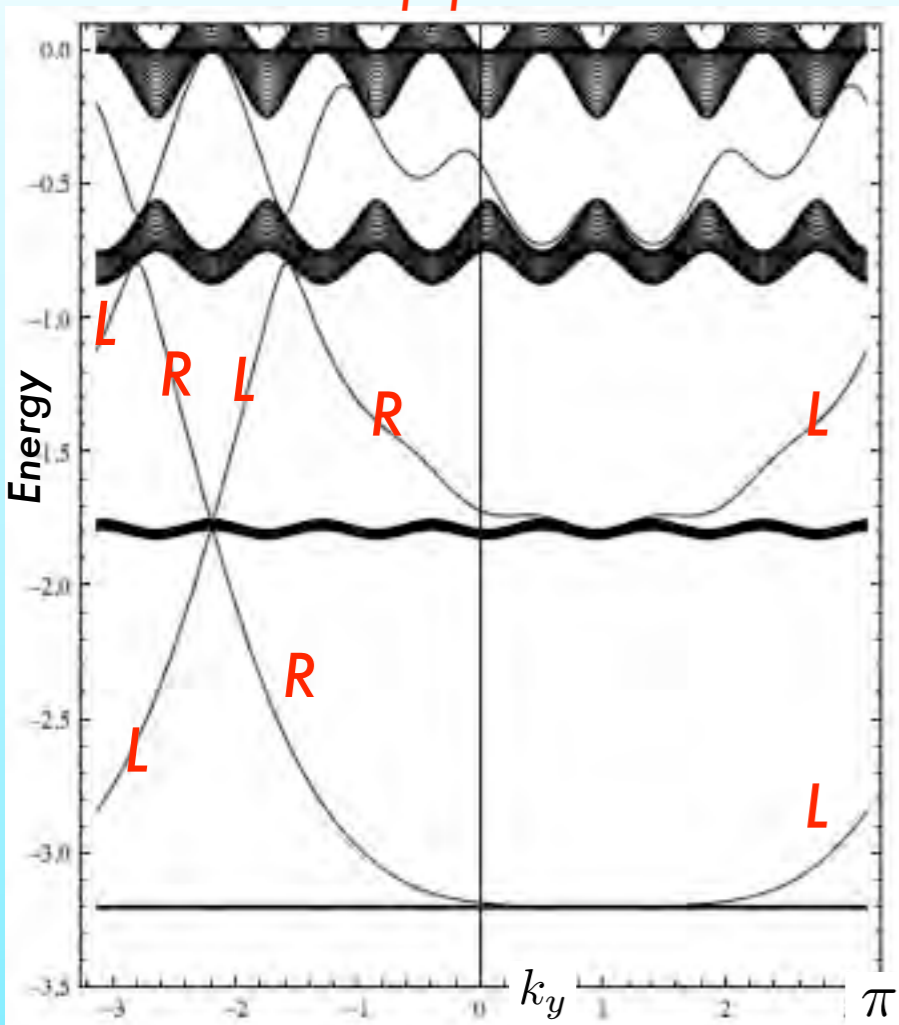
down spin



# Identification of edge states (QSHE)

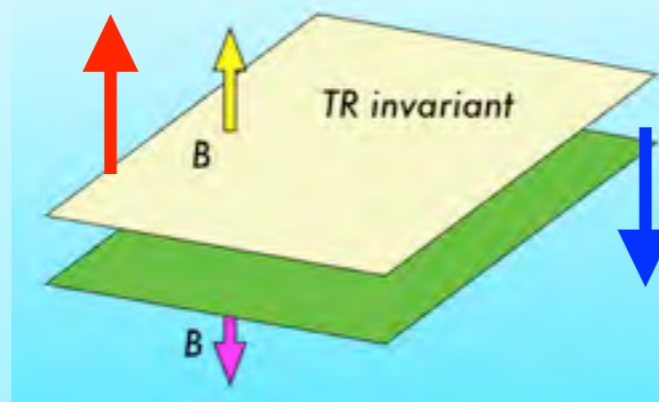
$Z_2$  edge states

upspin

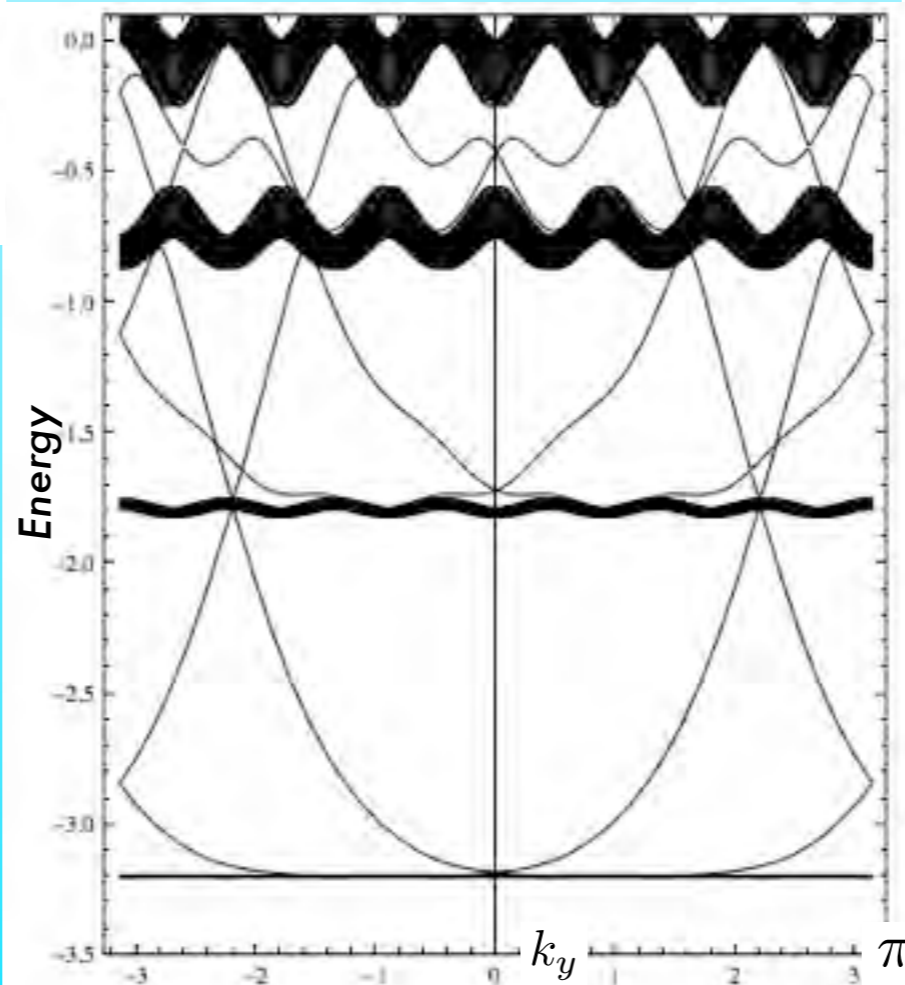
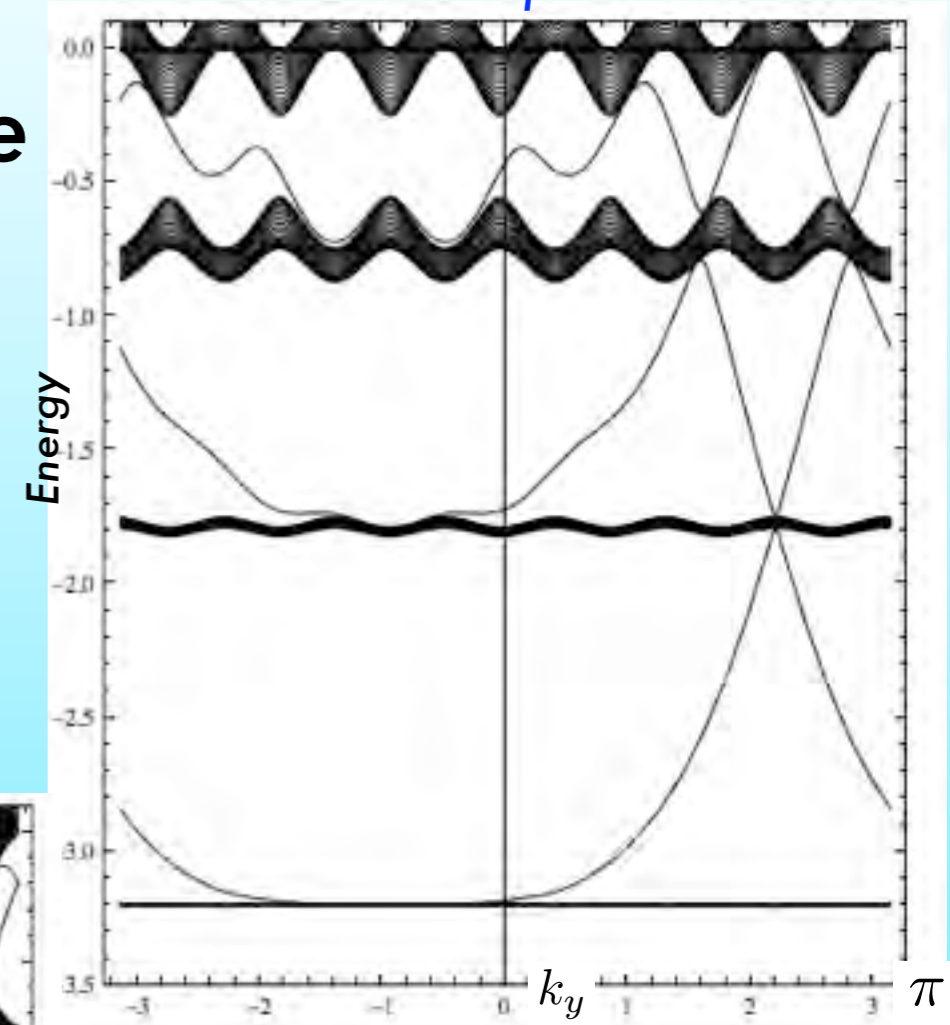


spin conserved case

decompose into up & down



down spin

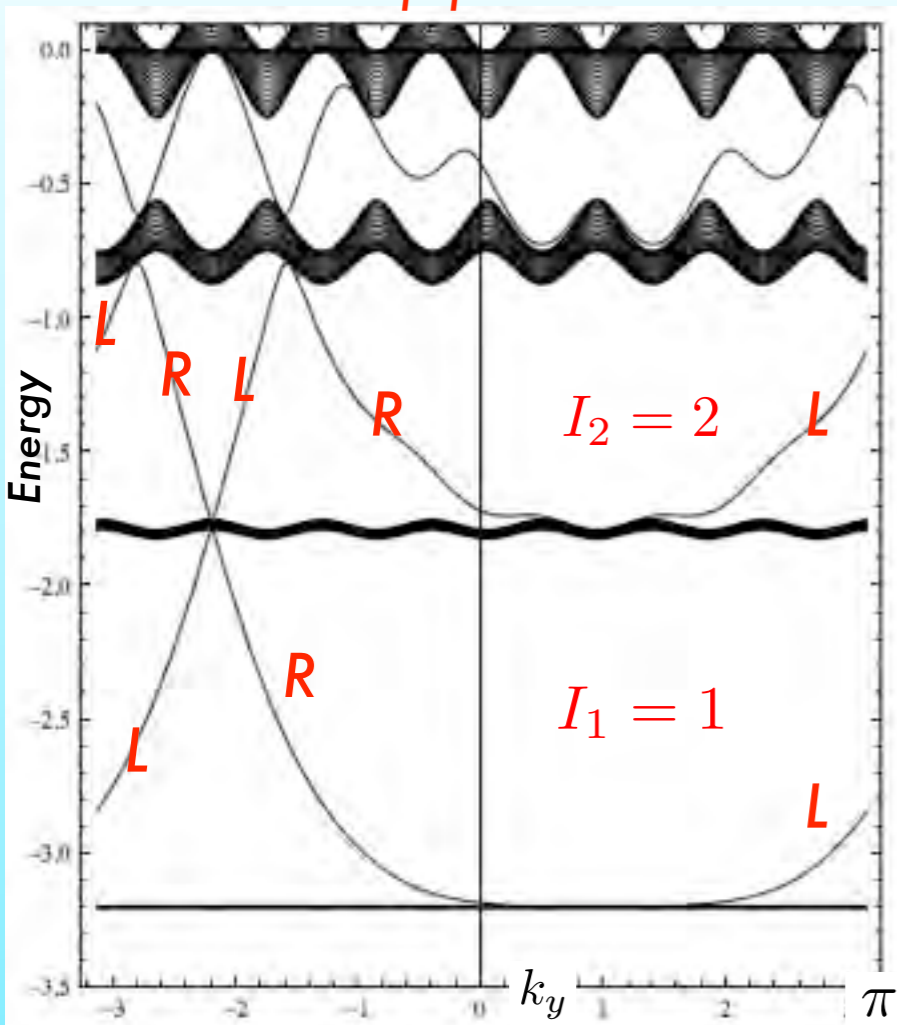


# Identification of edge states (QSHE)

$Z_2$  edge states

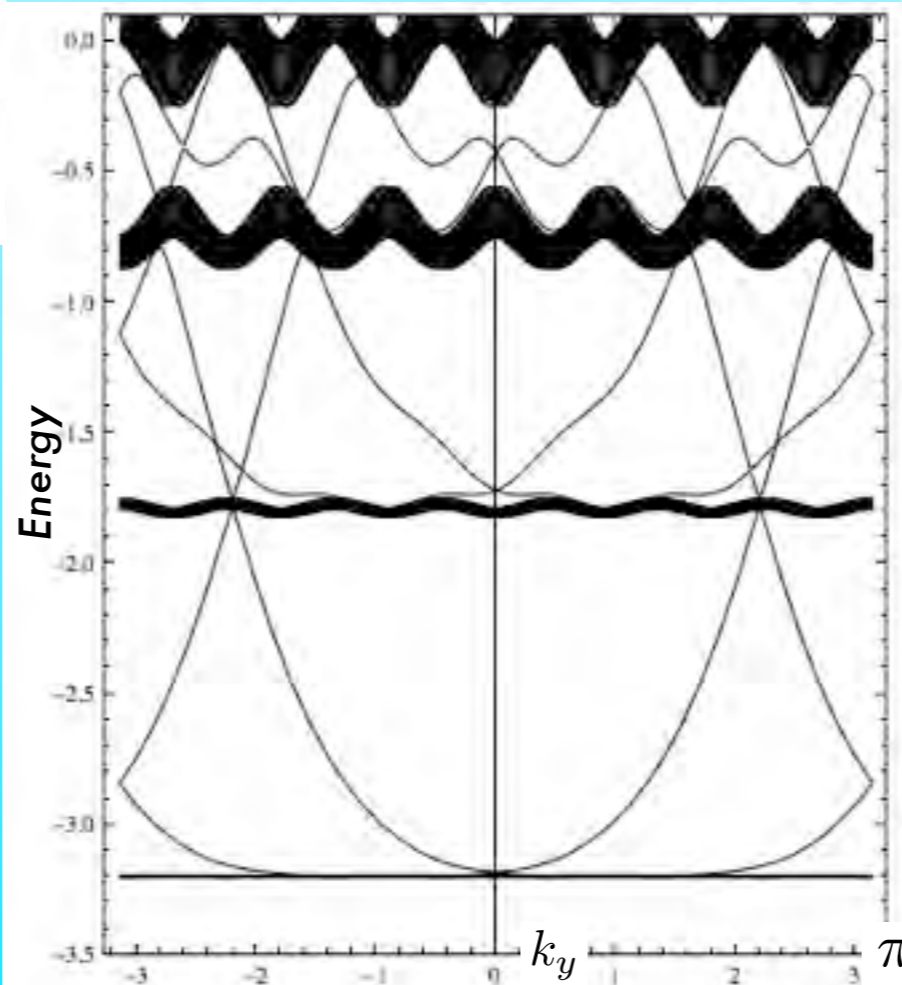
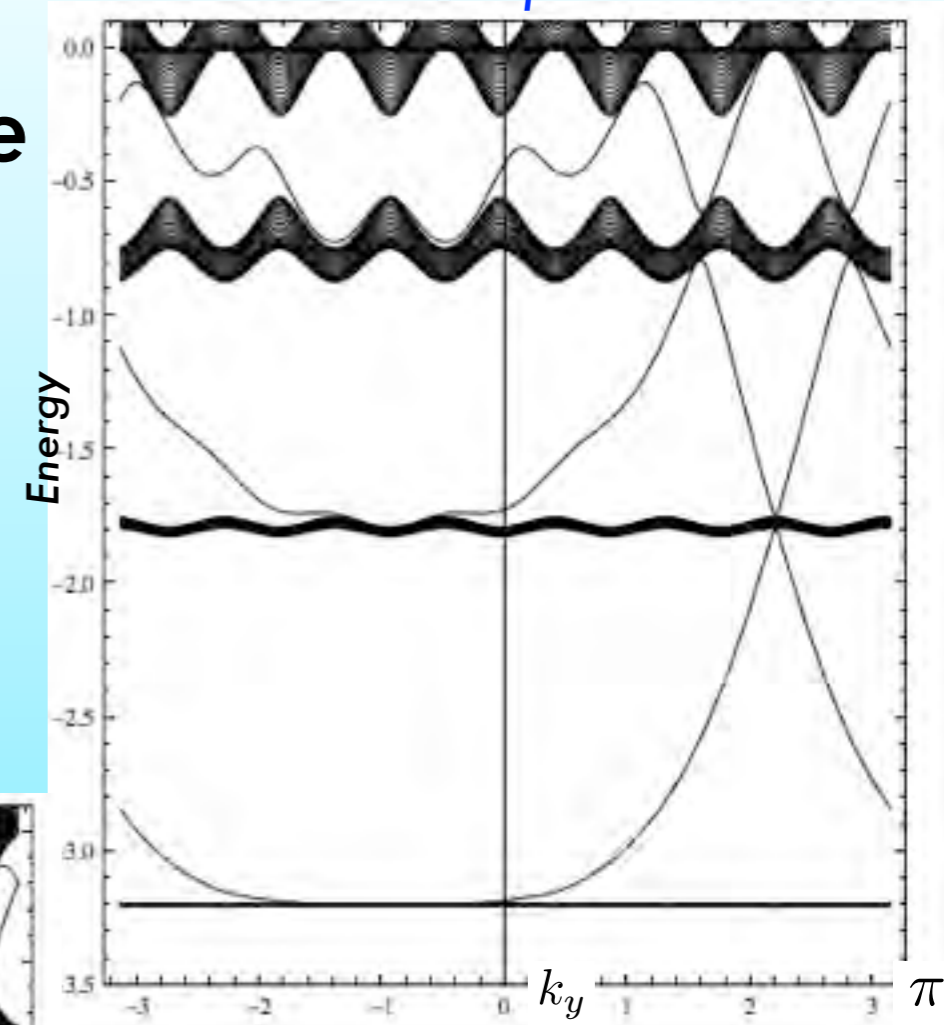
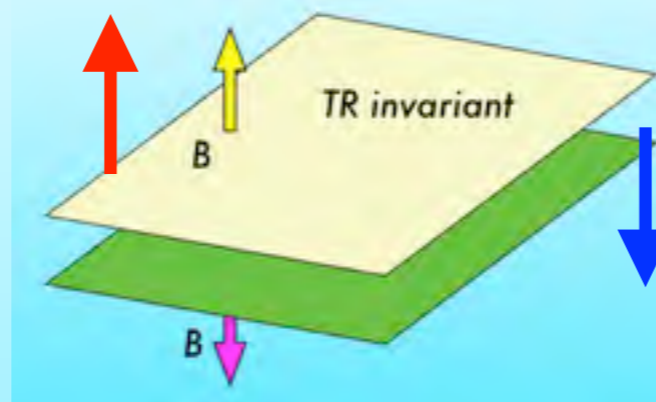
upspin

down spin



spin conserved case

decompose into up & down

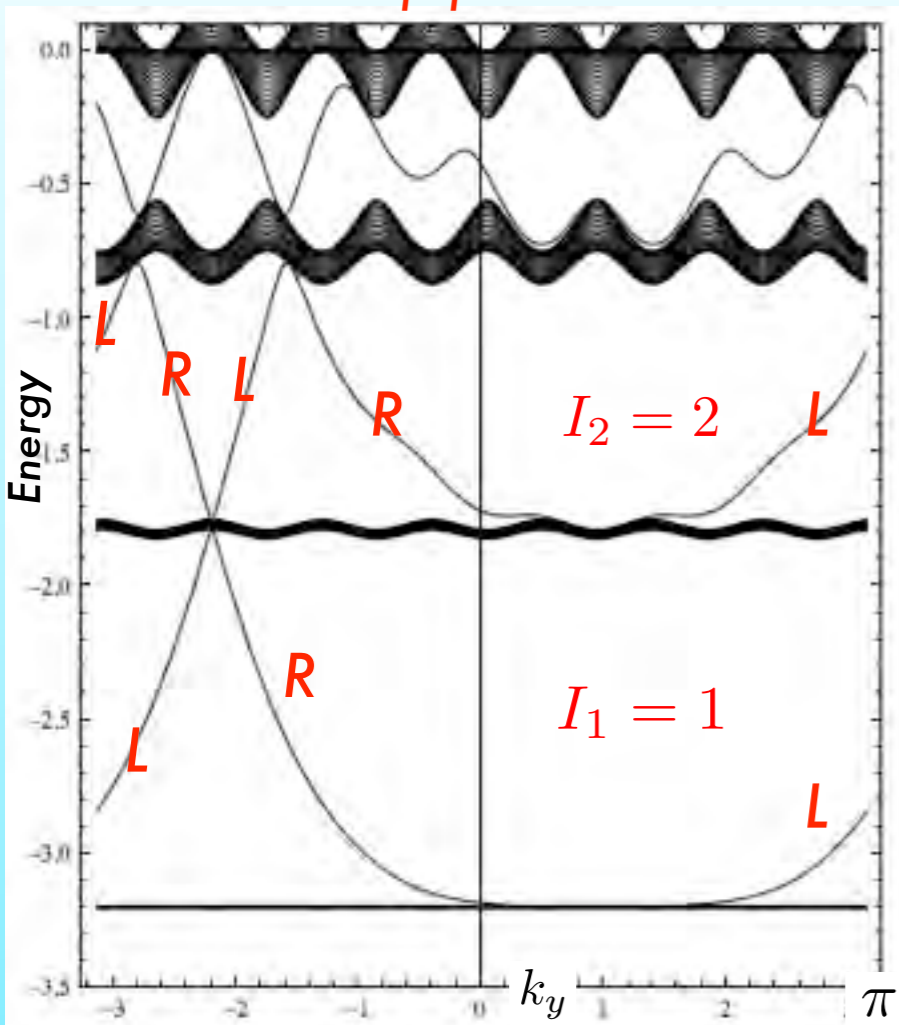




# Identification of edge states (QSHE)

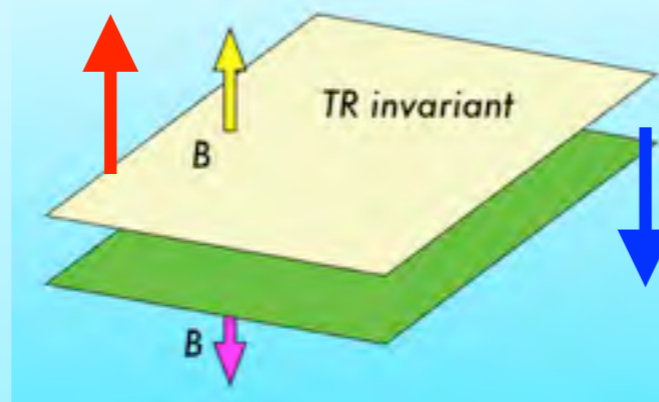
$Z_2$  edge states

upspin

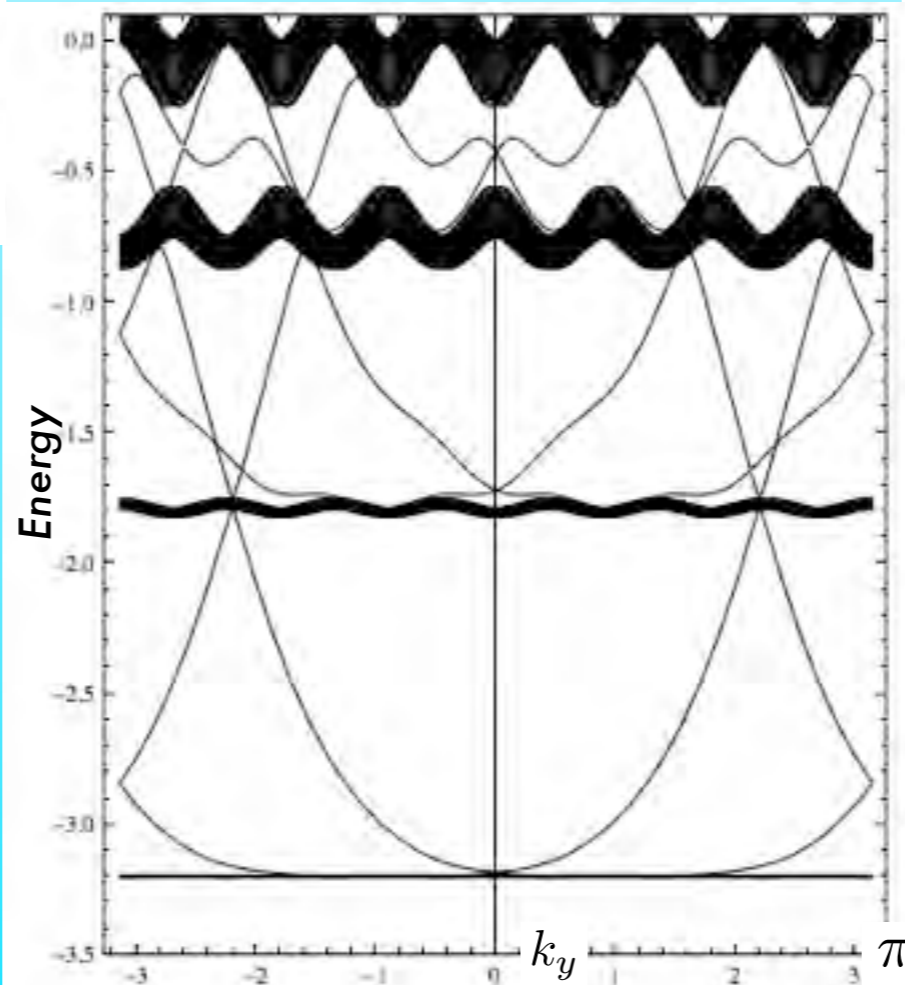
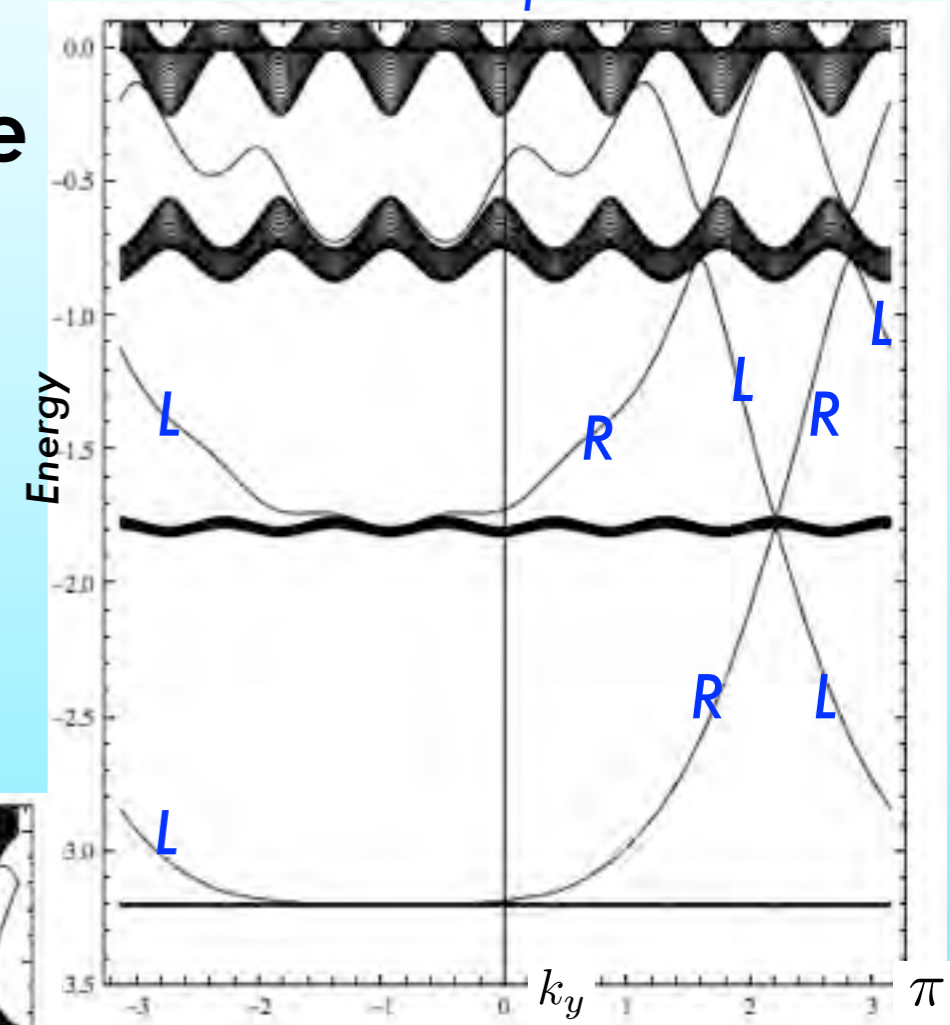


spin conserved case

decompose into up & down



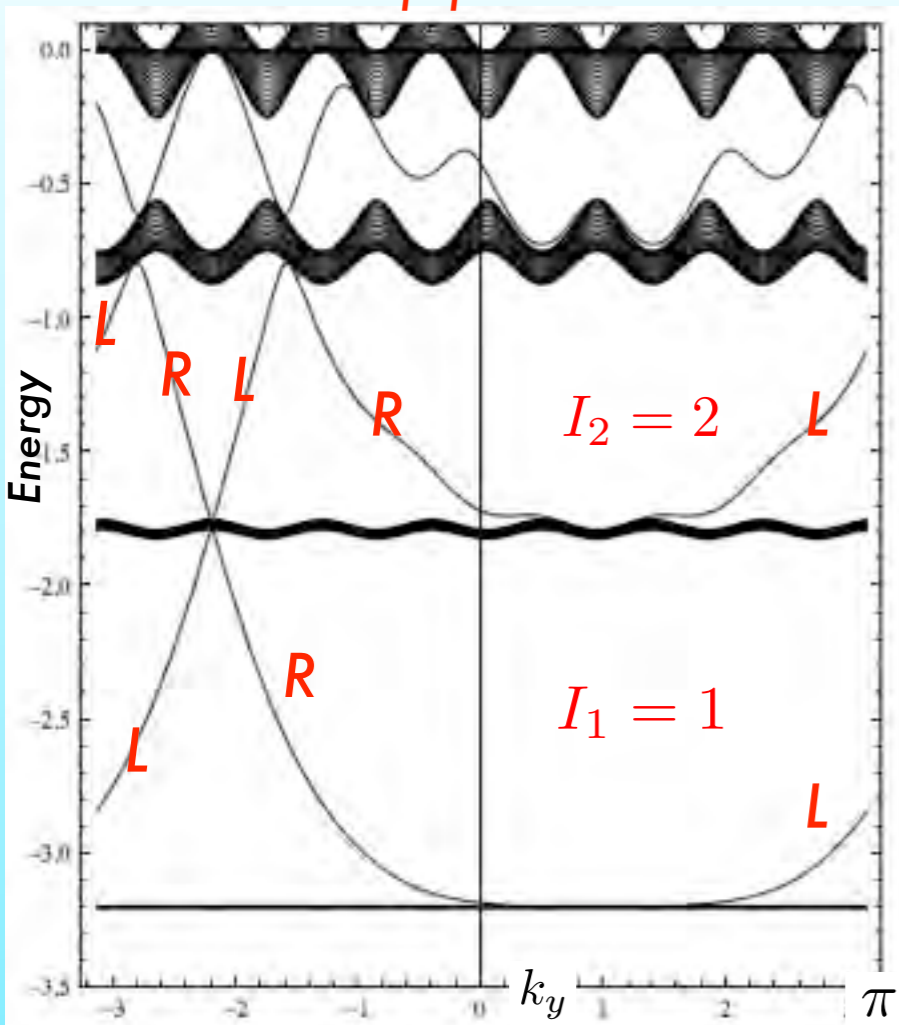
down spin



# Identification of edge states (QSHE)

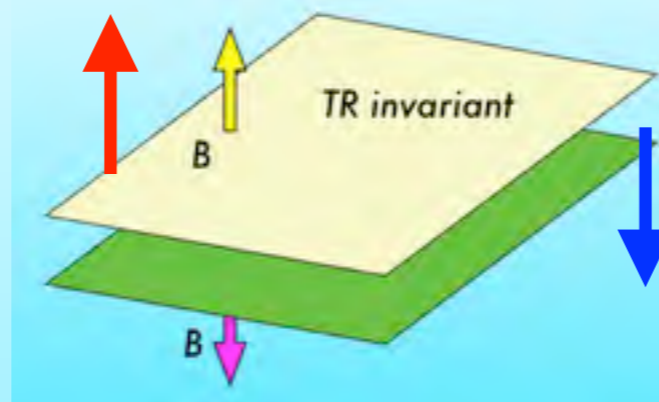
$Z_2$  edge states

upspin

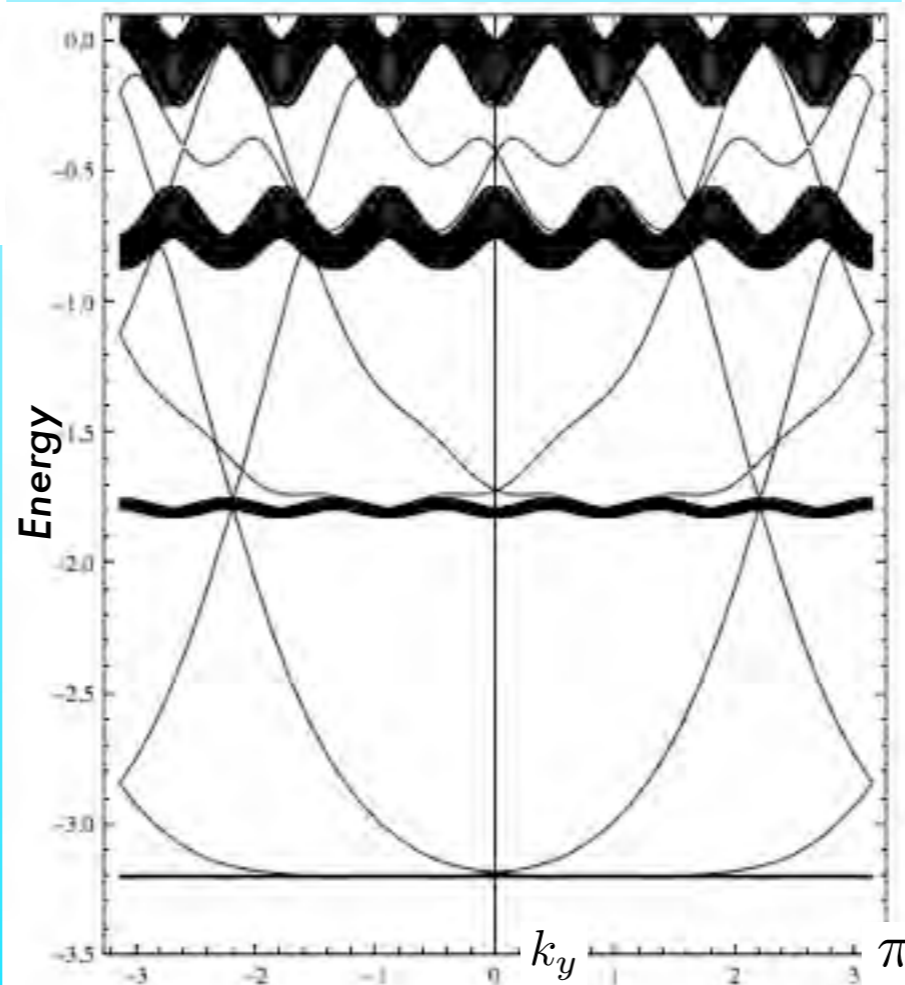
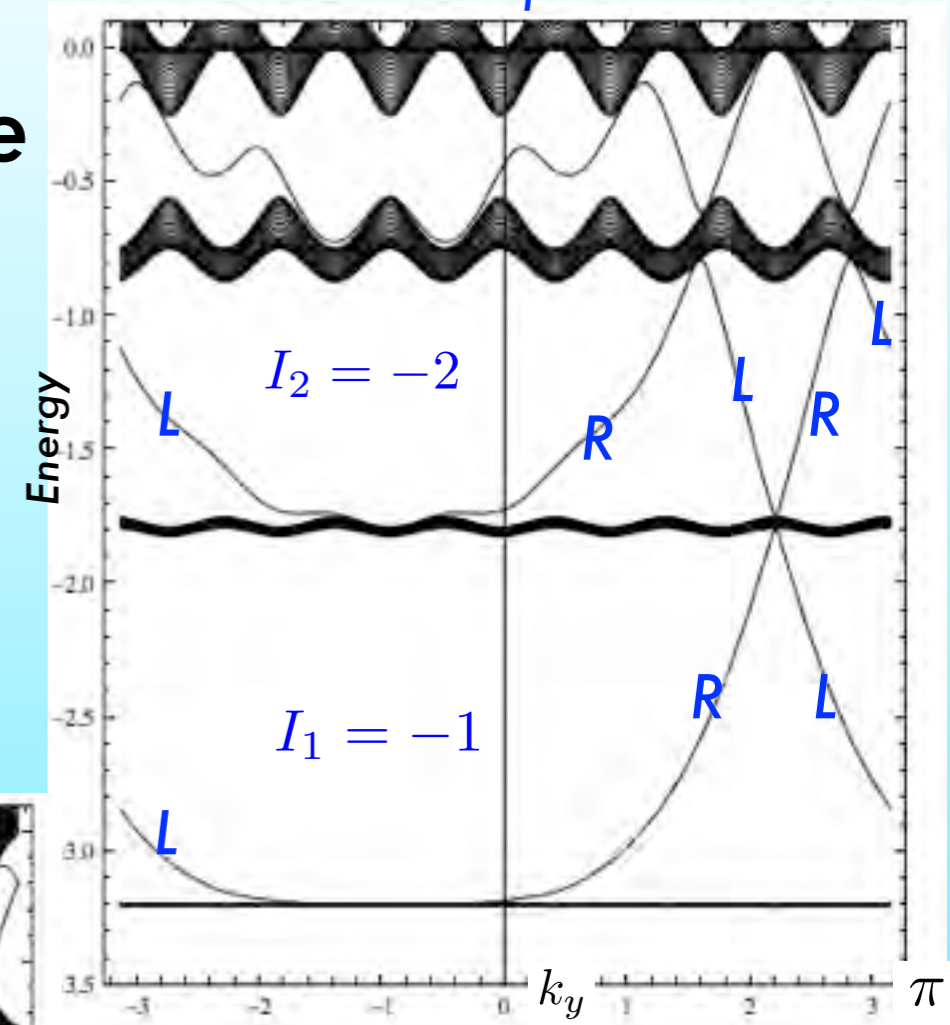


spin conserved case

decompose into up & down



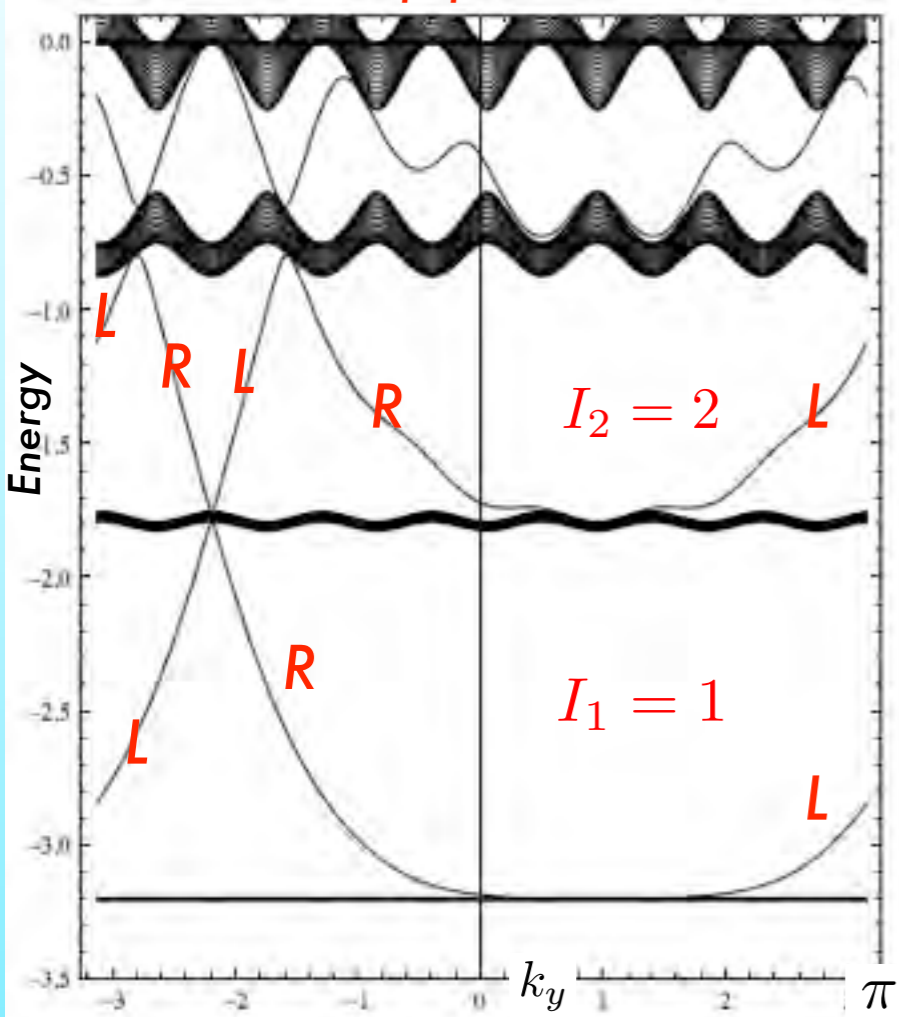
down spin



# Identification of edge states (QSHE)

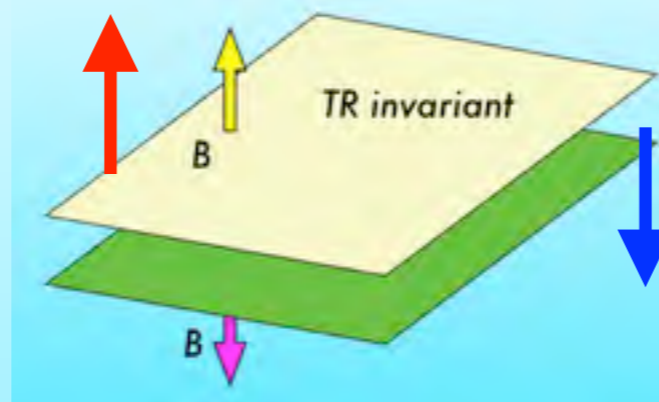
$Z_2$  edge states

upspin

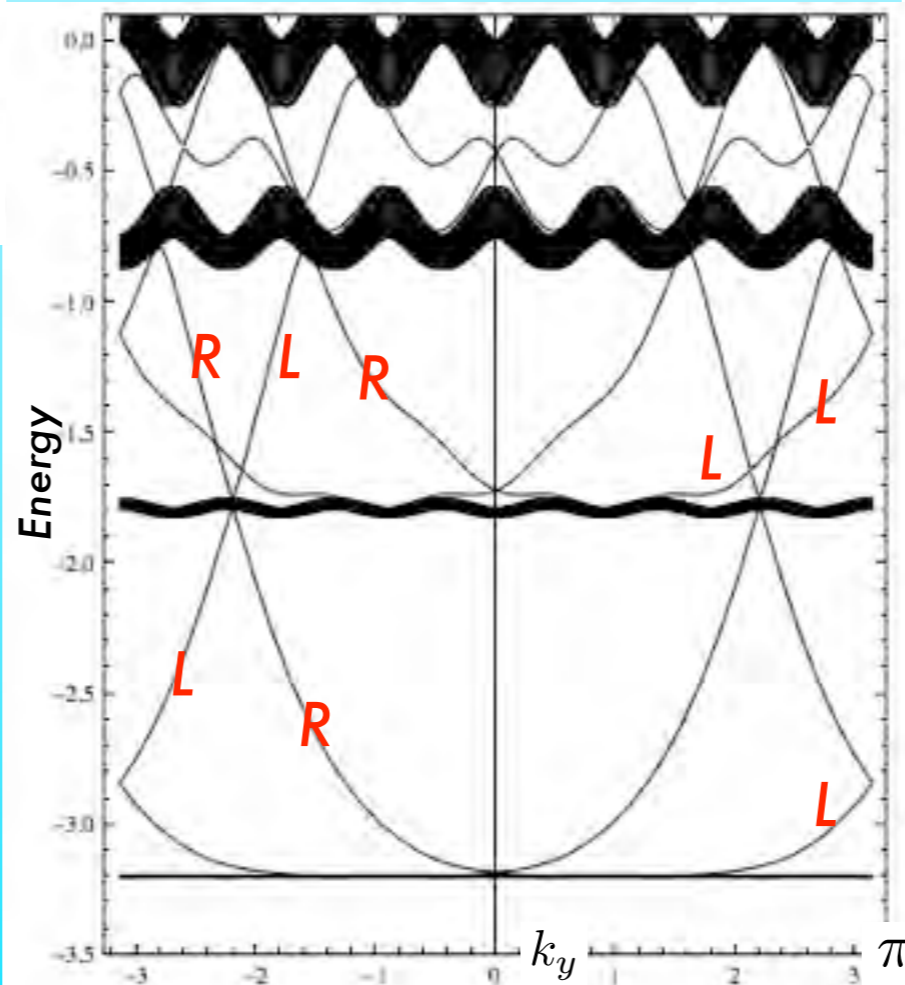
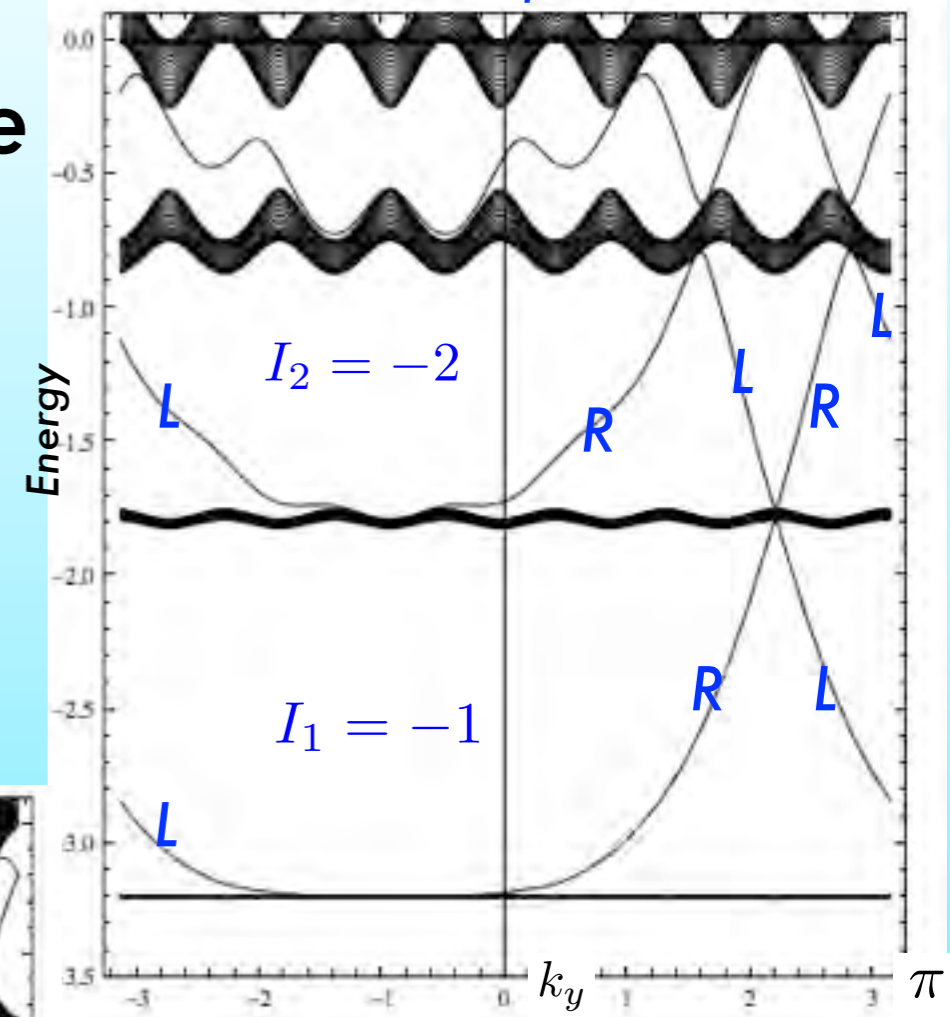


spin conserved case

decompose into up & down



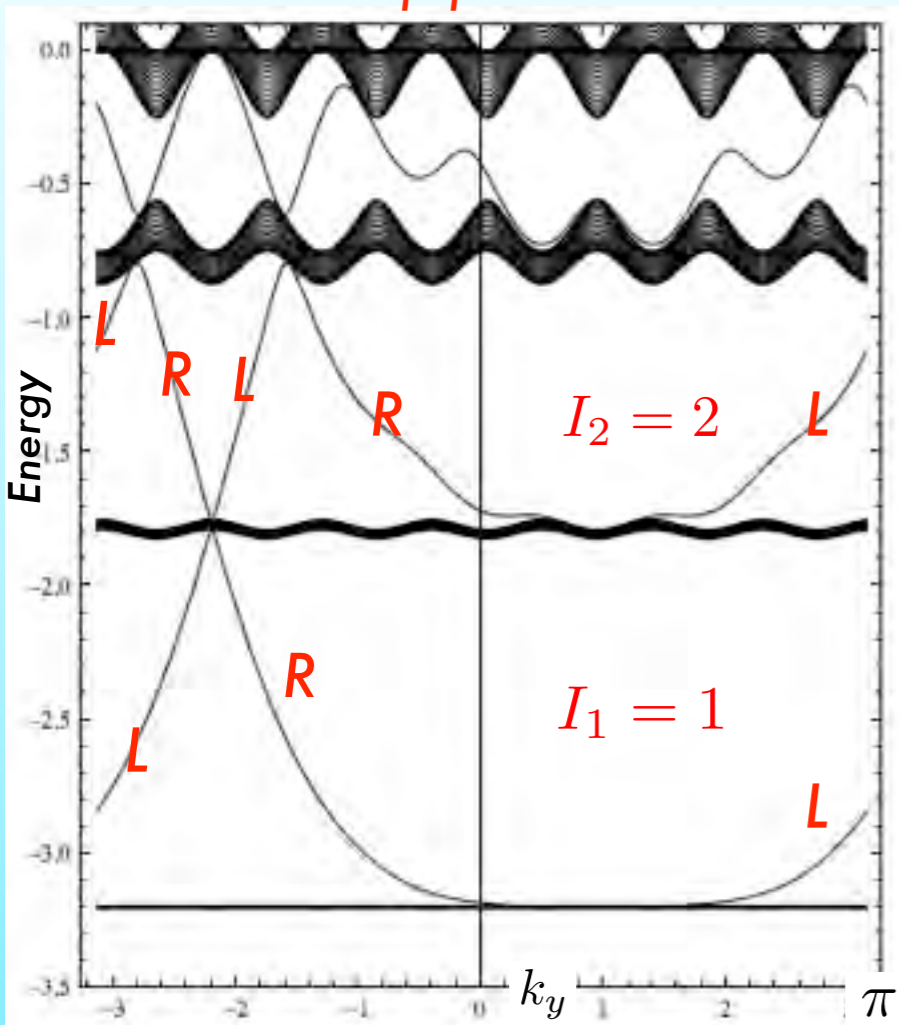
down spin



# Identification of edge states (QSHE)

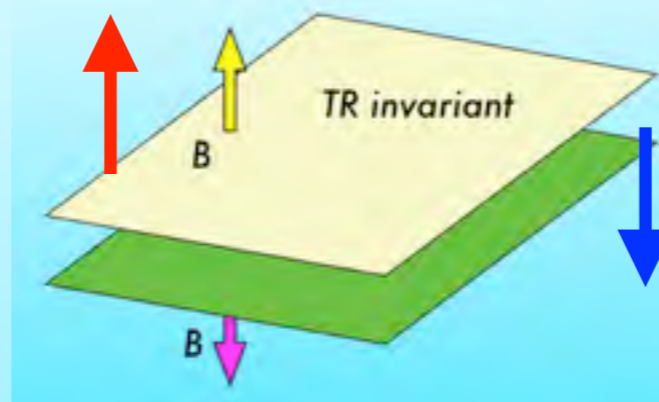
$Z_2$  edge states

upspin

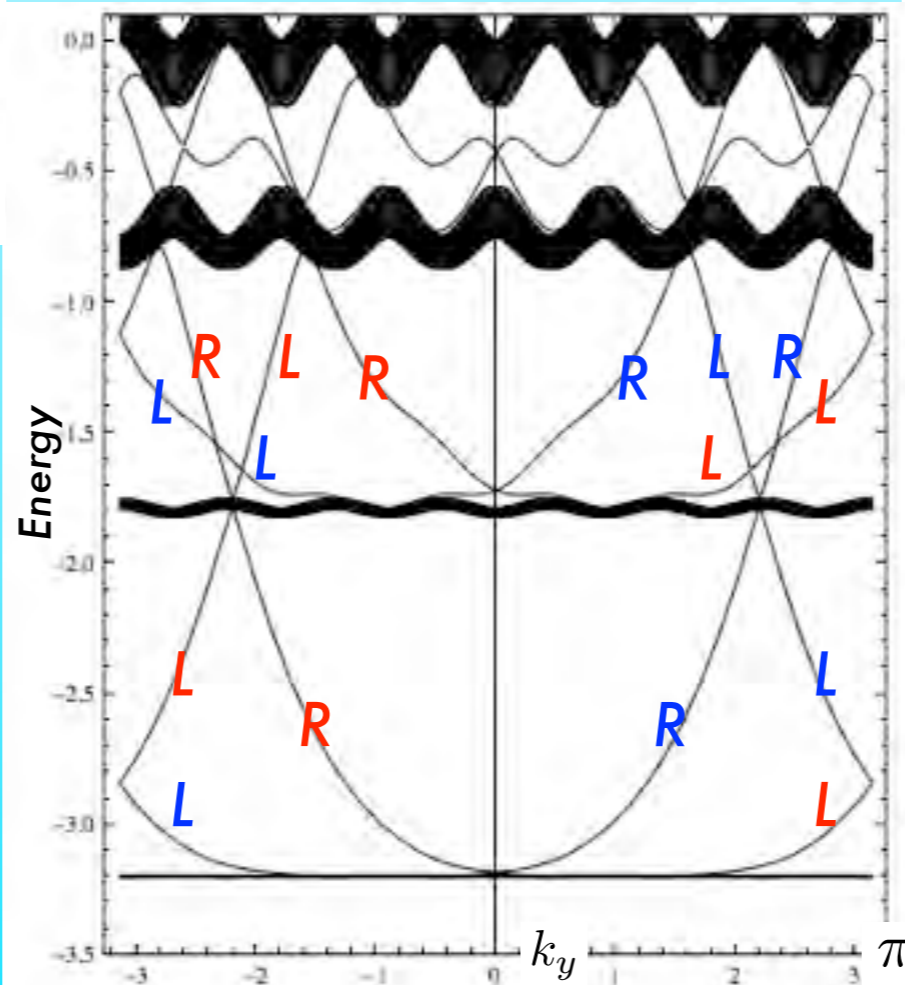
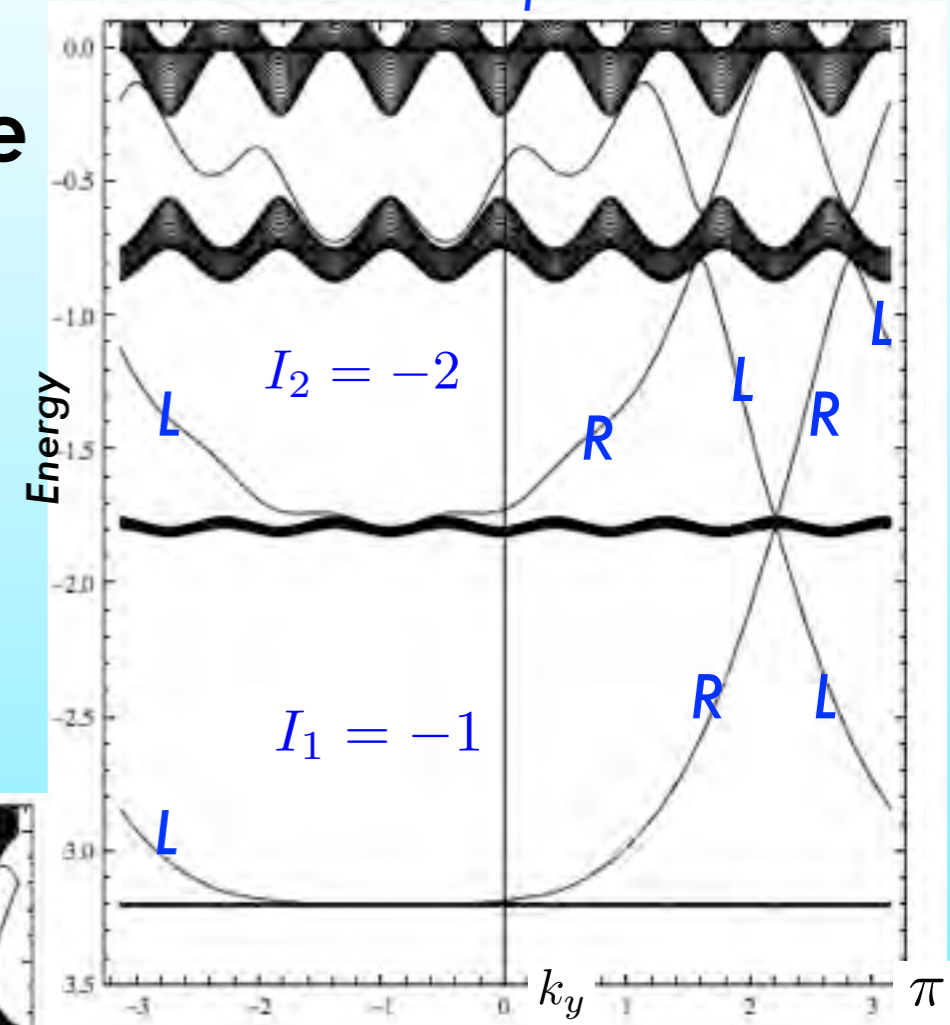


spin conserved case

decompose into up & down



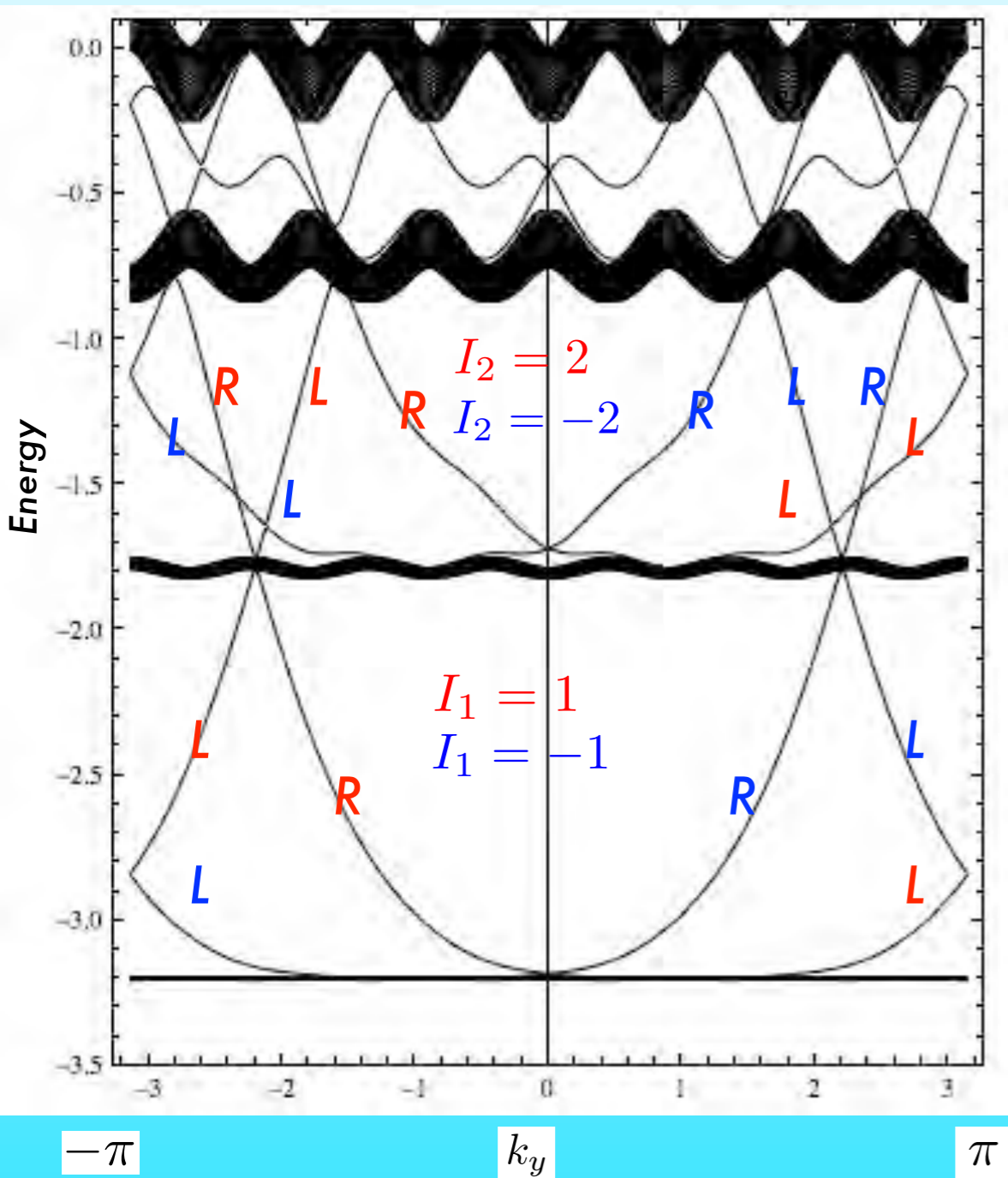
down spin



# Identification of edge states (QSHE)

$Z_2$  edge states

Spin Conserved

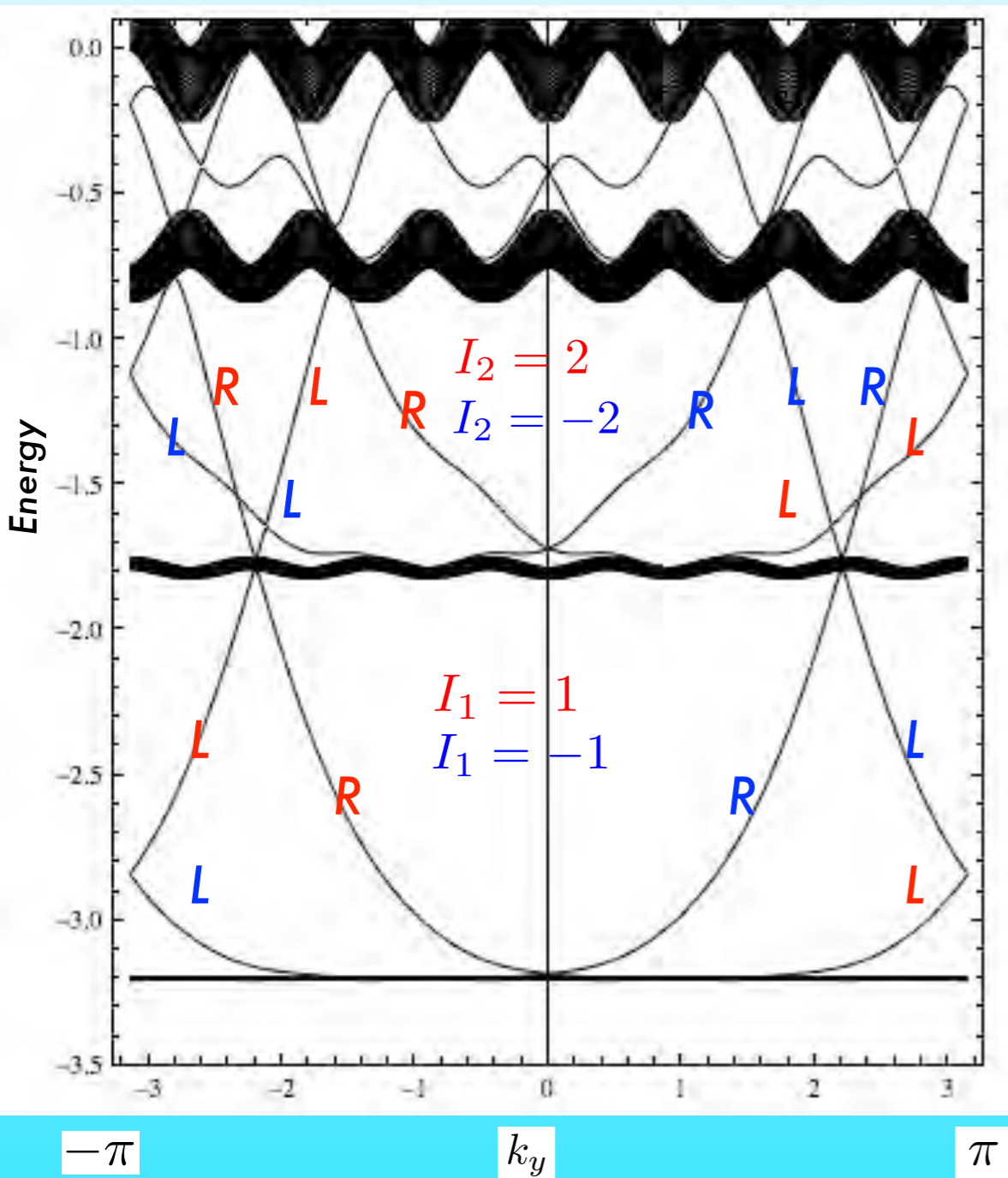


# Identification of edge states (QSHE)

$Z_2$  edge states

Spin Conserved

With Spin Orbit

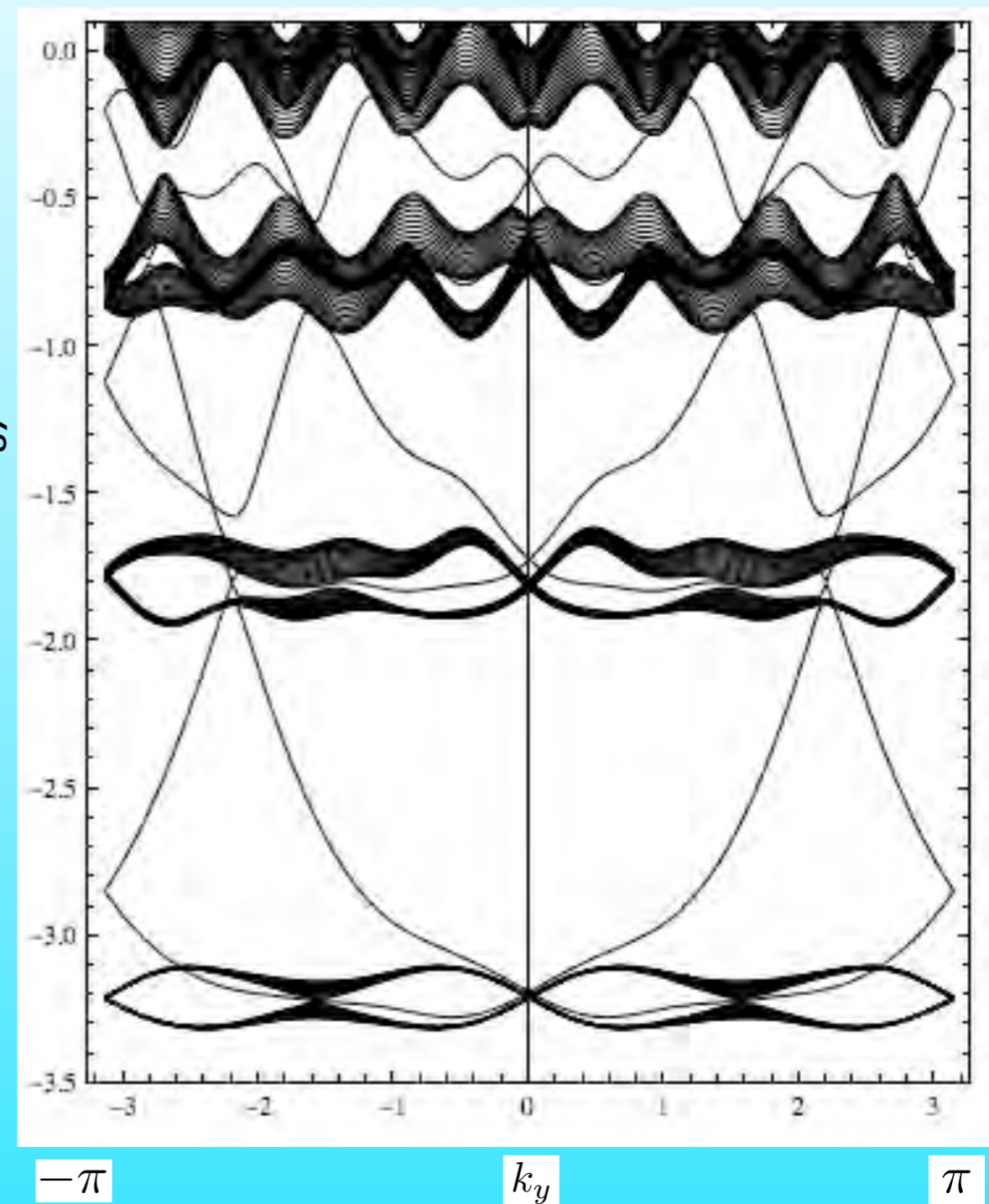
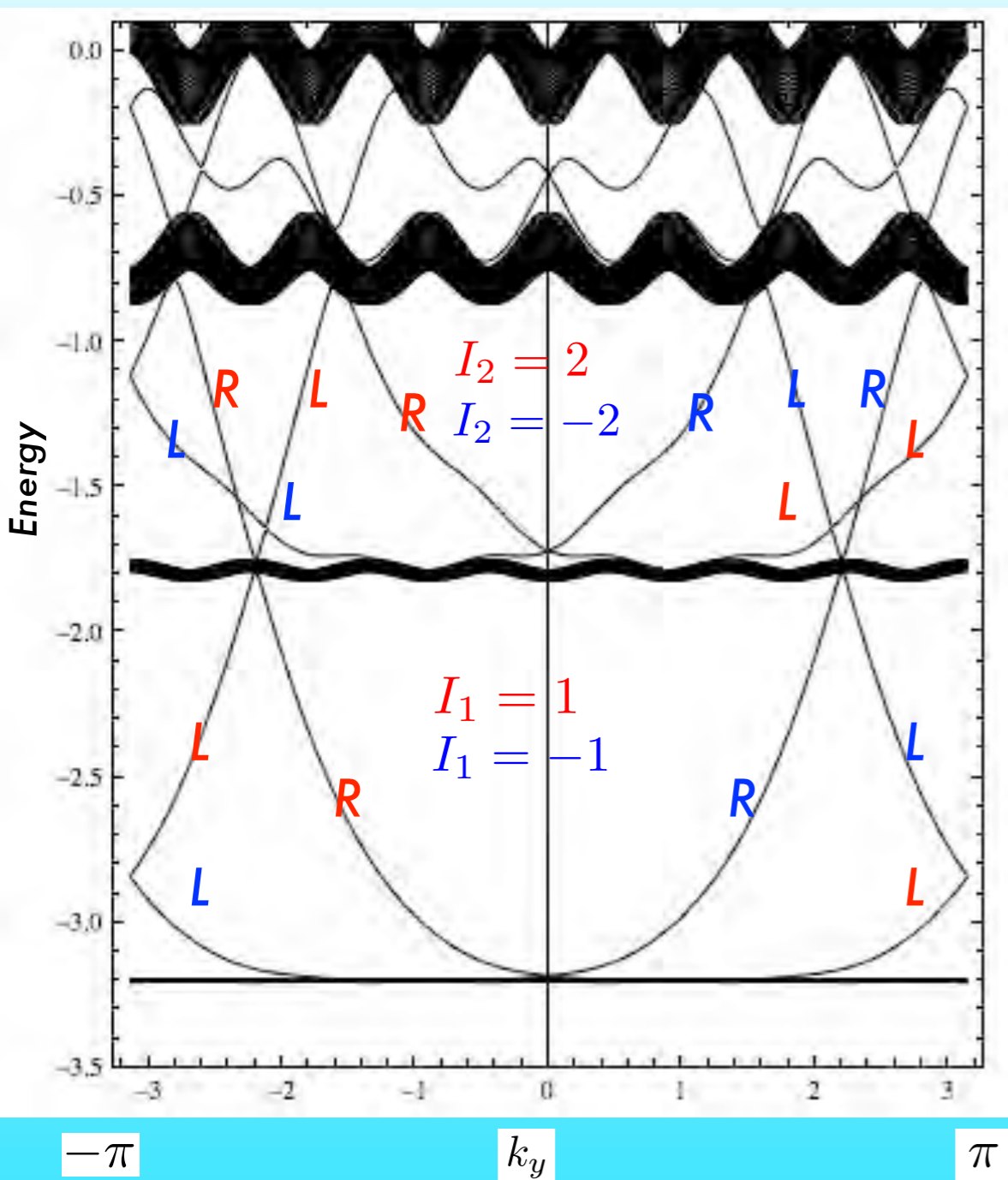


# Identification of edge states (QSHE)

$Z_2$  edge states

Spin Conserved

With Spin Orbit



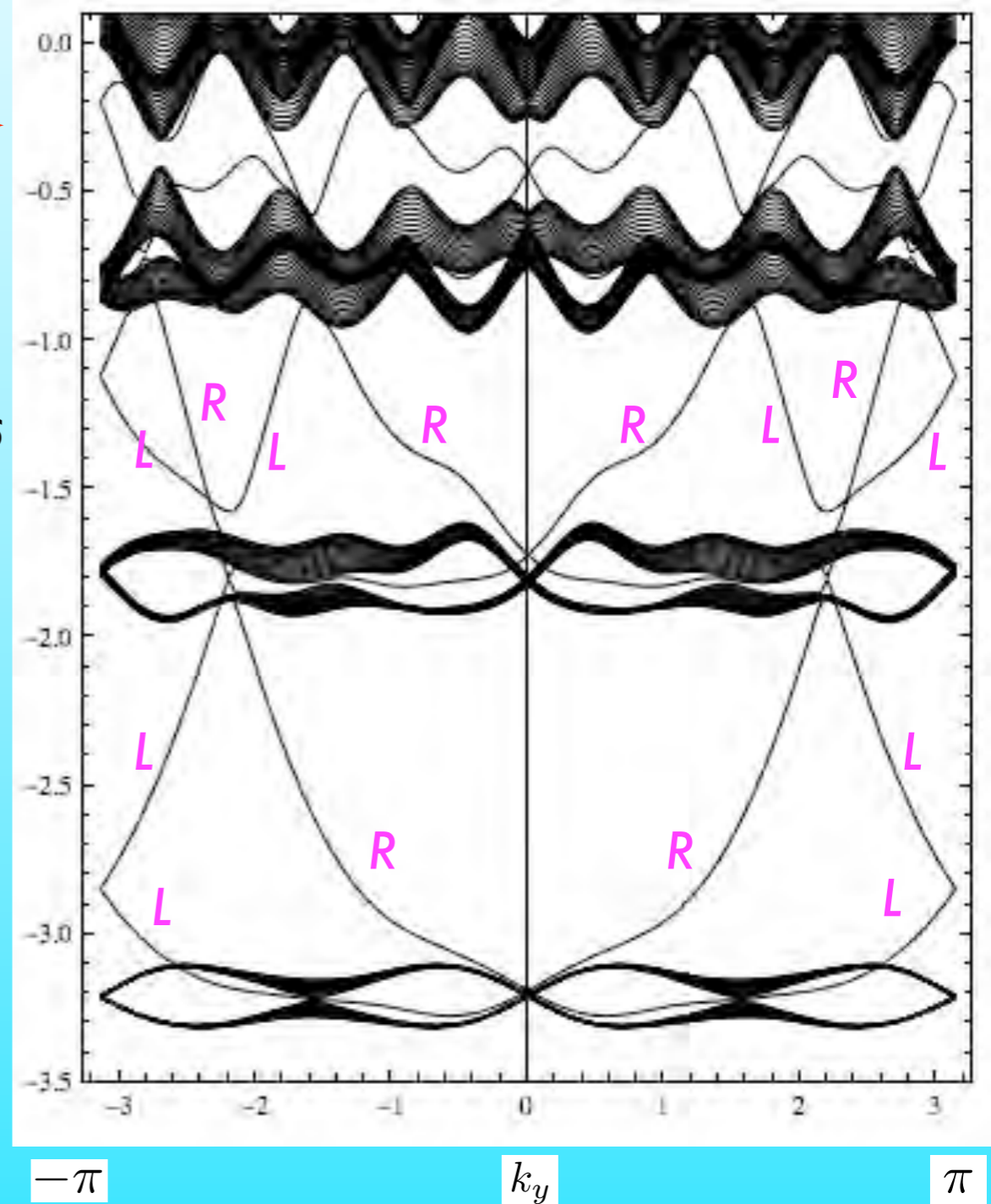
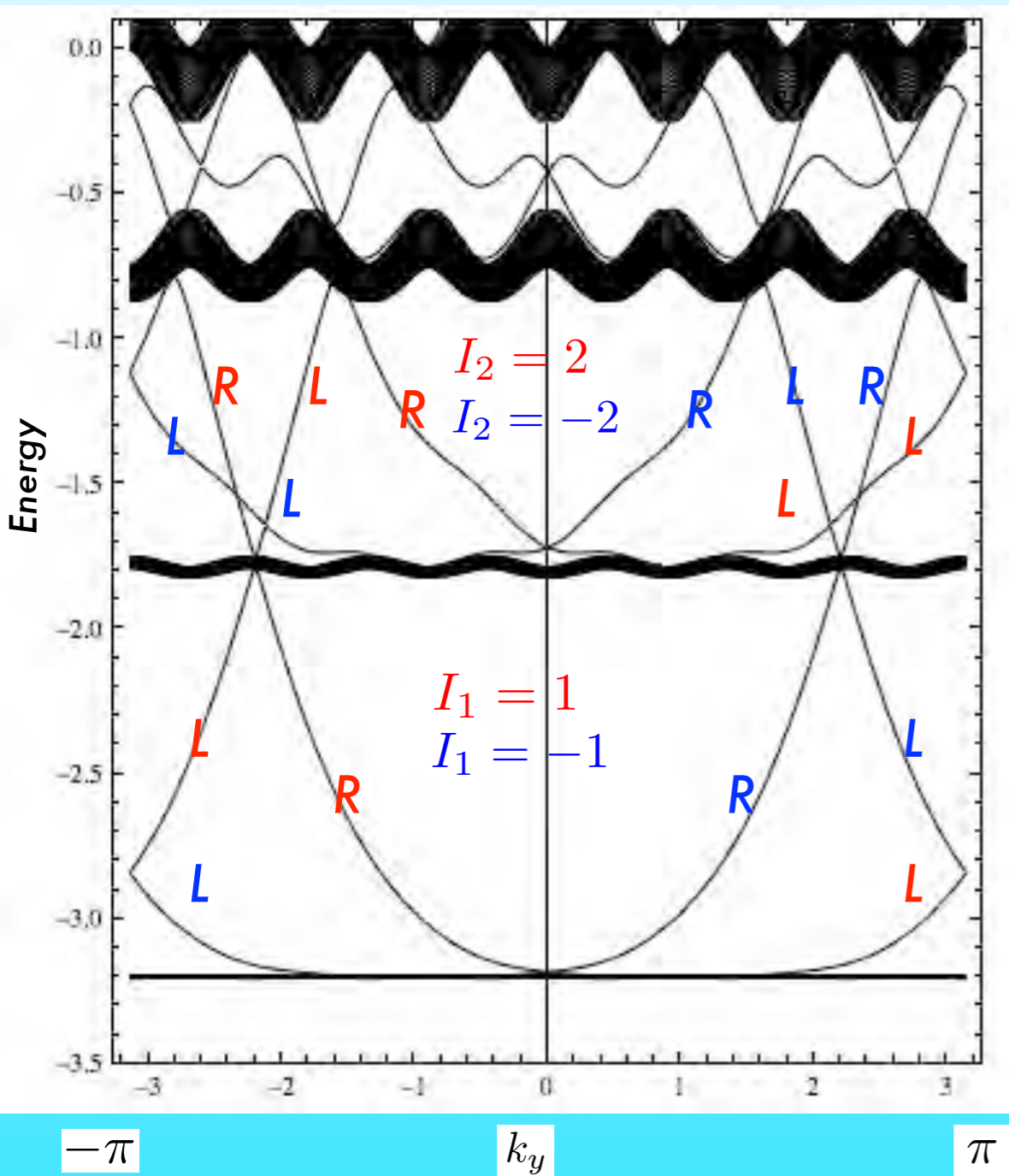
# Identification of edge states (QSHE)

$Z_2$  edge states

Spin Conserved

With Spin Orbit

Adiabatic modifications





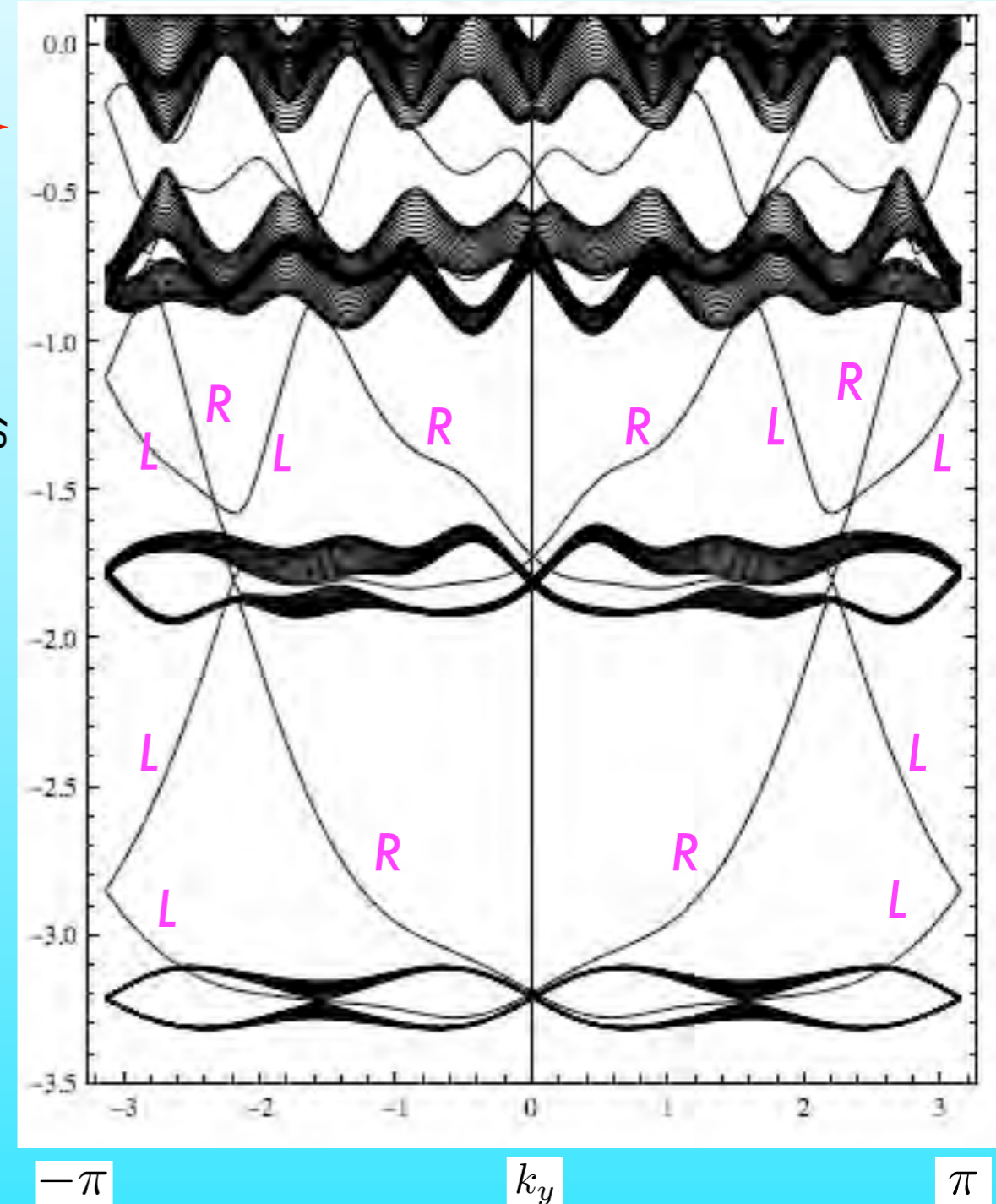
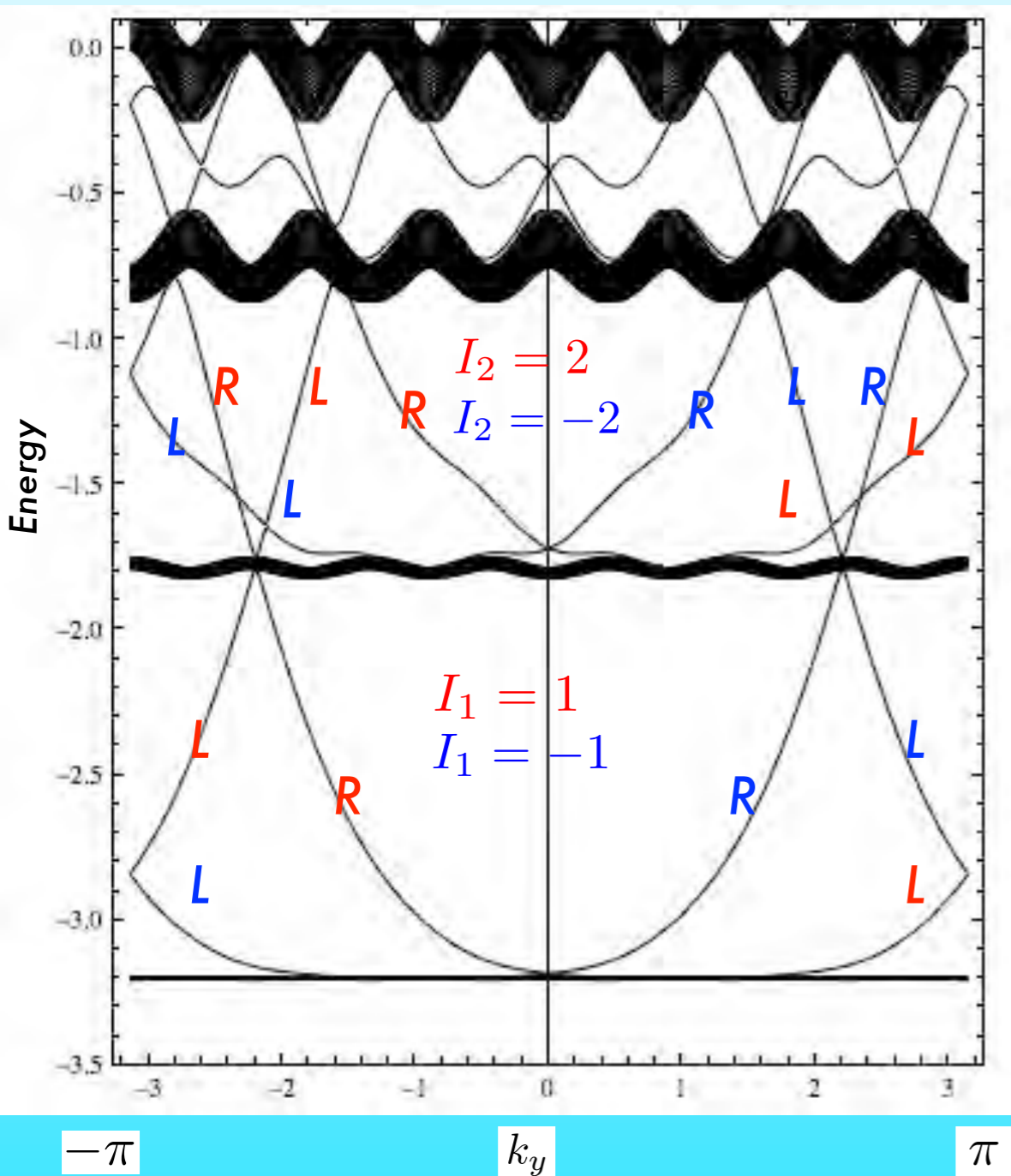
# Identification of edge states (QSHE)

$Z_2$  edge states

Spin Conserved

With Spin Orbit

Adiabatic modifications



Any topological protection ?

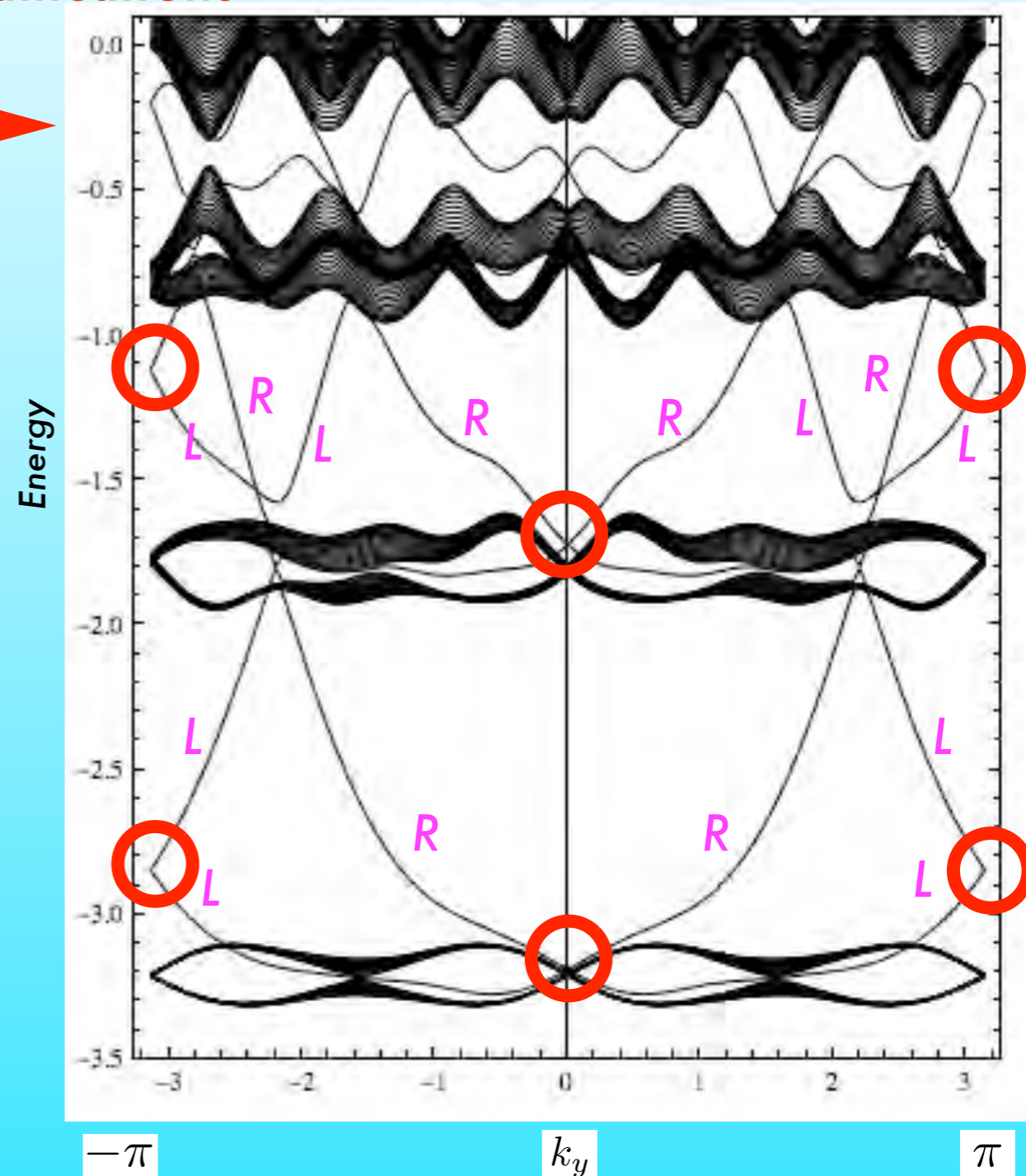
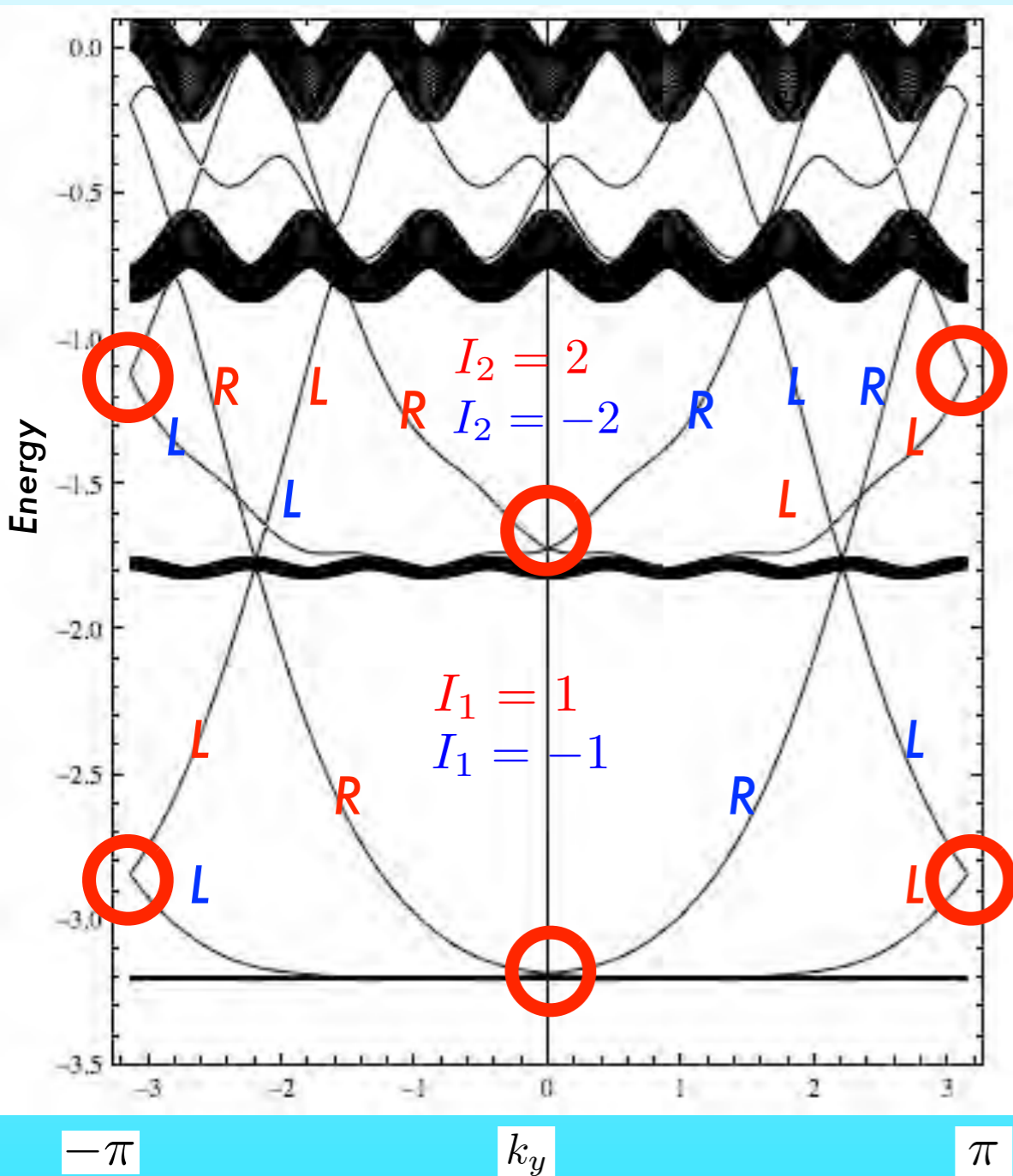
# Identification of edge states (QSHE)

$Z_2$  edge states

Spin Conserved

With Spin Orbit

Adiabatic modifications



Kramers degeneracy at TR invariant momenta

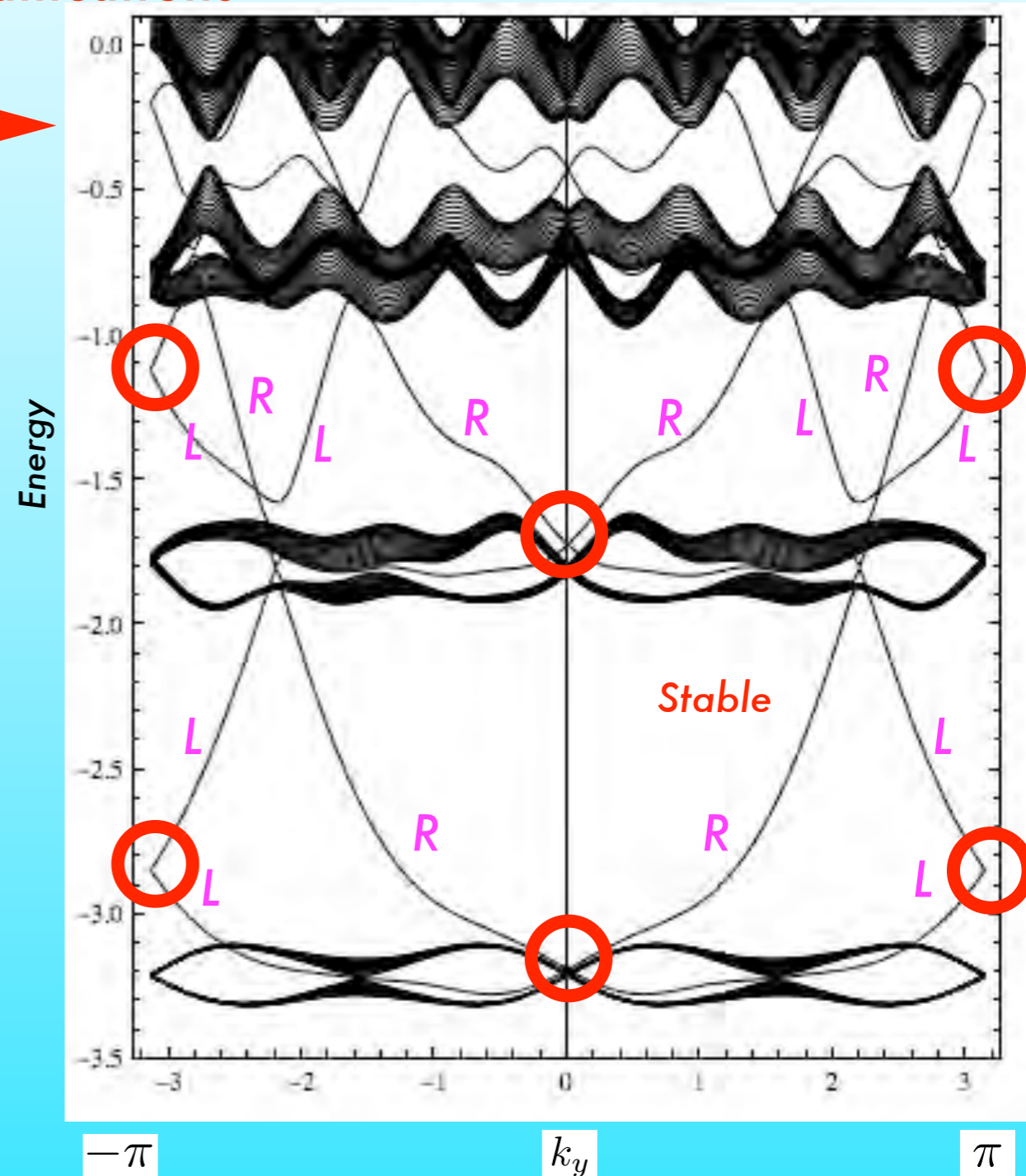
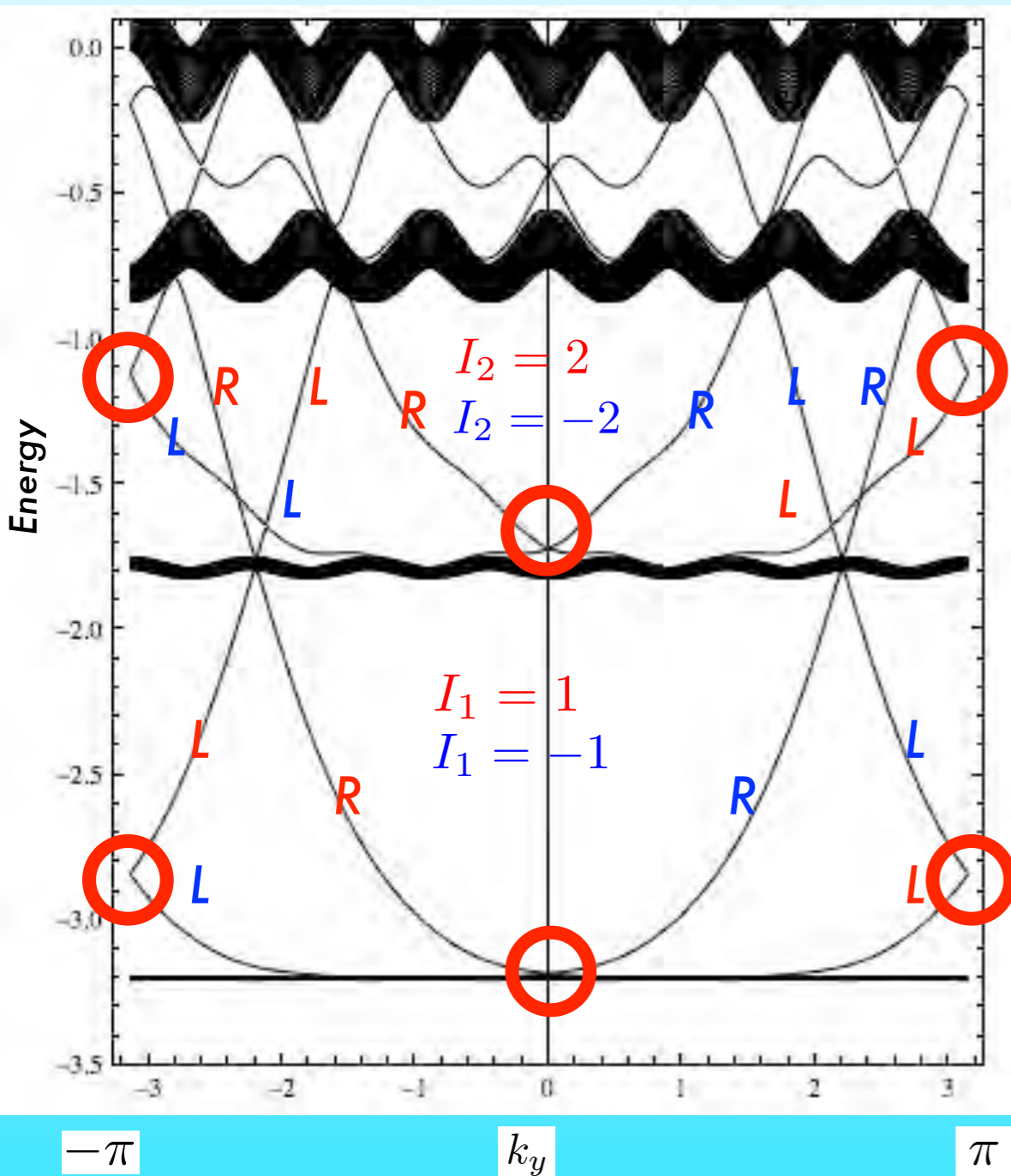
# Identification of edge states (QSHE)

$Z_2$  edge states

Spin Conserved

With Spin Orbit

Adiabatic modifications



Kramers degeneracy at TR invariant momenta

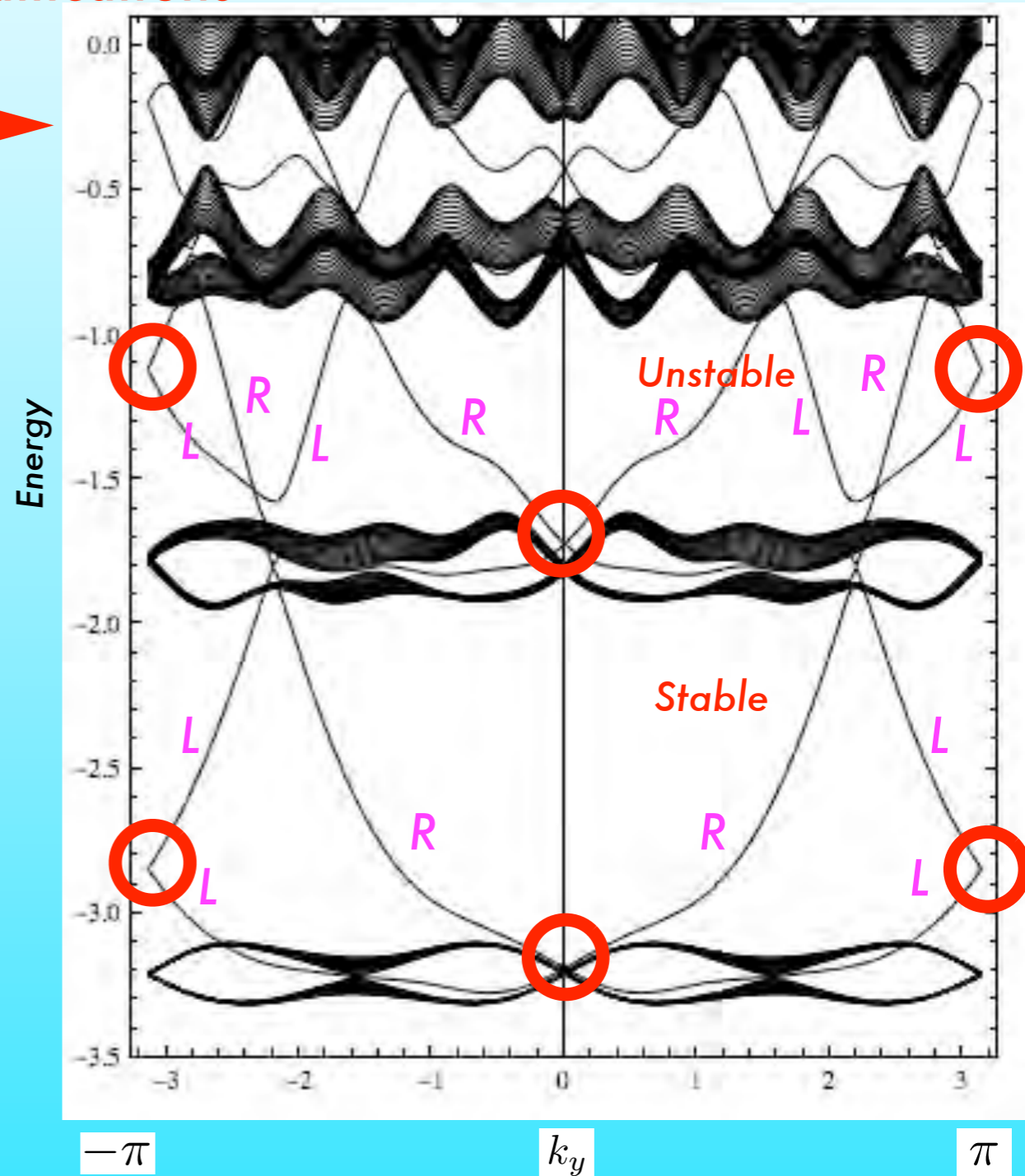
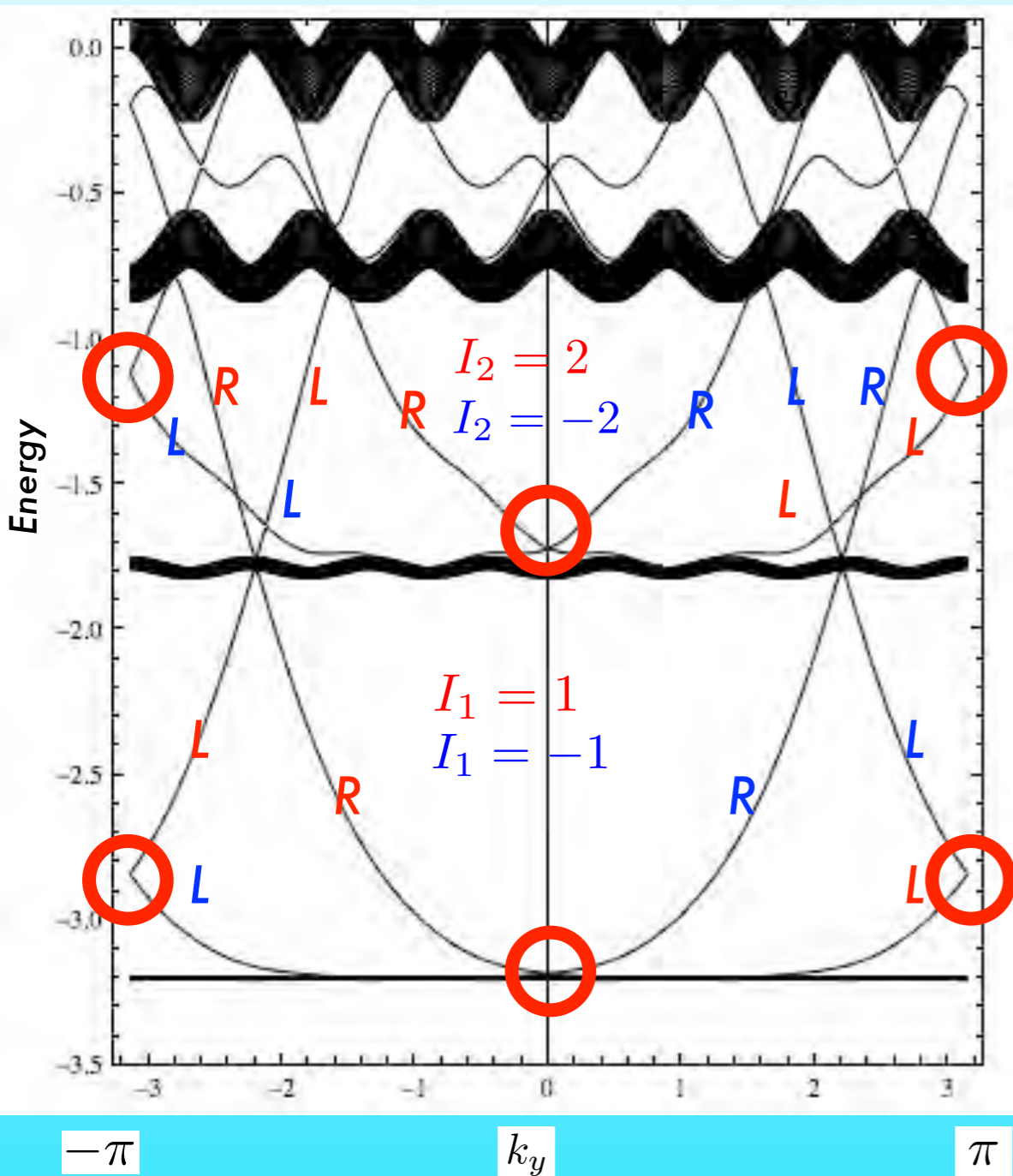
# Identification of edge states (QSHE)

$Z_2$  edge states

Spin Conserved

With Spin Orbit

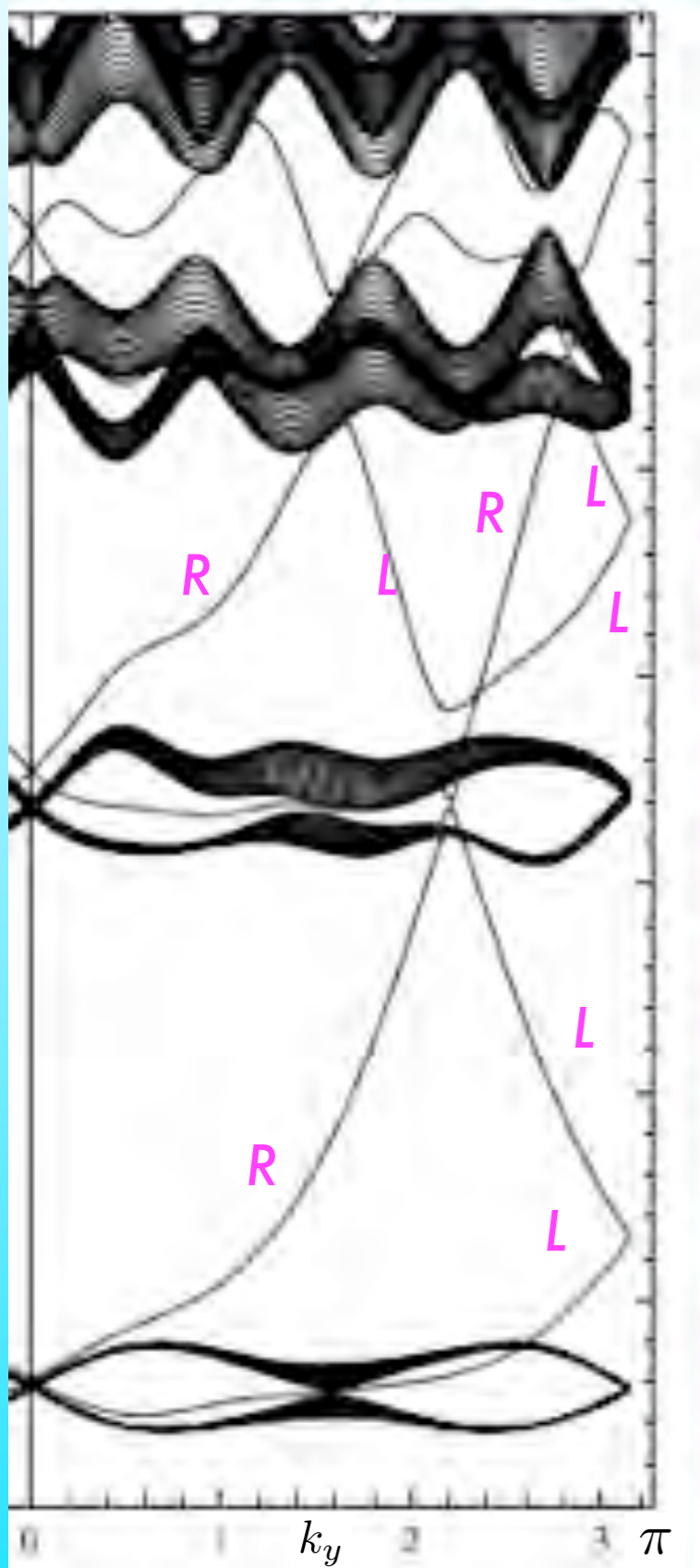
Adiabatic modifications



Kramers degeneracy at TR invariant momenta

# Identification of edge states (QSHE) With Spin Orbit

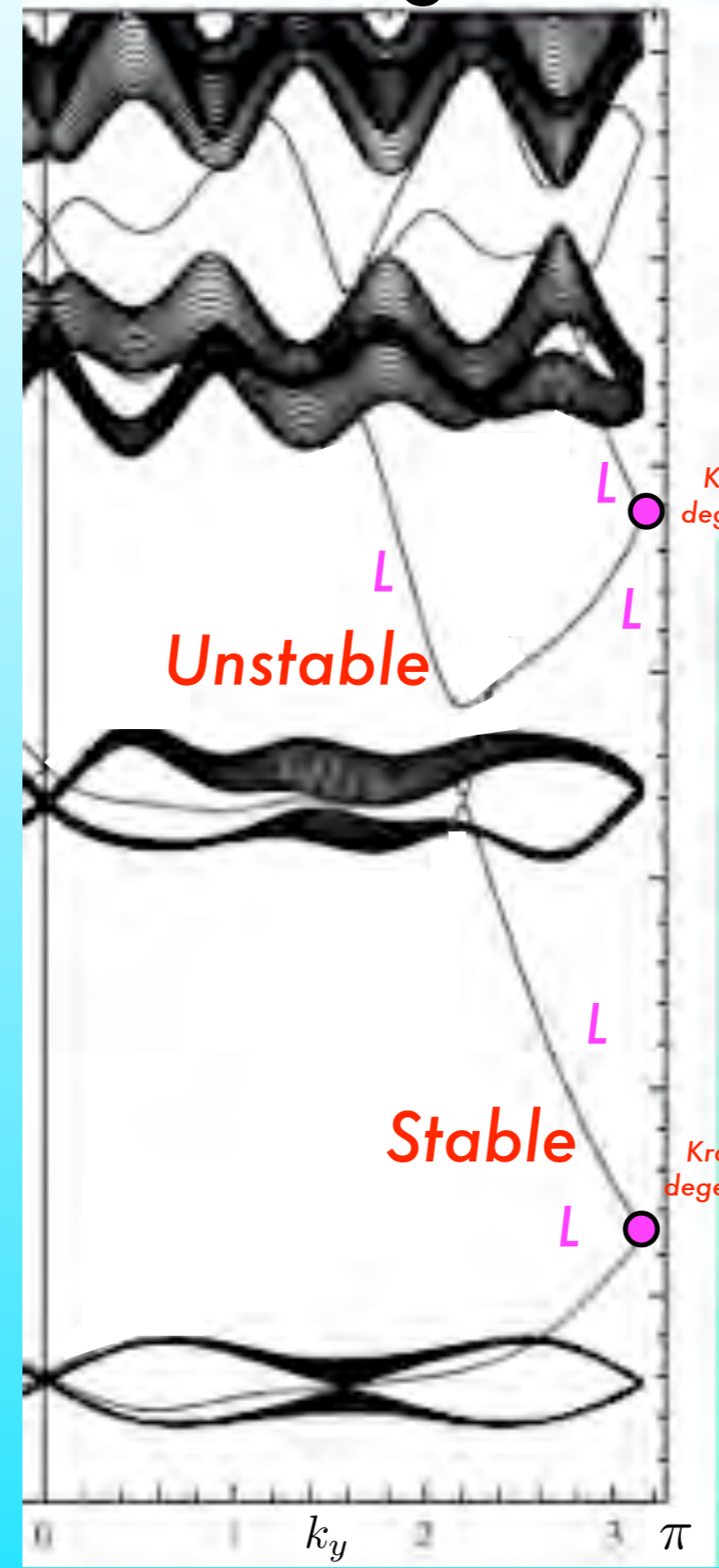
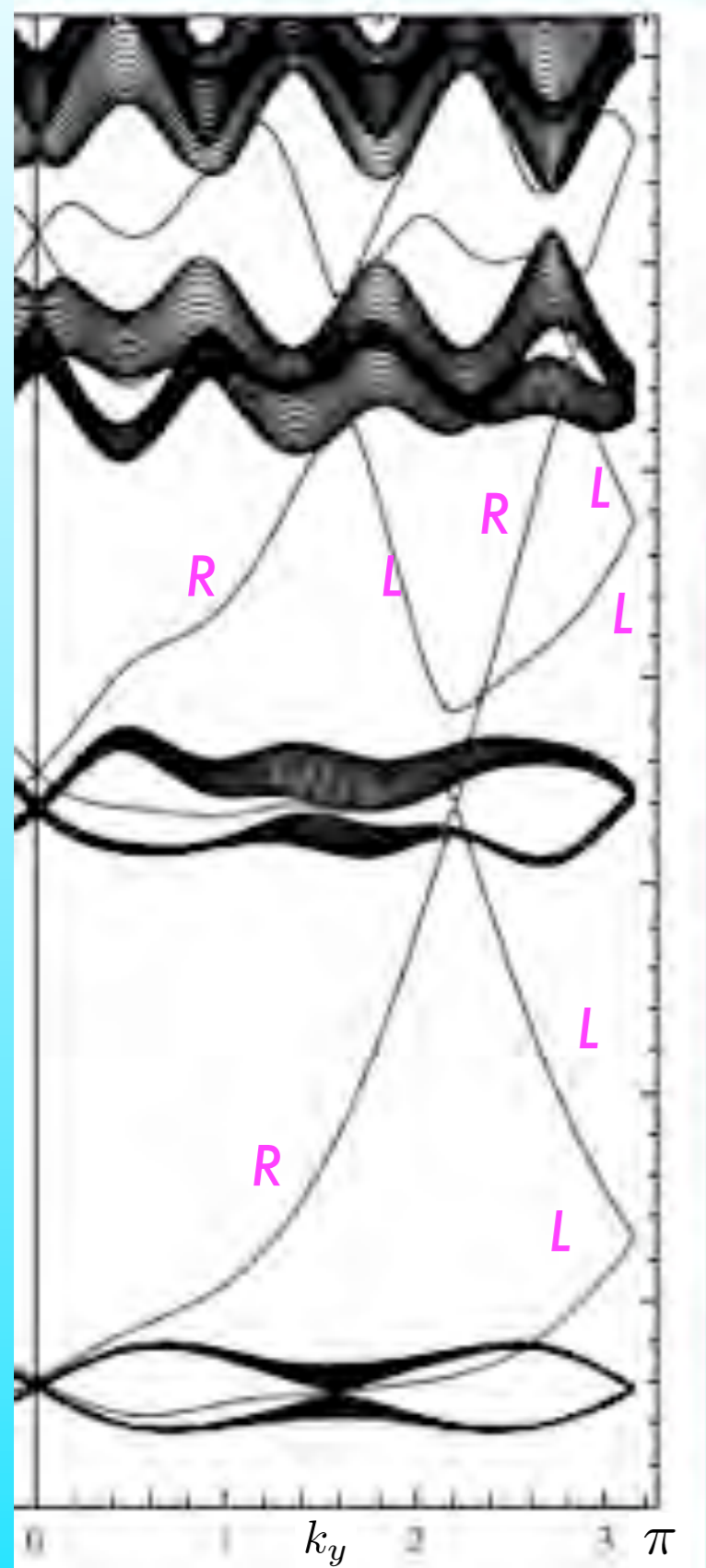
$Z_2$  edge states



# Identification of edge states (QSHE) With Spin Orbit

$Z_2$  edge states

L edge

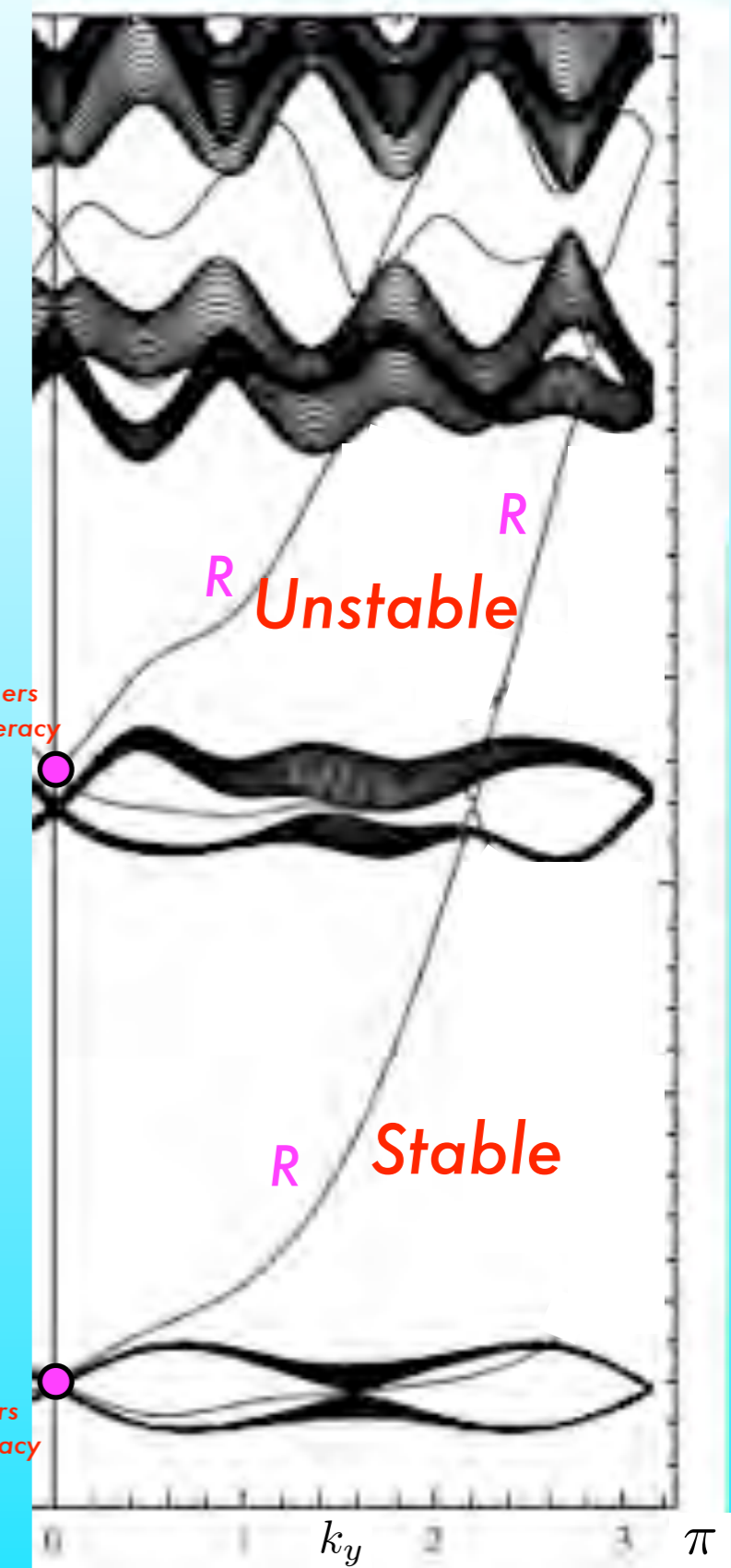
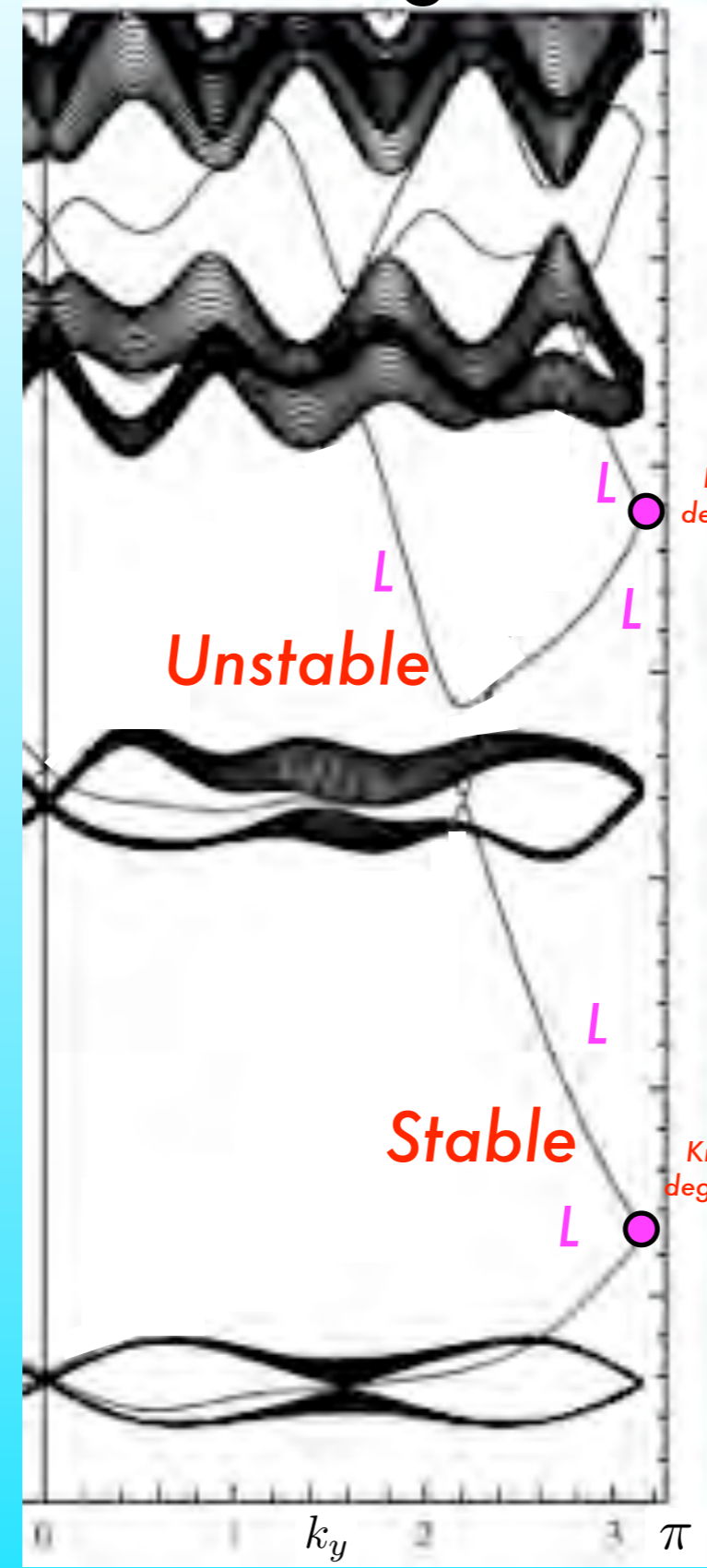
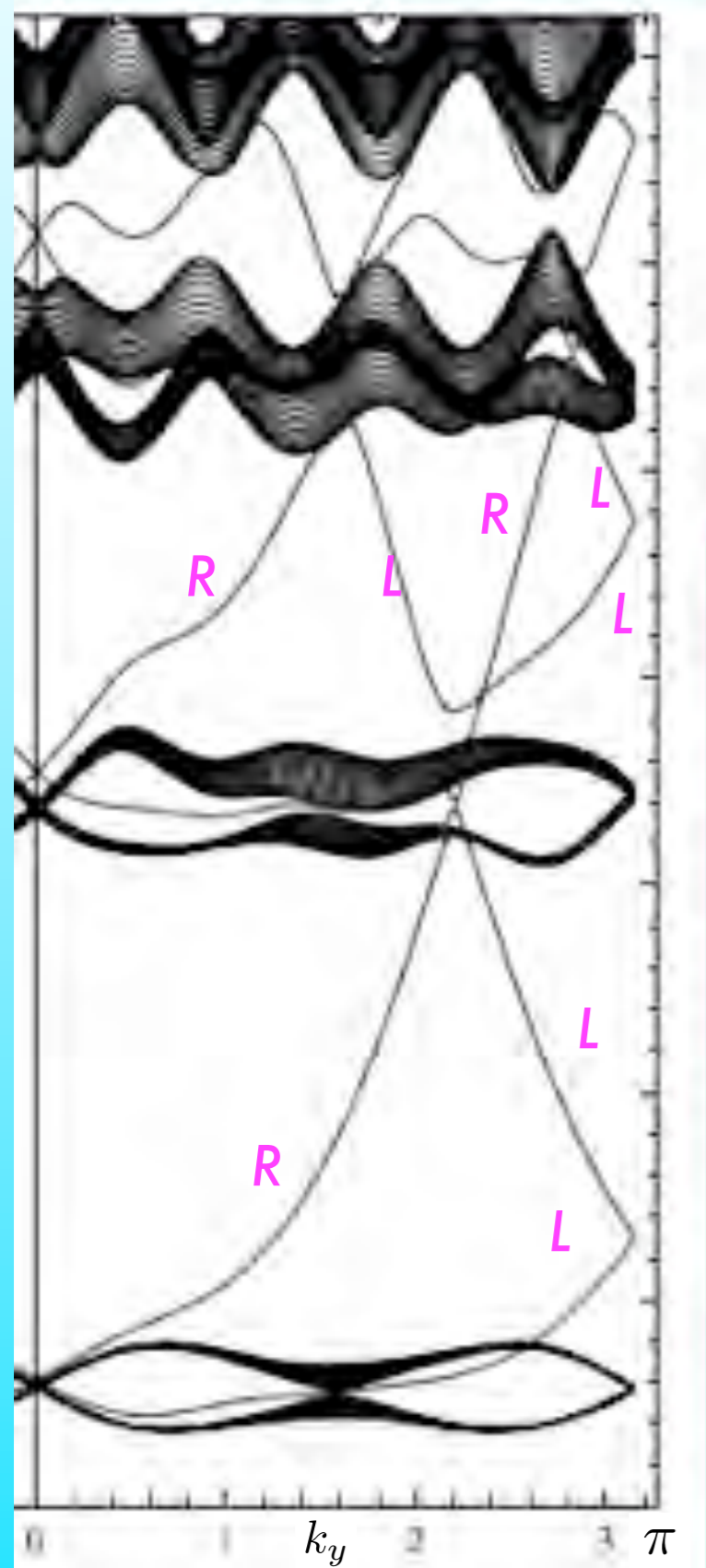


# Identification of edge states (QSHE) With Spin Orbit

$Z_2$  edge states

L edge

R edge

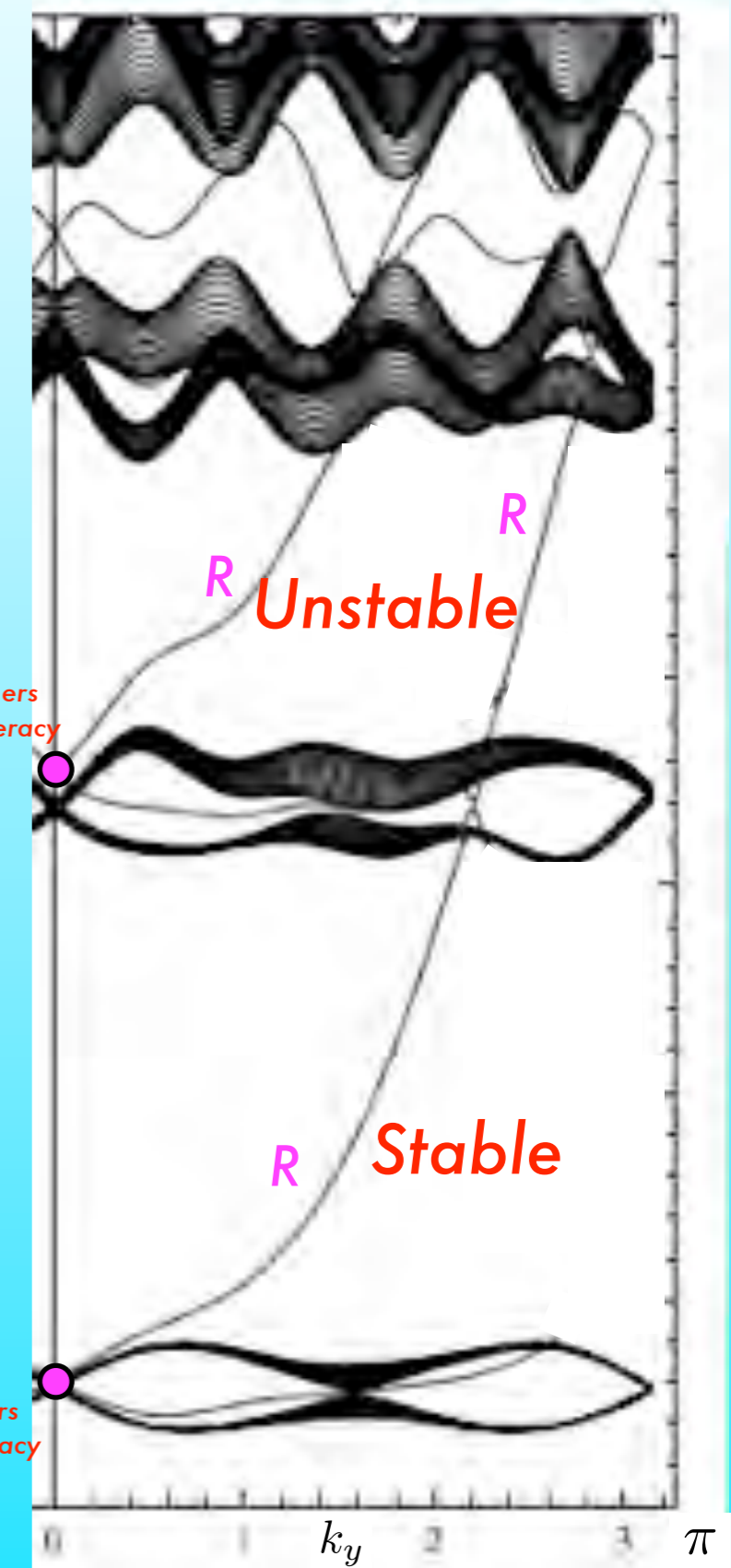
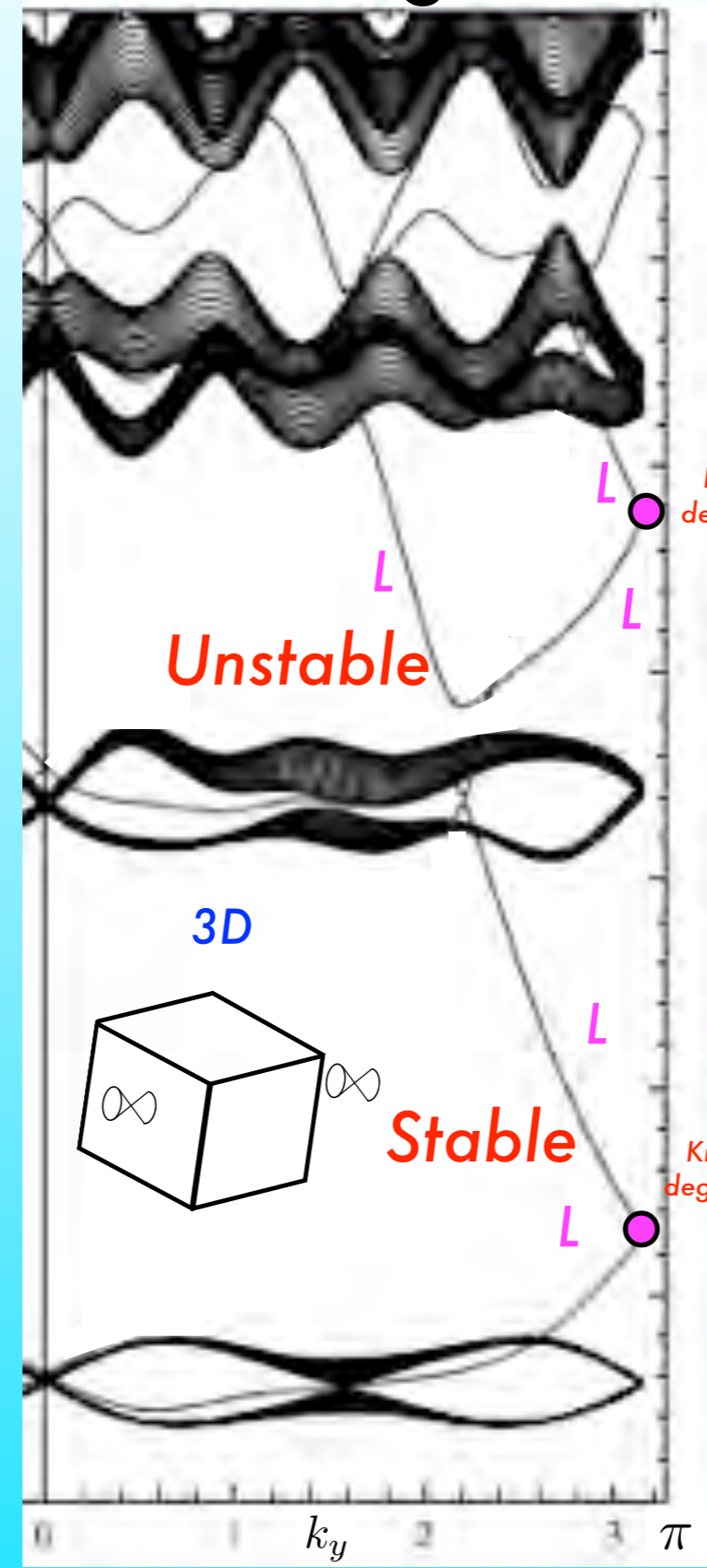
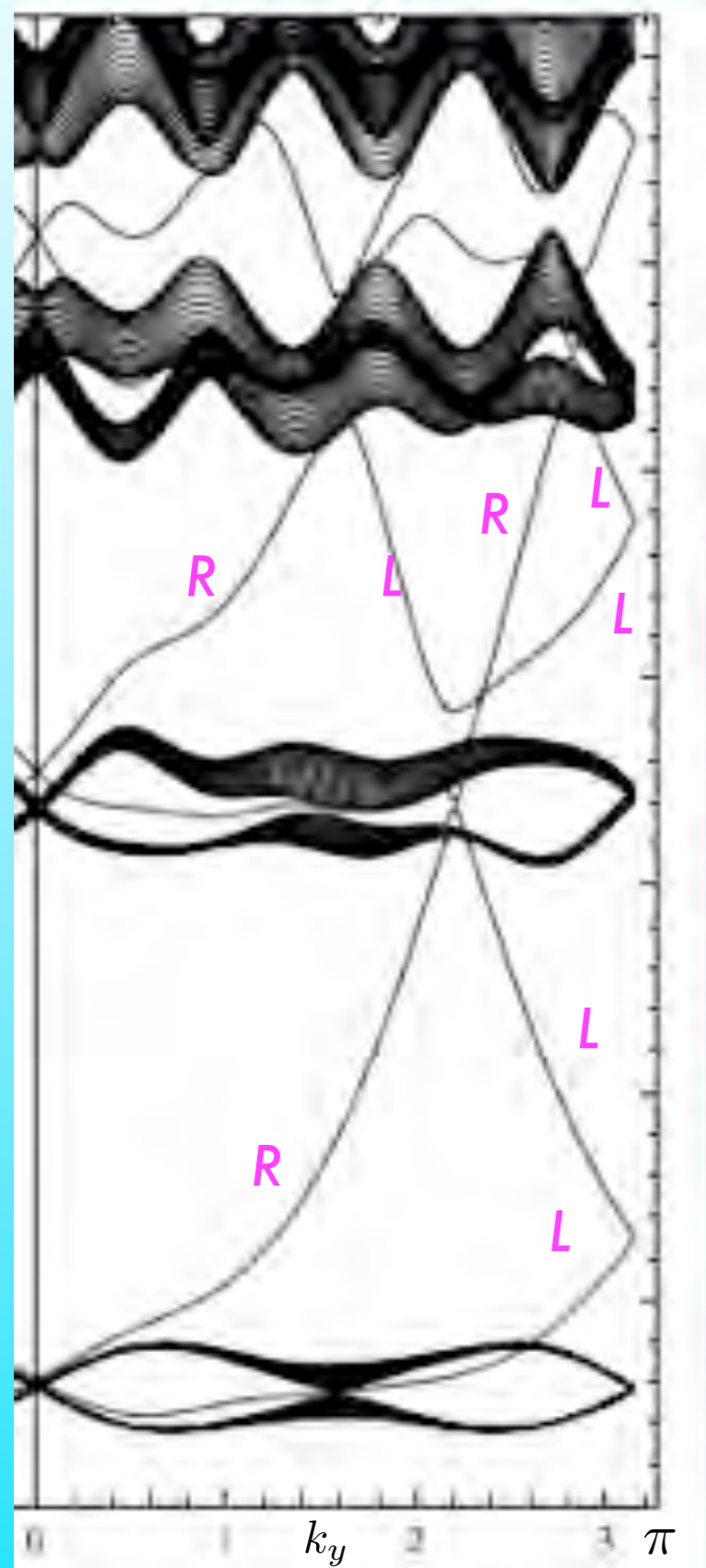


# Identification of edge states (QSHE) With Spin Orbit

Z<sub>2</sub> edge states

L edge

R edge





$Z_2$  edge states

# Topologically protected edge states

TR invariant  
Point

TR invariant  
Point

TR invariant  
Point

TR invariant  
Point



0  $k_y$   $\pi$

0  $k_y$   $\pi$

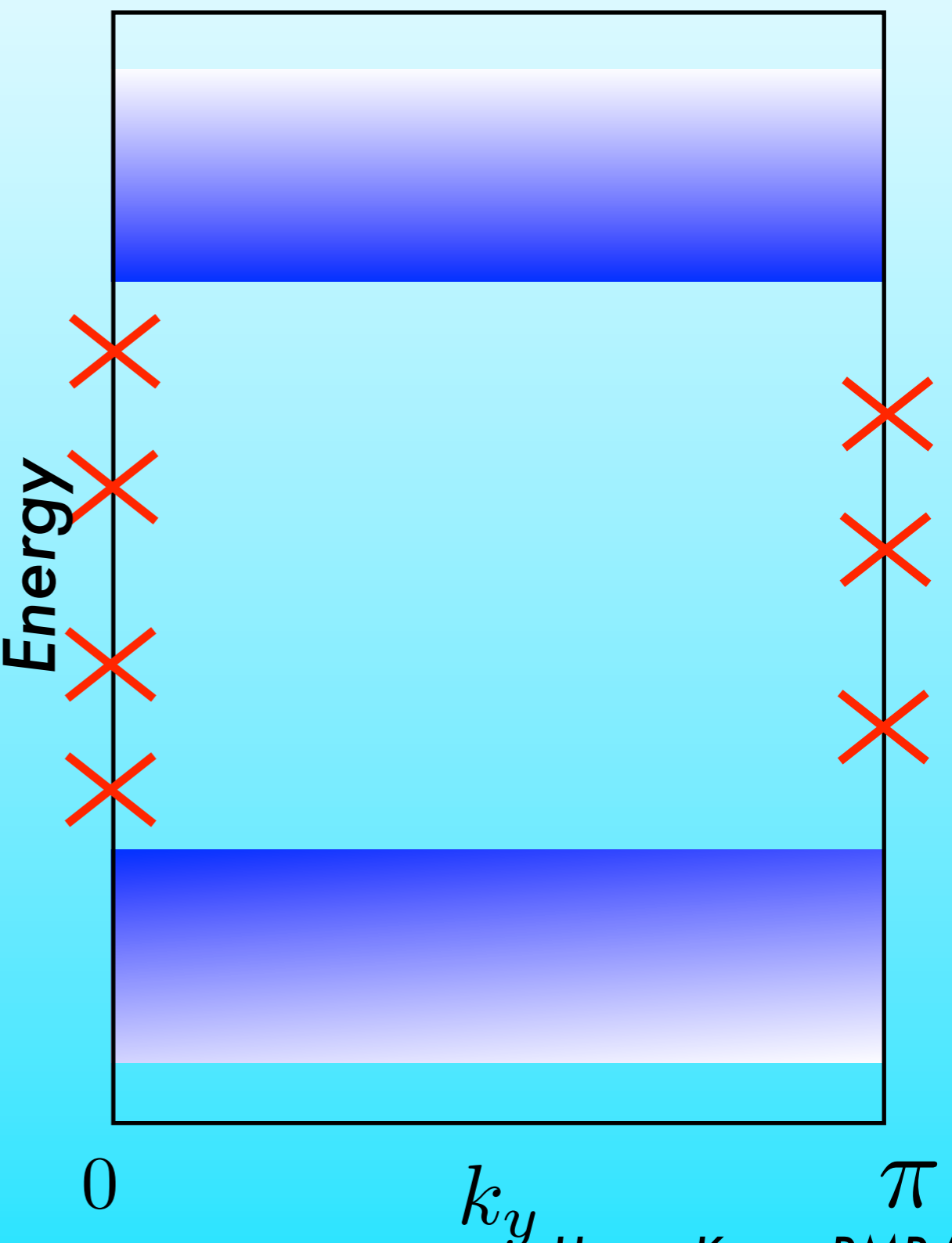
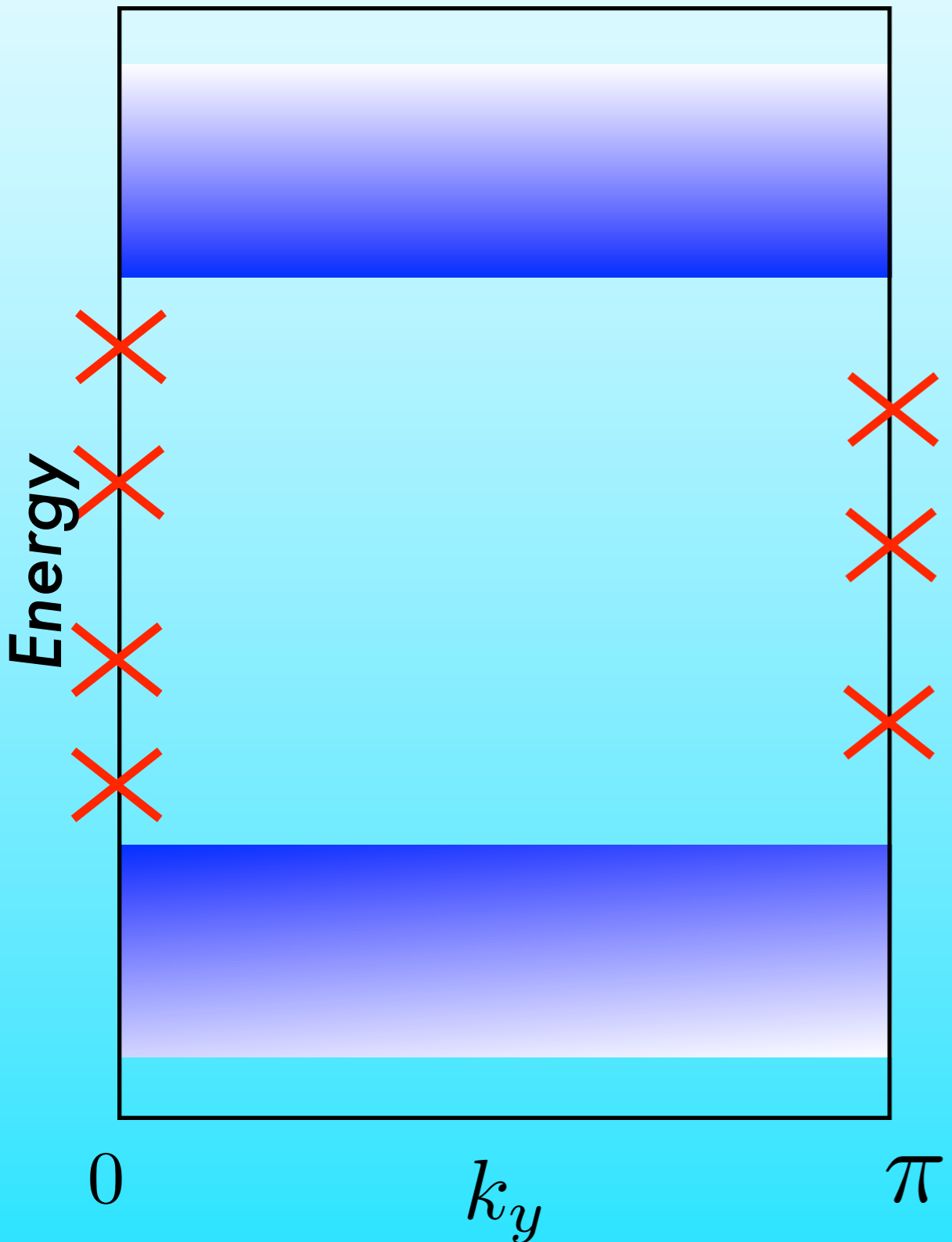
# Topologically protected edge states

TR invariant Point

TR invariant Point

TR invariant Point

TR invariant Point



$Z_2$  edge states

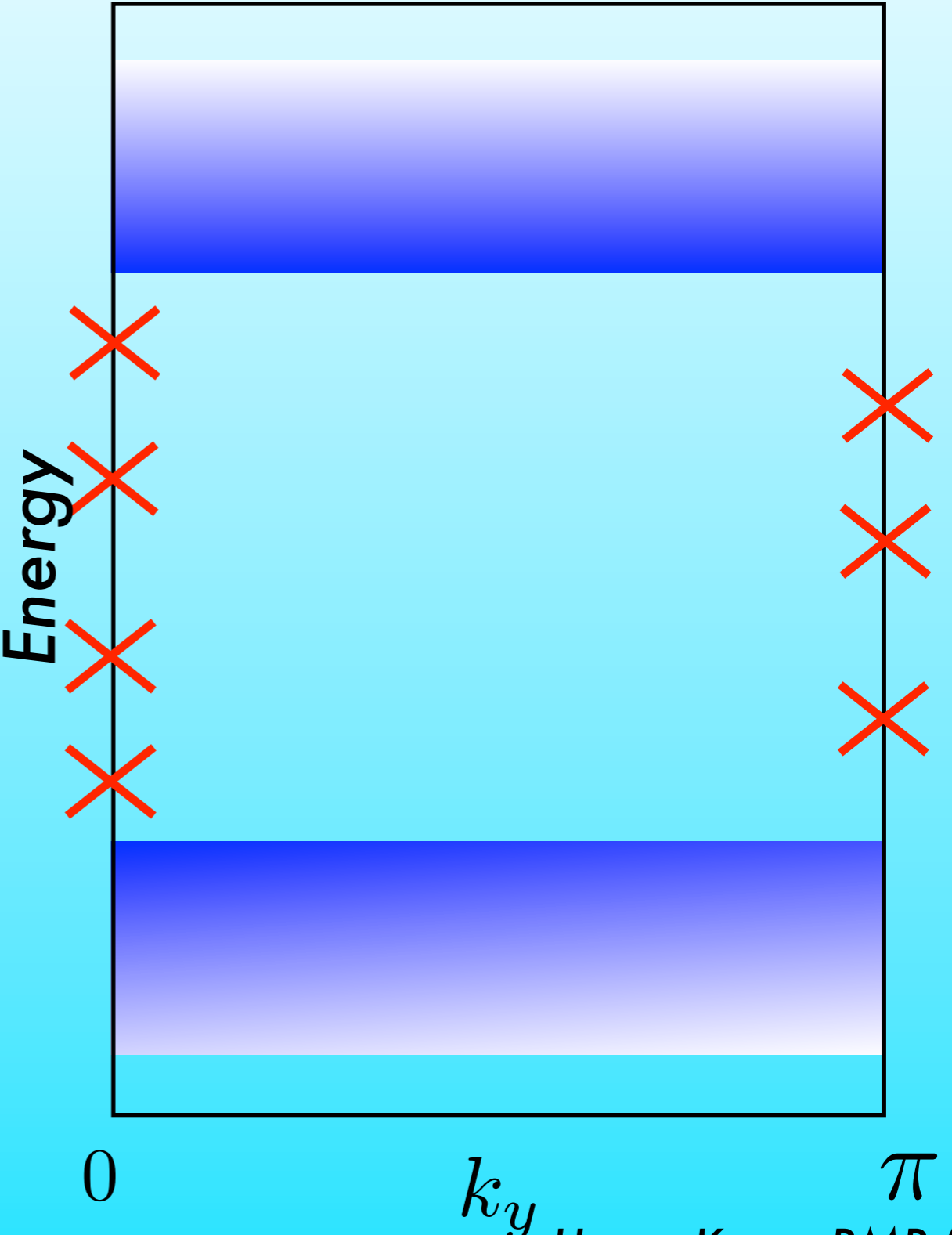
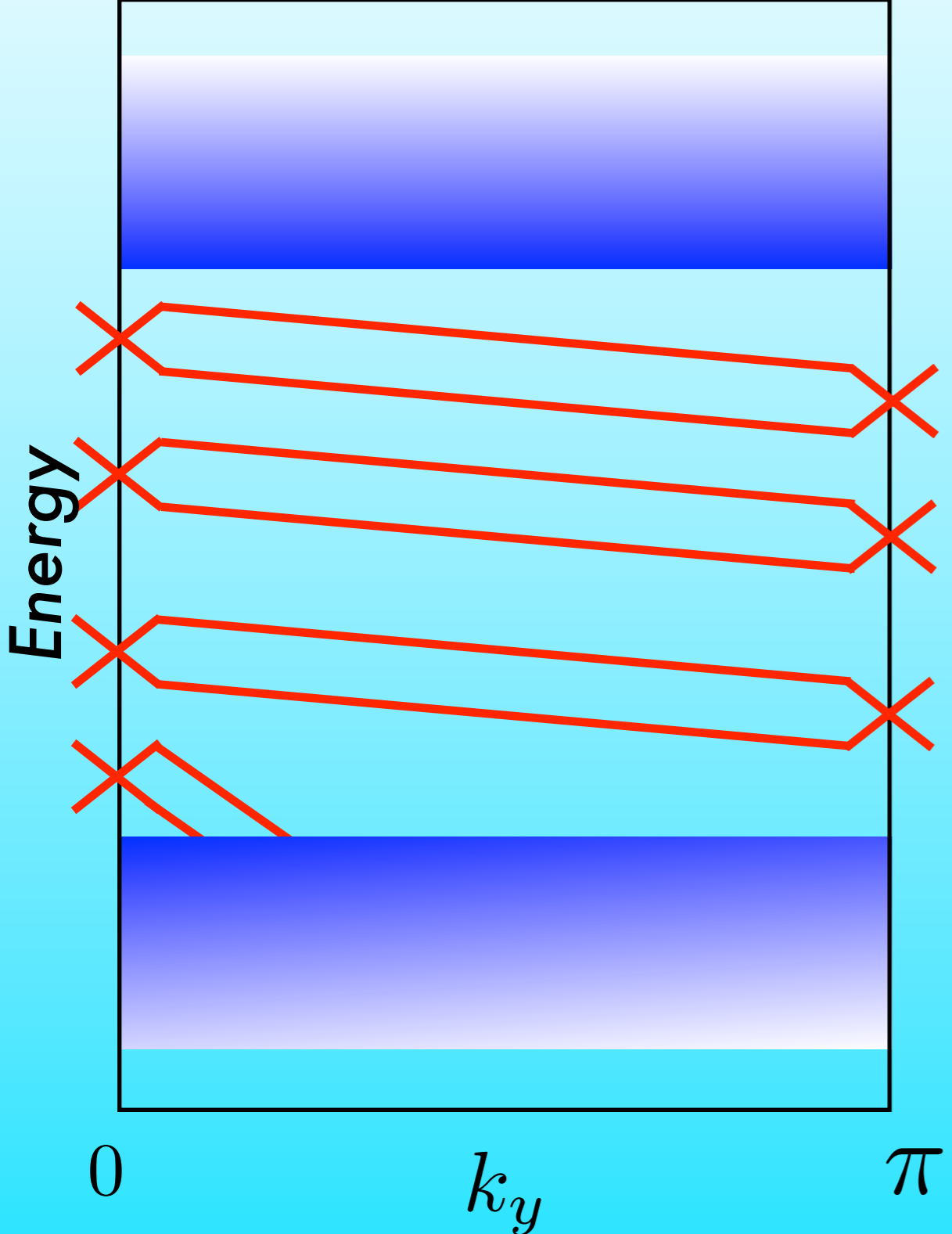
# Topologically protected edge states

TR invariant Point

TR invariant Point

TR invariant Point

TR invariant Point



# Topologically protected edge states

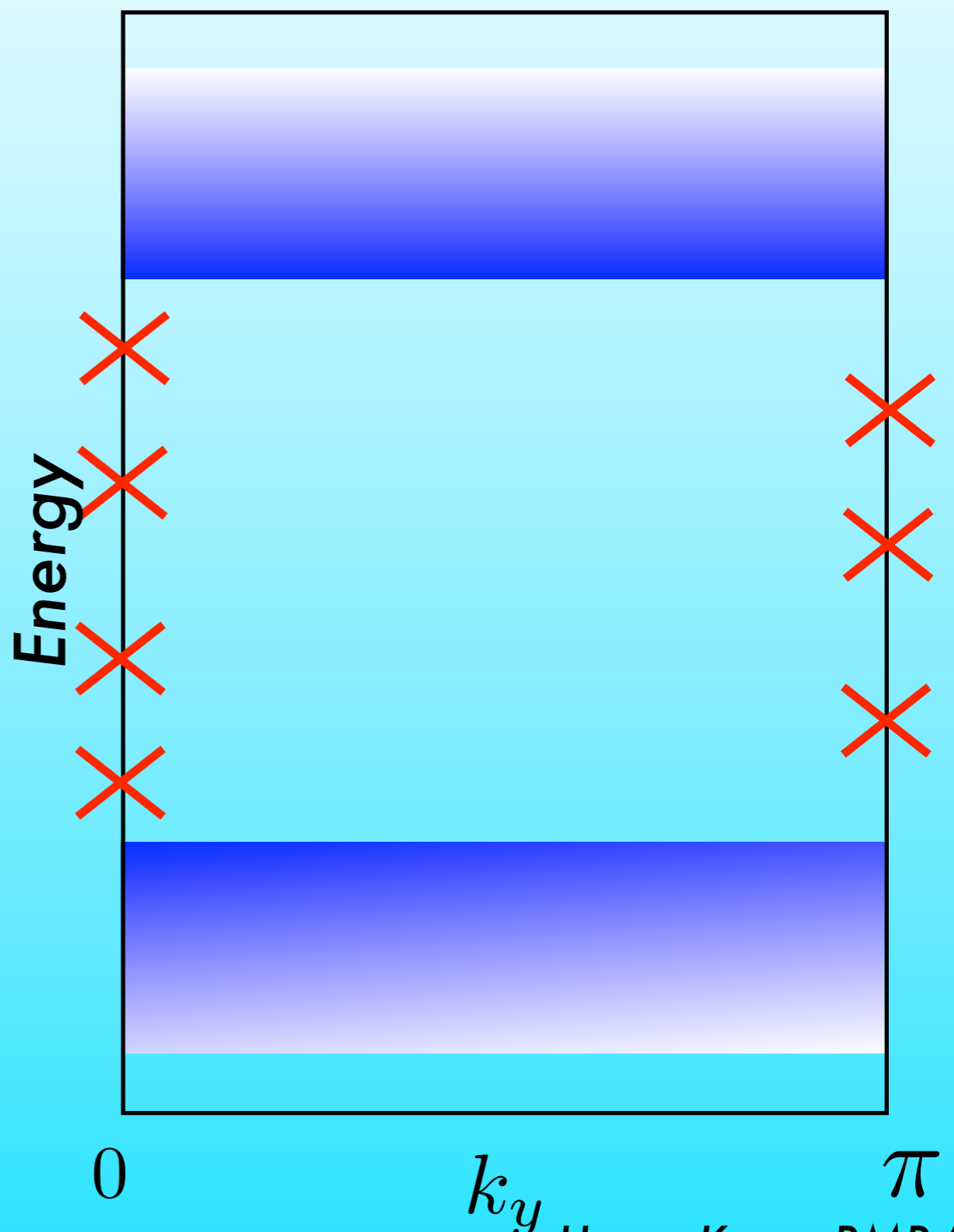
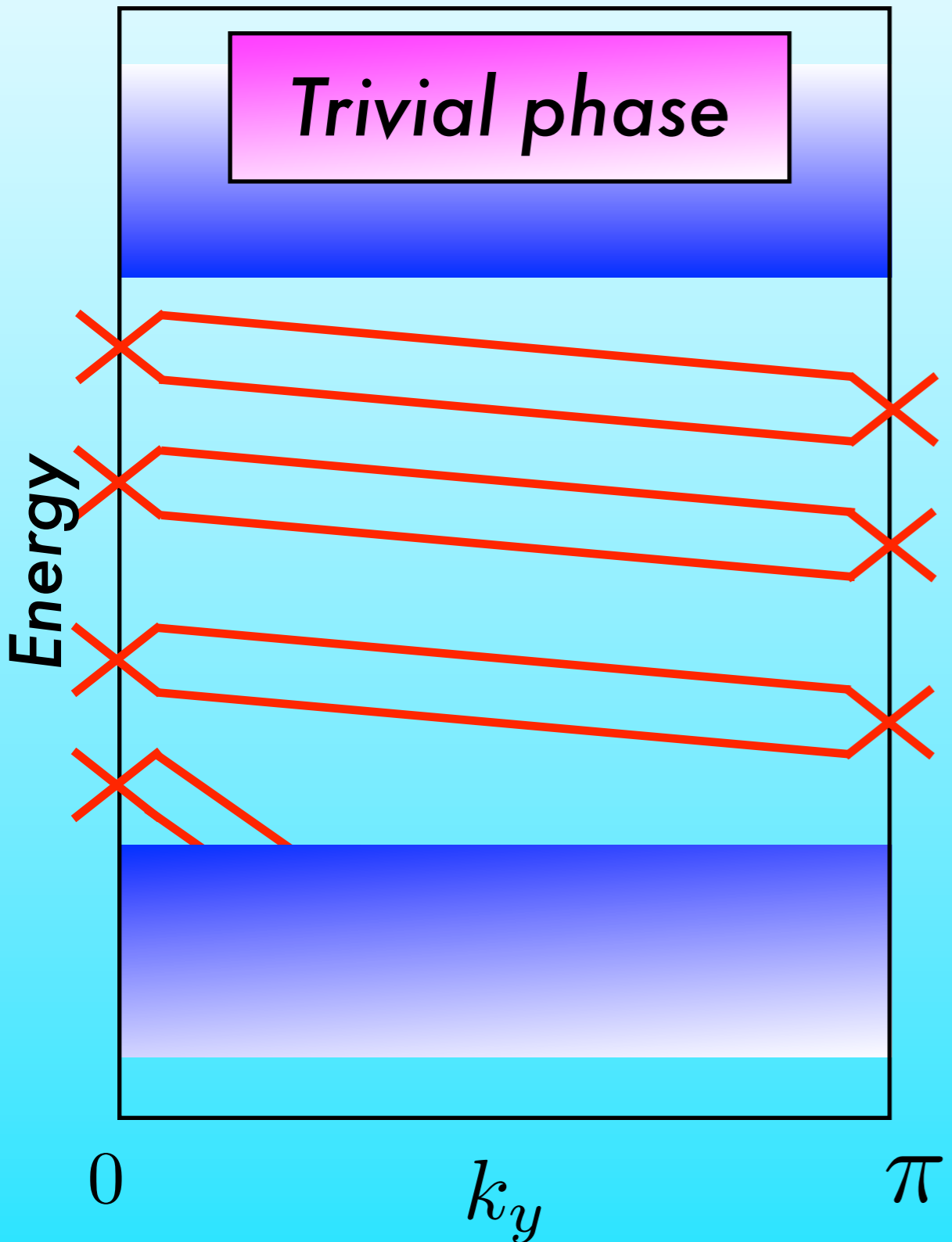
TR invariant  
Point

TR invariant  
Point

TR invariant  
Point

TR invariant  
Point

Trivial phase



# Topologically protected edge states

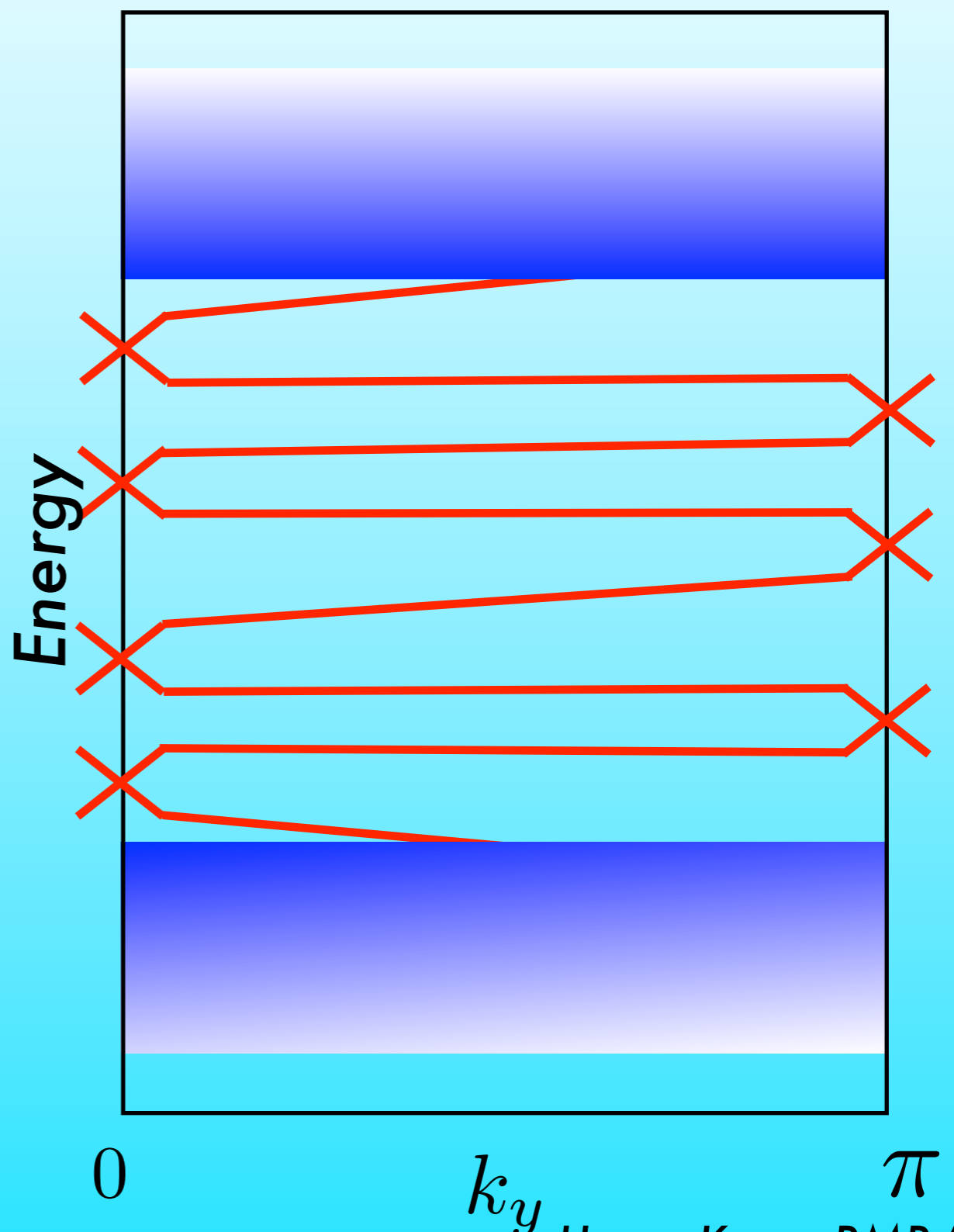
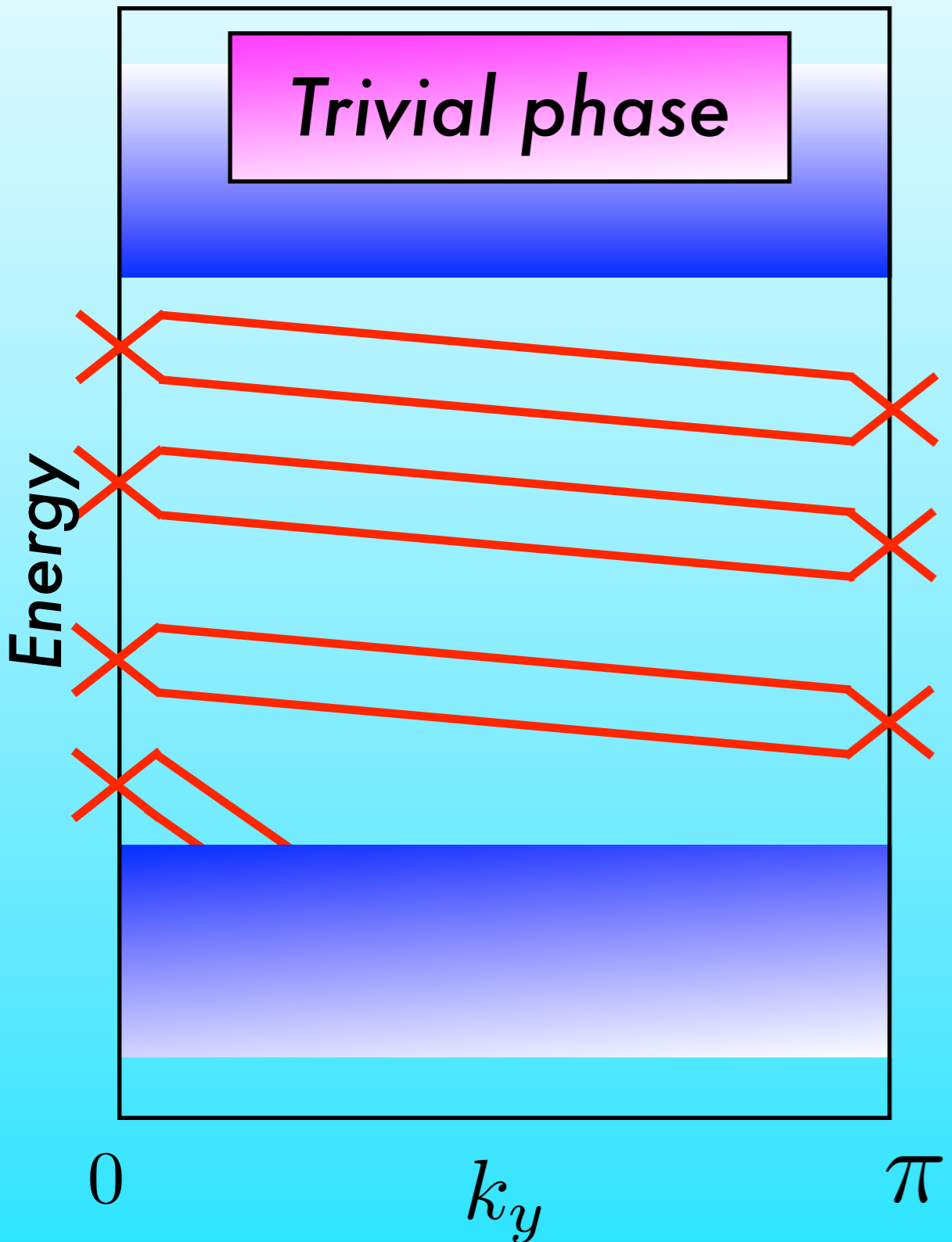
TR invariant Point

TR invariant Point

TR invariant Point

TR invariant Point

Trivial phase



# Topologically protected edge states

TR invariant  
Point

TR invariant  
Point

TR invariant  
Point

TR invariant  
Point

Trivial phase

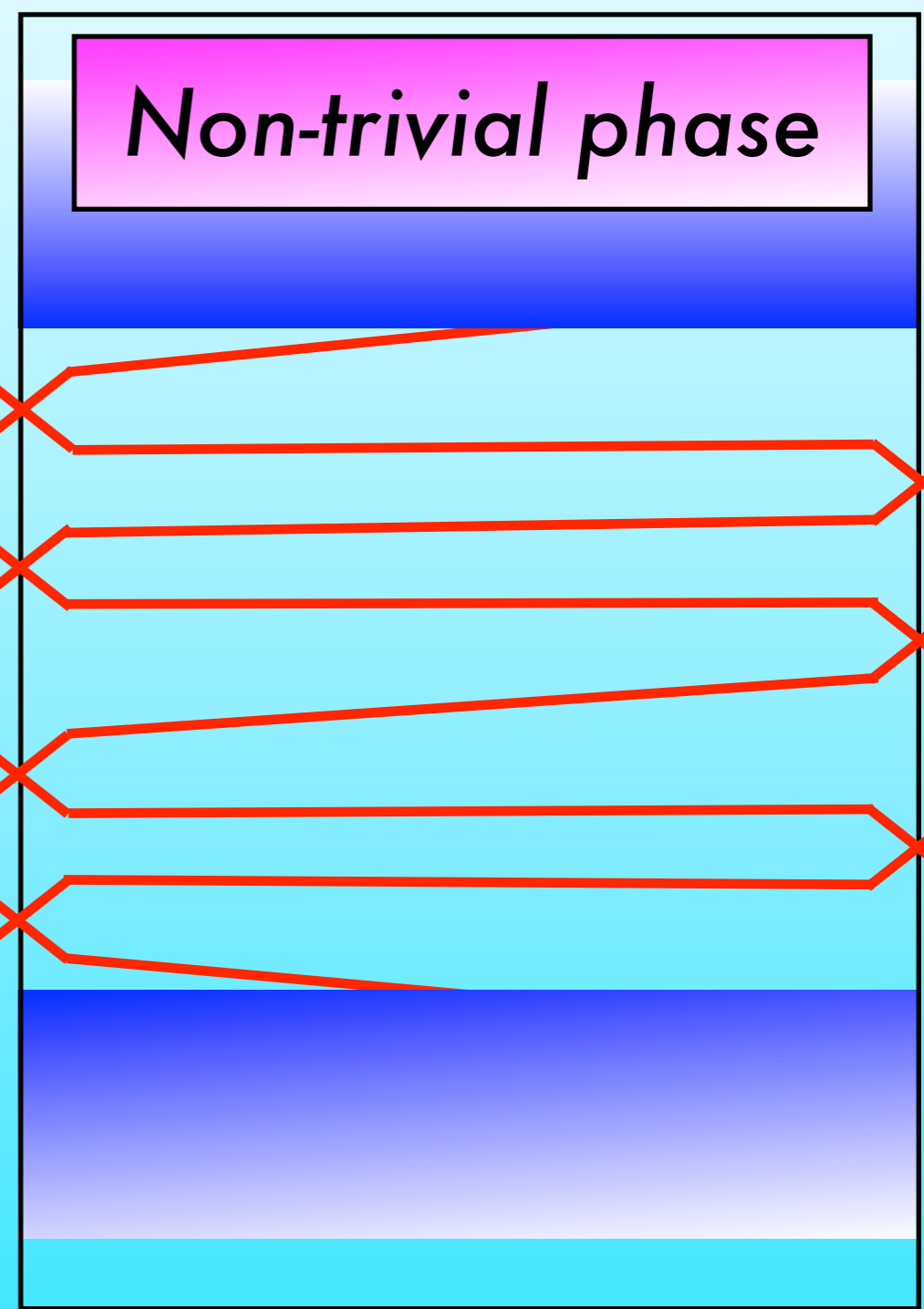
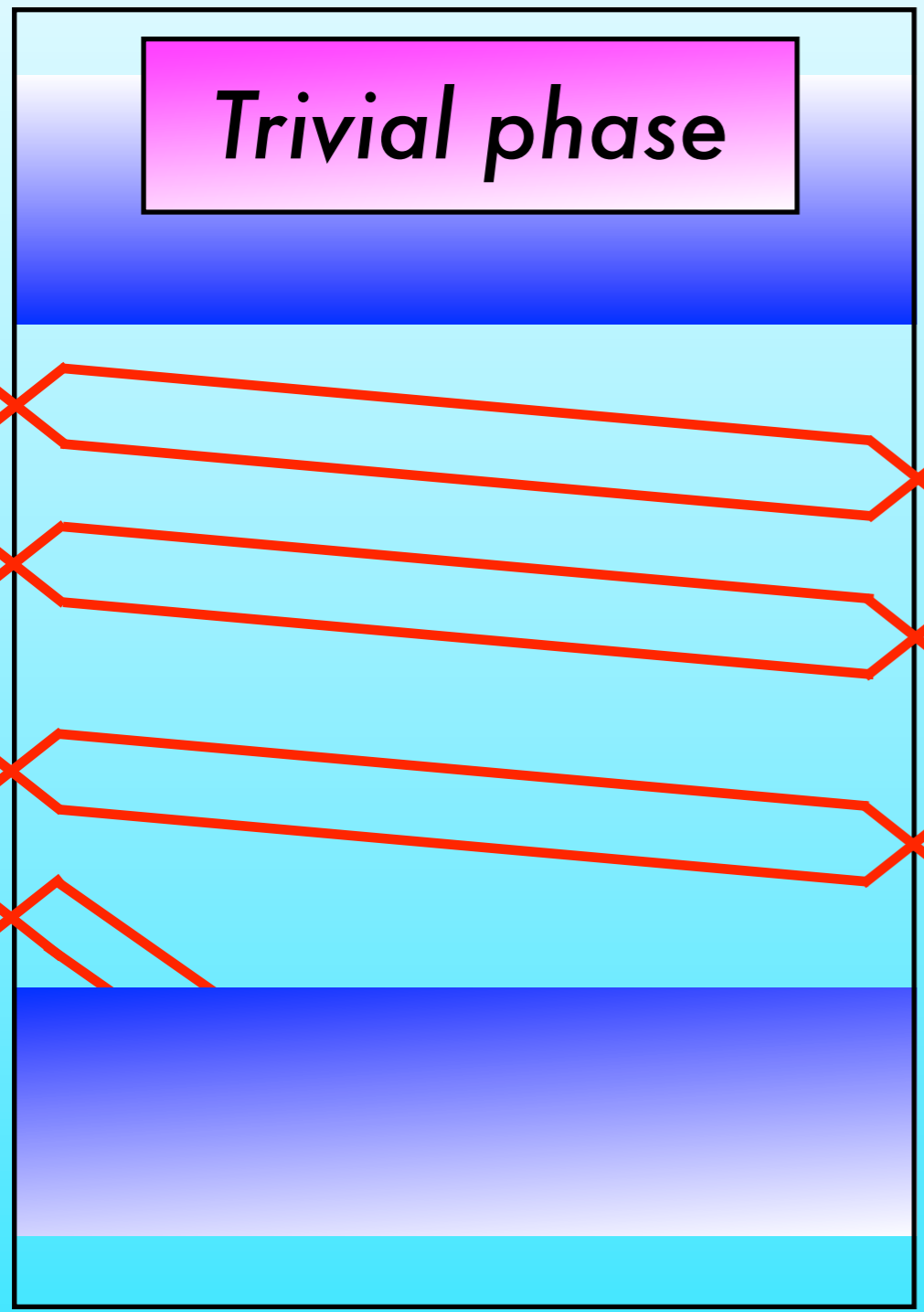
Non-trivial phase

Energy

Energy

0  $k_y$   $\pi$

0  $k_y$   $\pi$



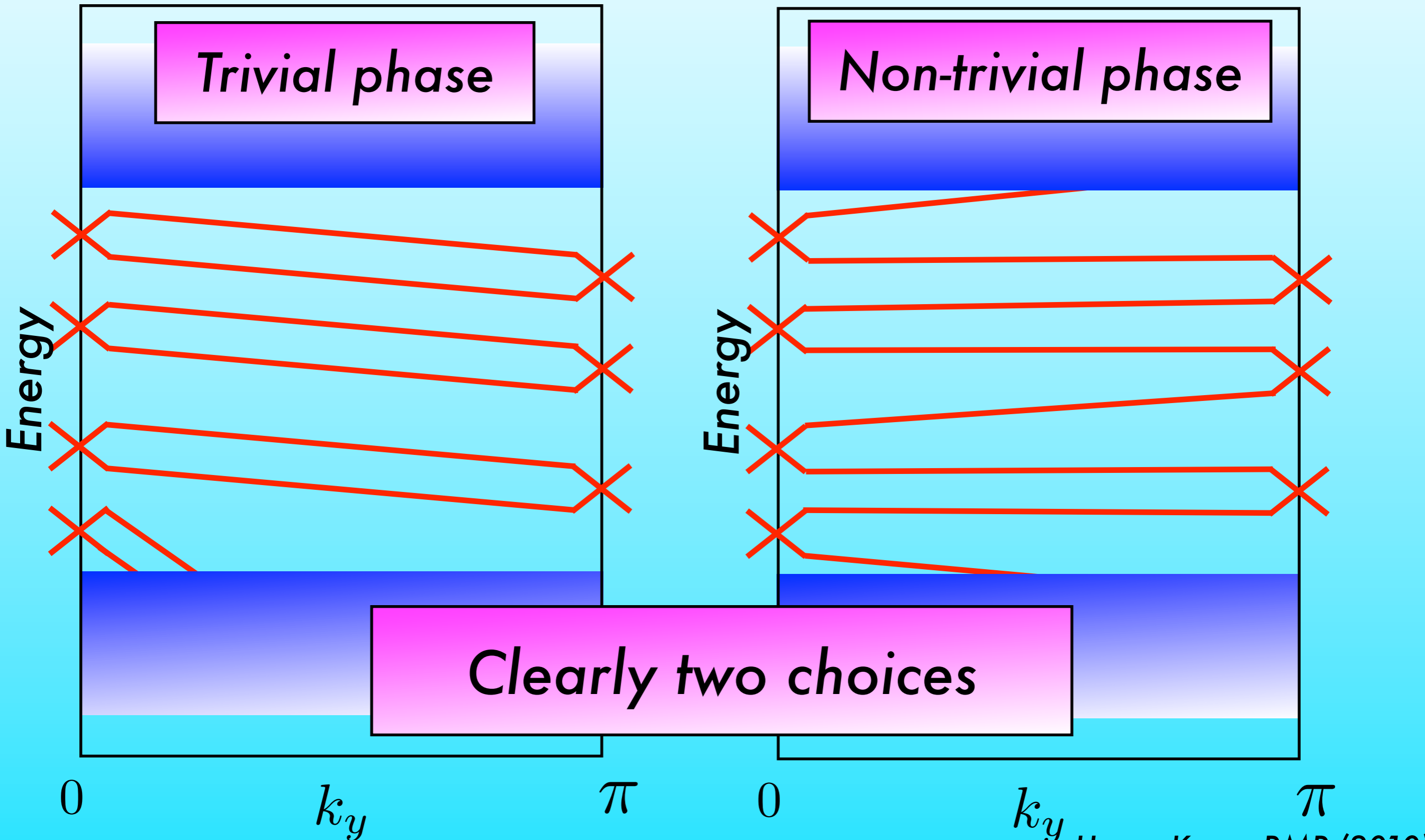
# Topologically protected edge states

TR invariant  
Point

TR invariant  
Point

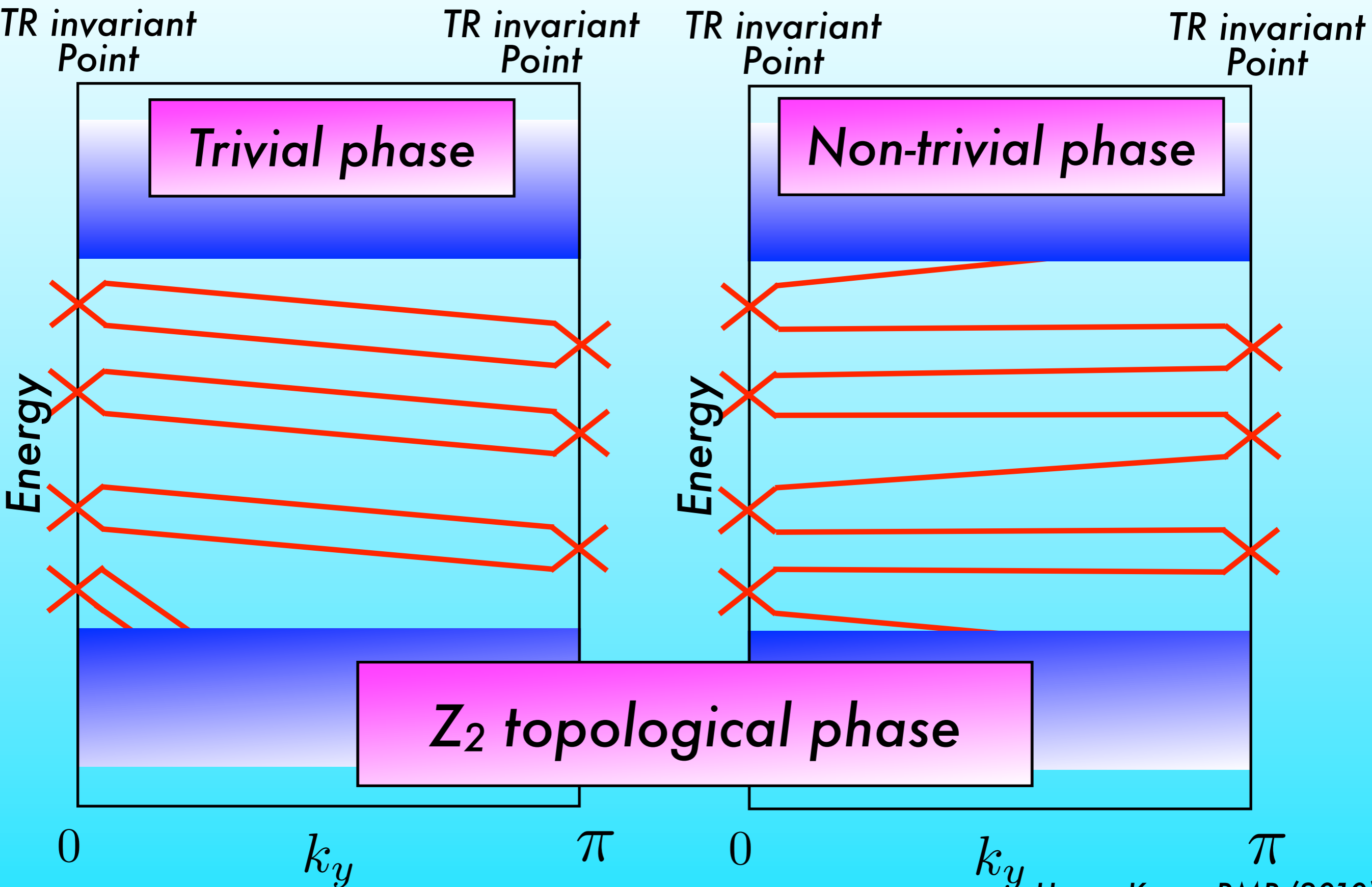
TR invariant  
Point

TR invariant  
Point



Clearly two choices

# Topologically protected edge states





$Z_2$  edge states

# Topology

Edges characterize the featureless bulk

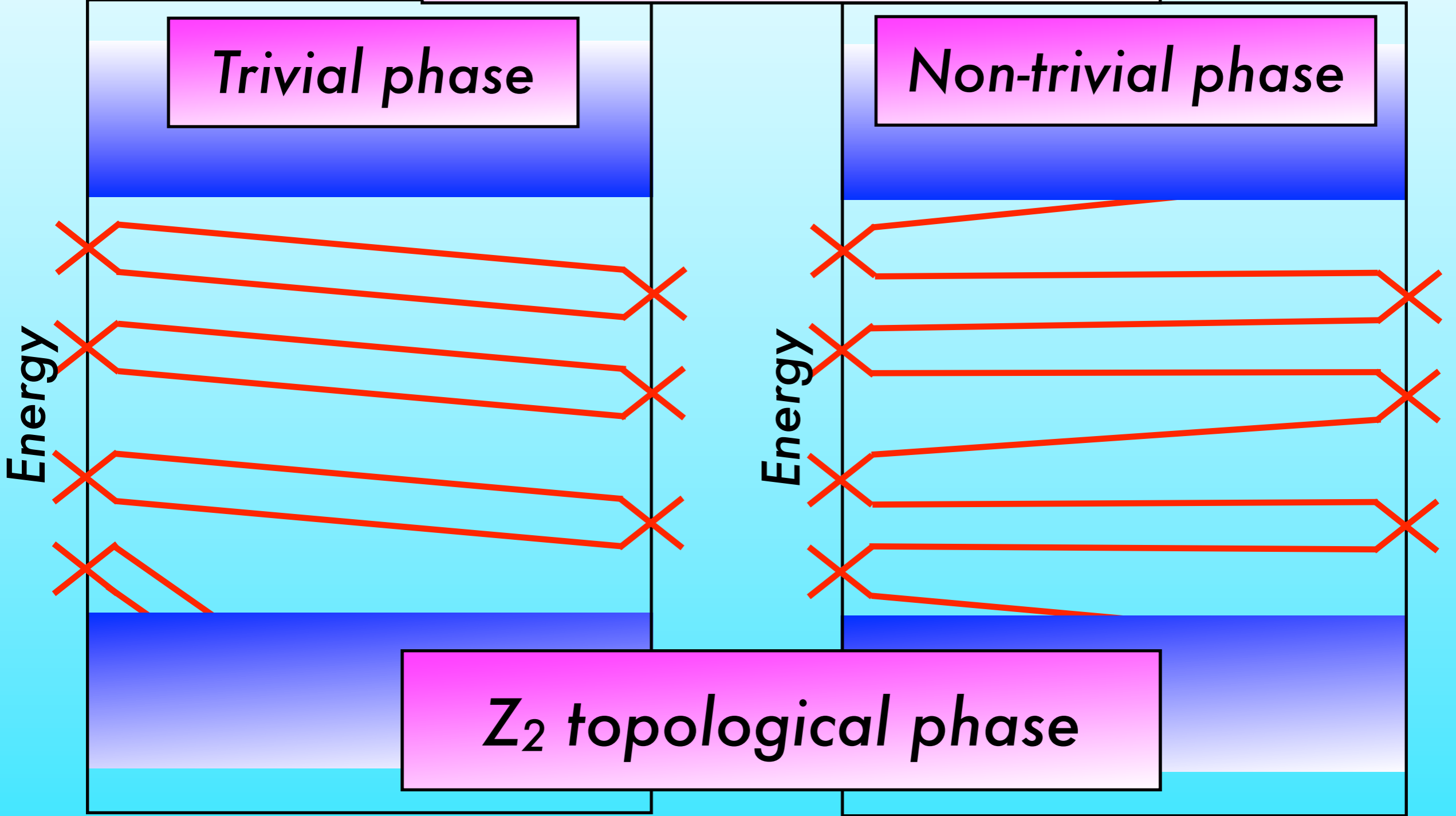
states

TR invariant Point

TR invariant Point

Trivial phase

Non-trivial phase



$Z_2$  topological phase

0  $k_y$   $\pi$  0  $k_y$   $\pi$

$Z_2$  edge states

Topology

Edges characterize the featureless bulk

states

TR invariant Point

TR invariant Point

Y. Hatsugai, Phys. Rev. Lett. 71, 3697 (1993)

Bulk-Edge correspondence

Universality



Bulk state  
(scattering state)  
Bulk Gap  
Non trivial Vacuum

Control with each other

Edge state  
(Bound state)  
Particles in the gap

QHE, Spin chains, Graphene, QSHE, ...

$Z_2$  topological phase

Energy

0

$k_y$

$\pi$

0

$k_y$

$\pi$

Hasan-Kane, RMP (2010)

$Z_2$  edge states

# Topologically protected edge states

TR invariant  
Point

TR invariant  
Point

TR invariant  
Point

TR invariant  
Point



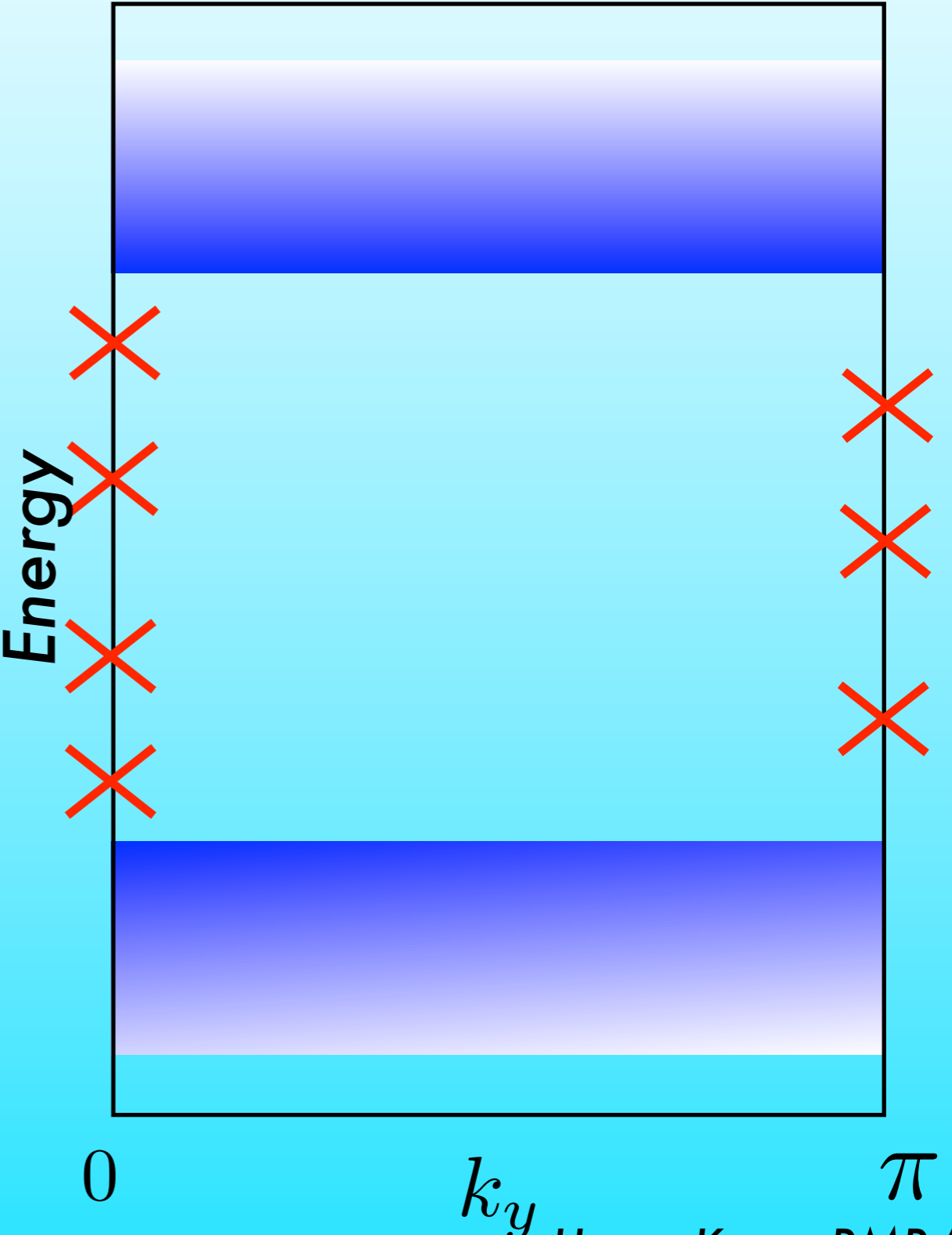
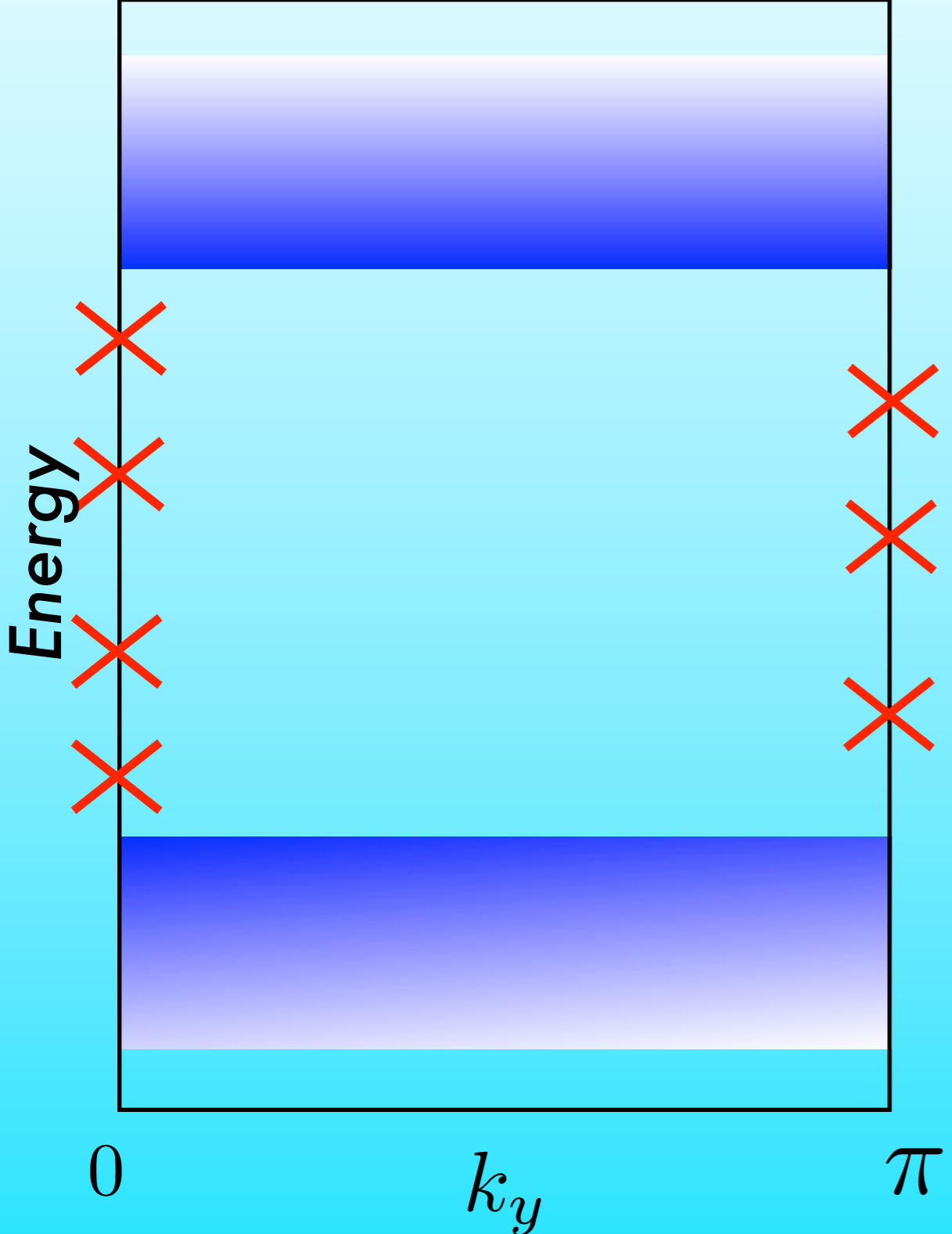
# Topologically protected edge states

TR invariant Point

TR invariant Point

TR invariant Point

TR invariant Point



$Z_2$  edge states

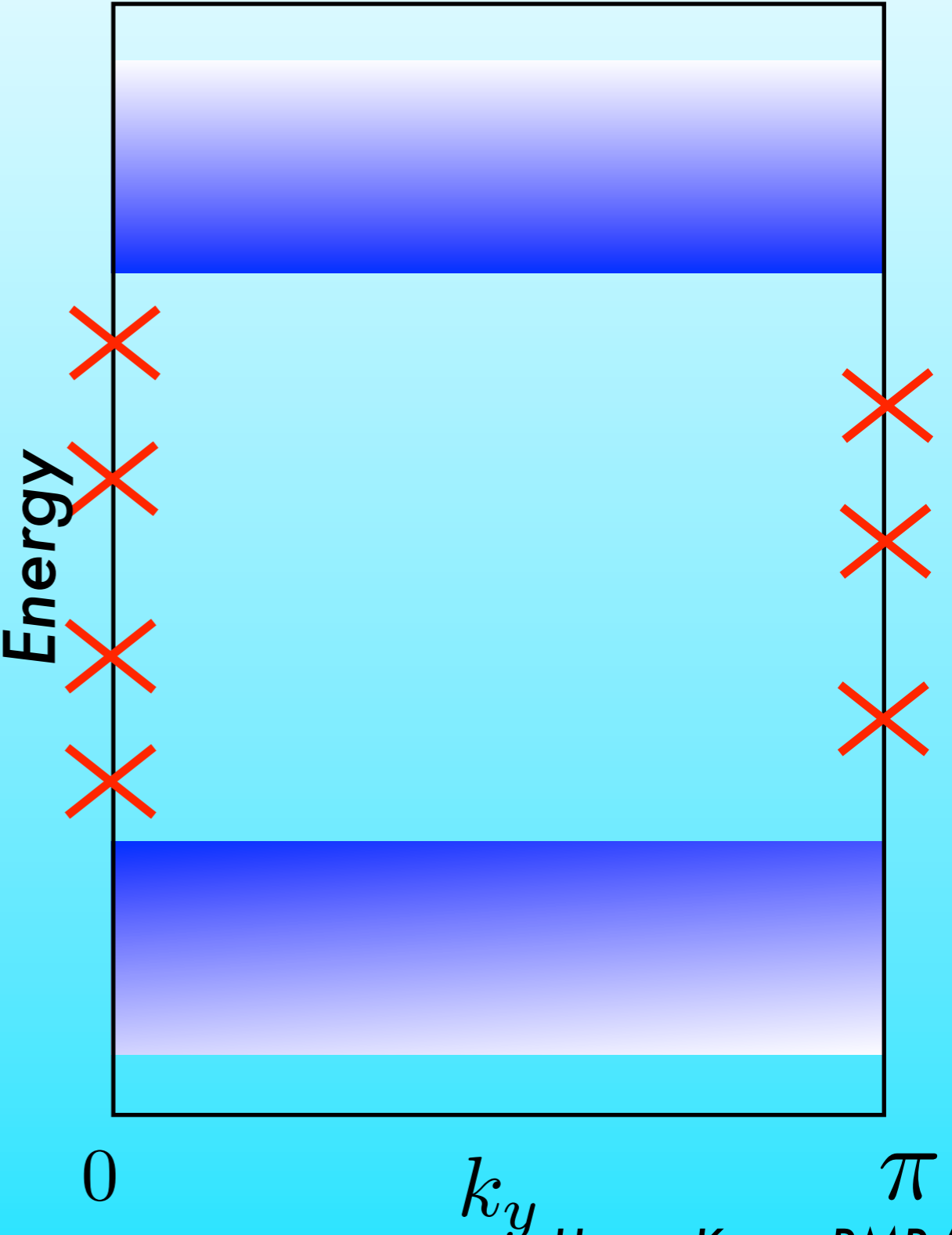
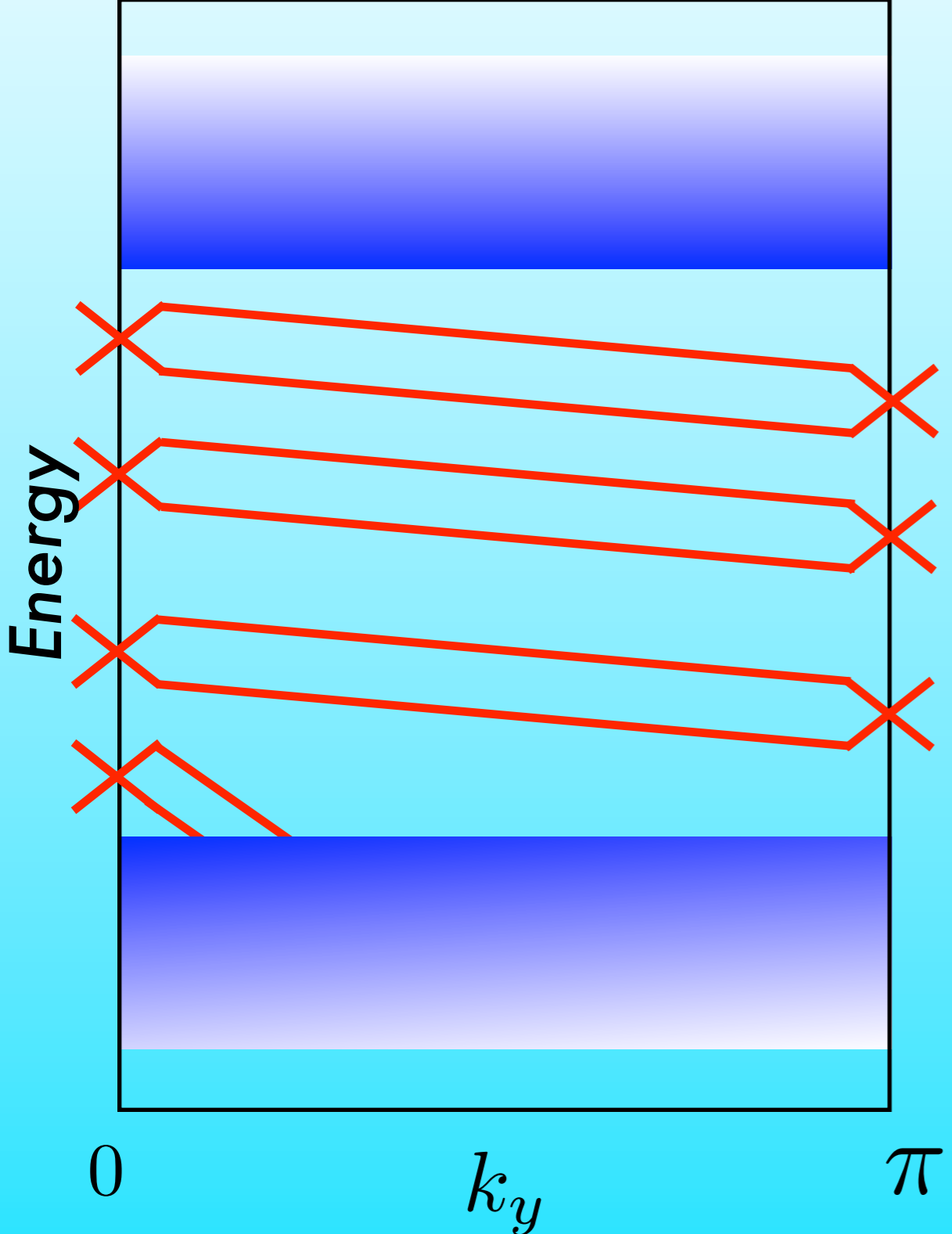
# Topologically protected edge states

TR invariant Point

TR invariant Point

TR invariant Point

TR invariant Point



# Topologically protected edge states

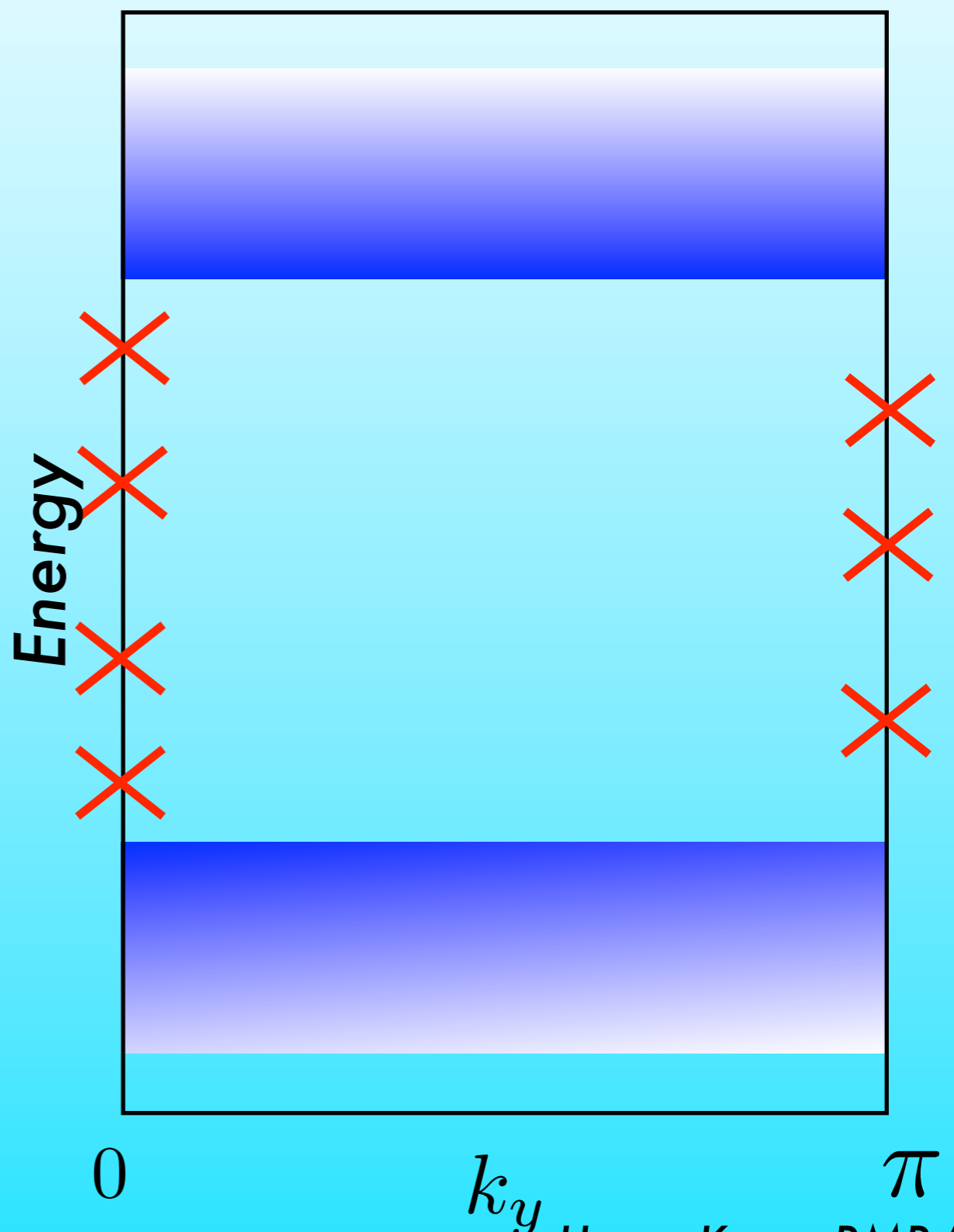
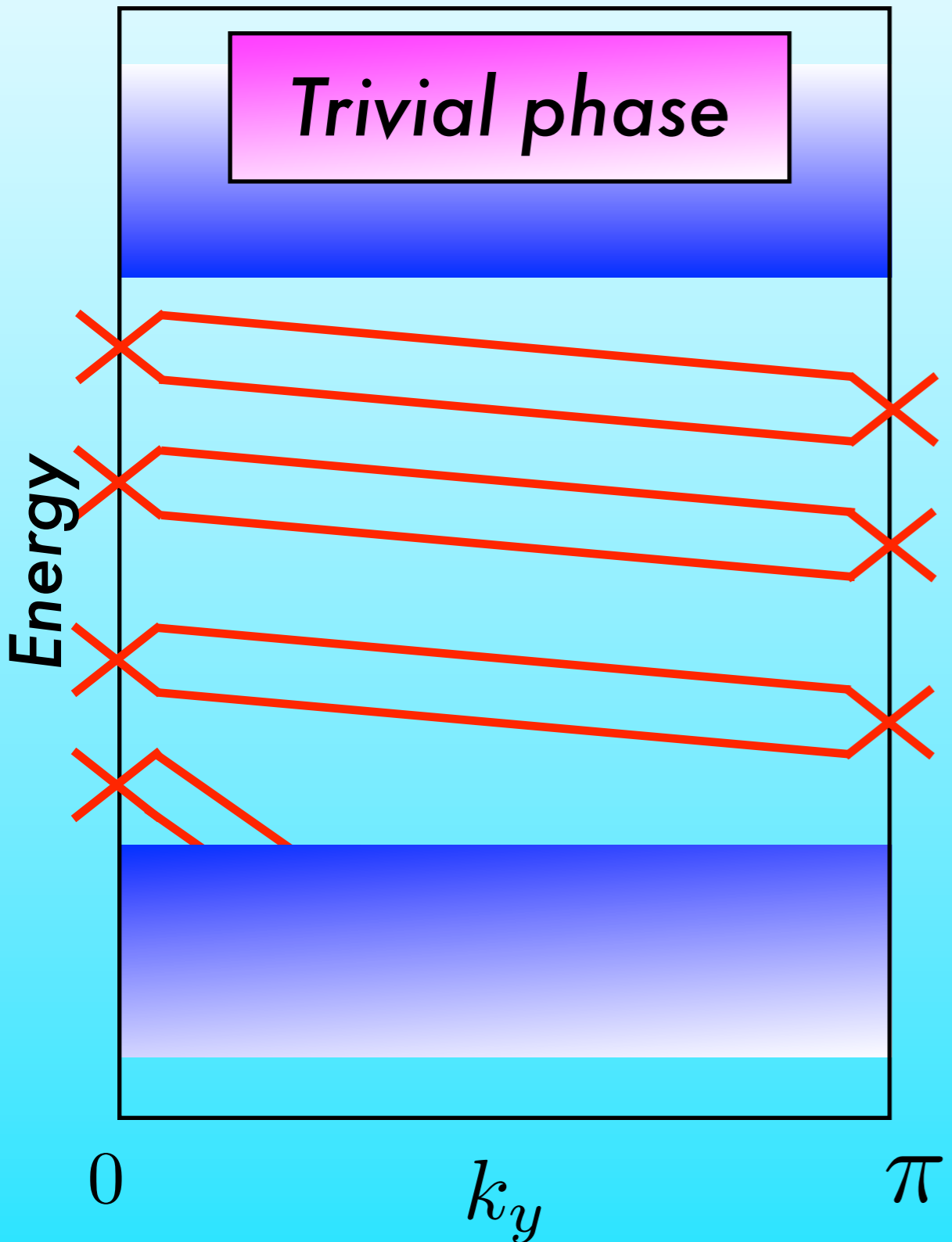
TR invariant Point

TR invariant Point

TR invariant Point

TR invariant Point

Trivial phase



# Topologically protected edge states

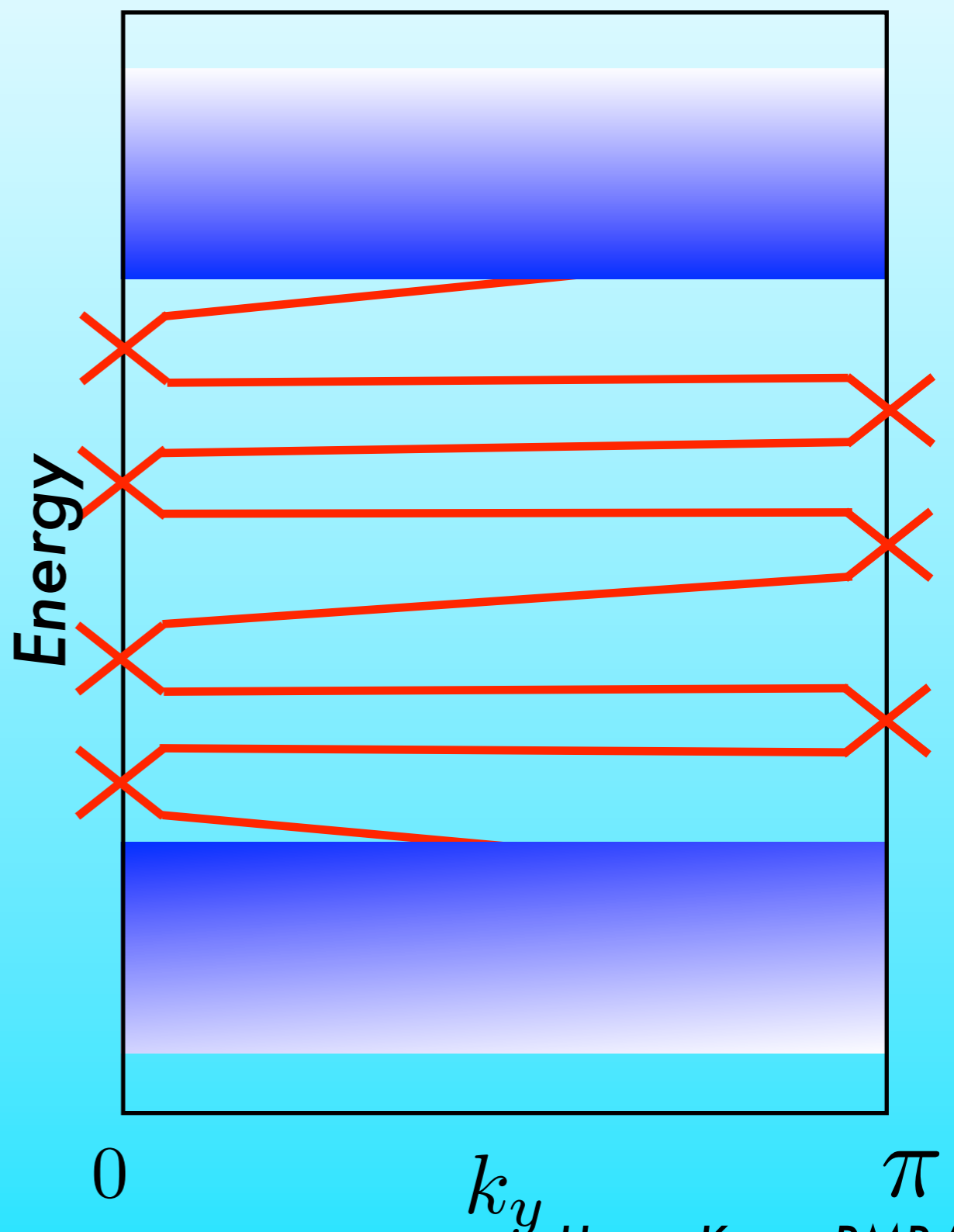
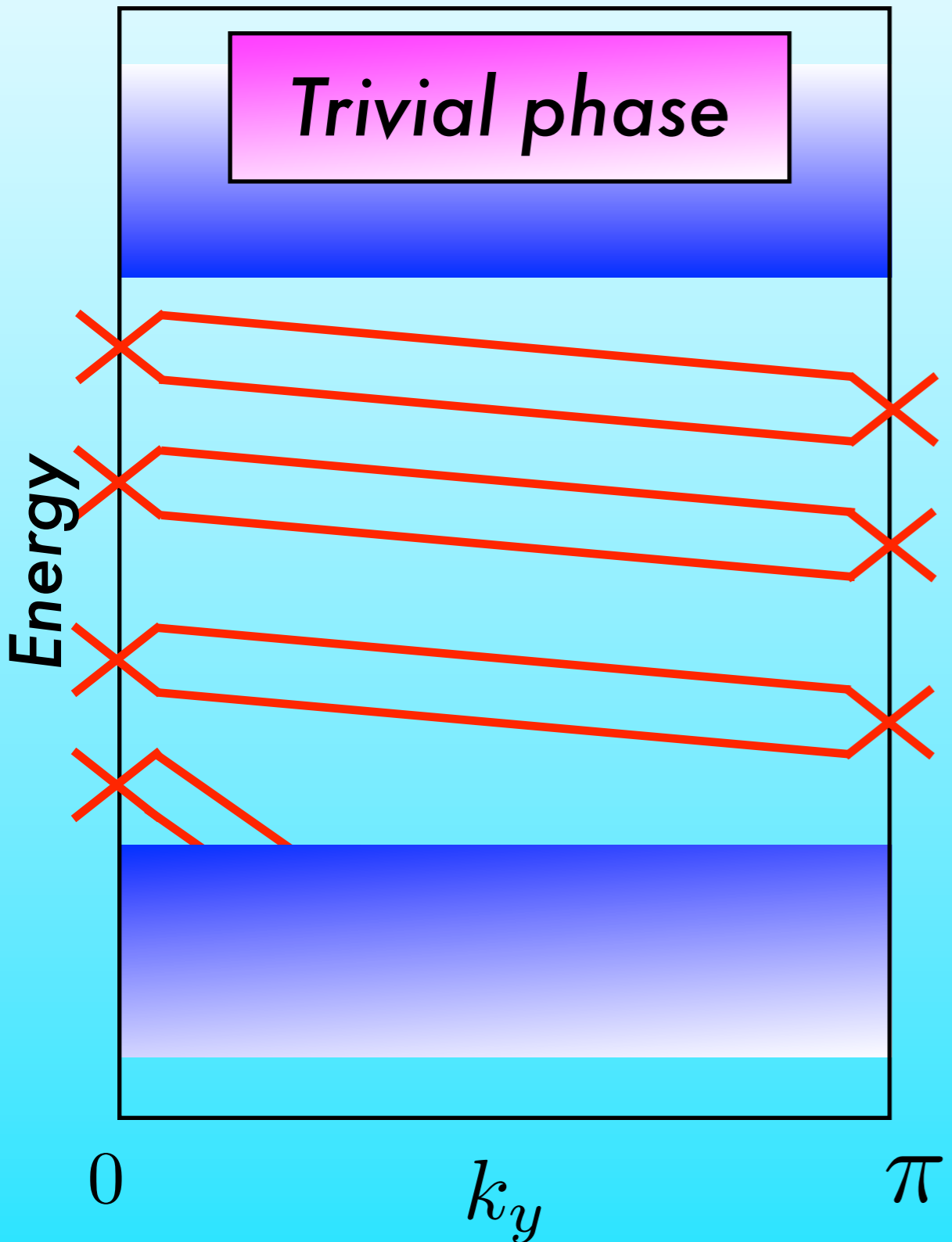
TR invariant Point

TR invariant Point

TR invariant Point

TR invariant Point

Trivial phase



# Topologically protected edge states

TR invariant Point

TR invariant Point

TR invariant Point

TR invariant Point

Trivial phase

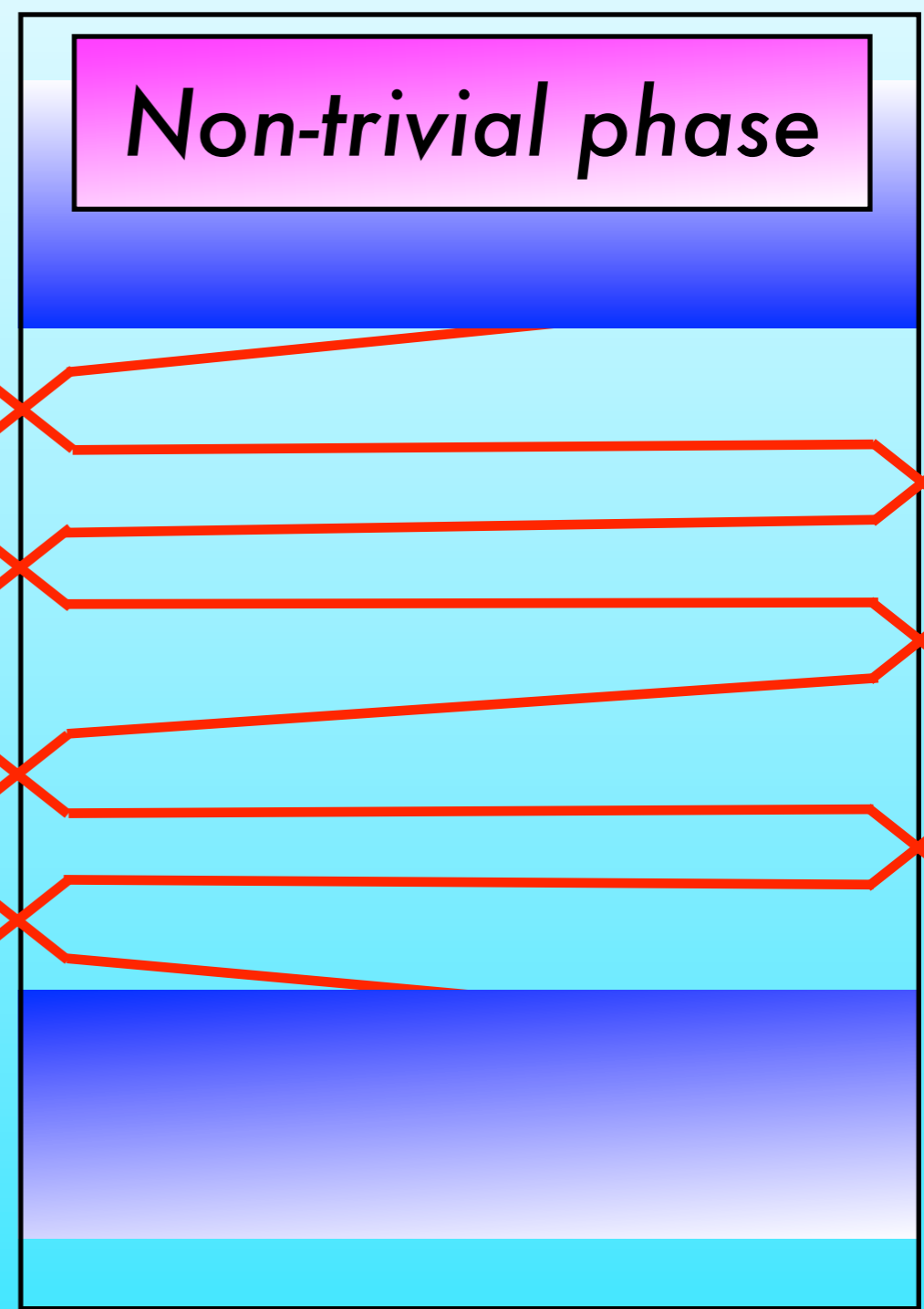
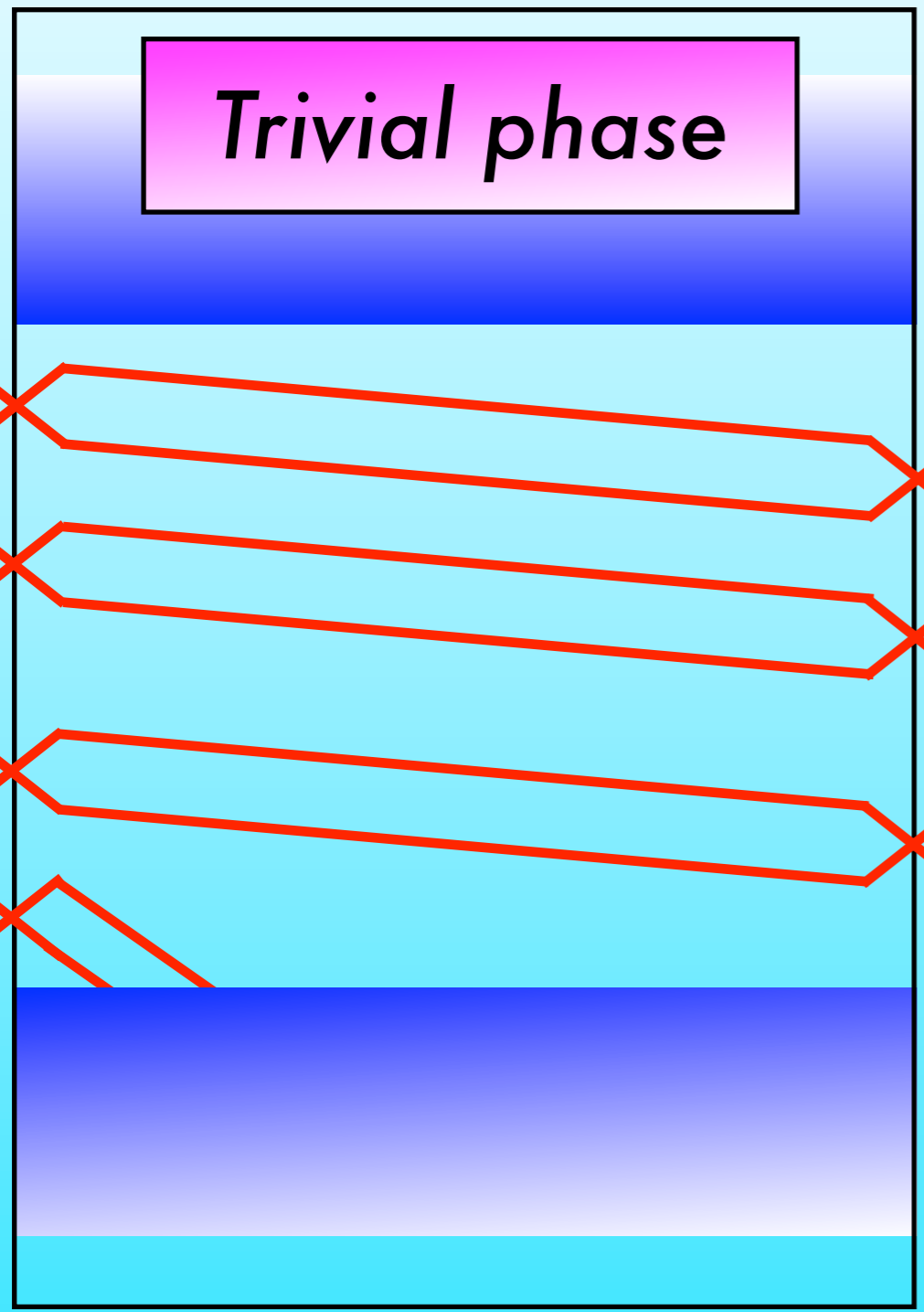
Non-trivial phase

Energy

Energy

0  $k_y$   $\pi$

0  $k_y$   $\pi$





# Topologically protected edge states

TR invariant Point

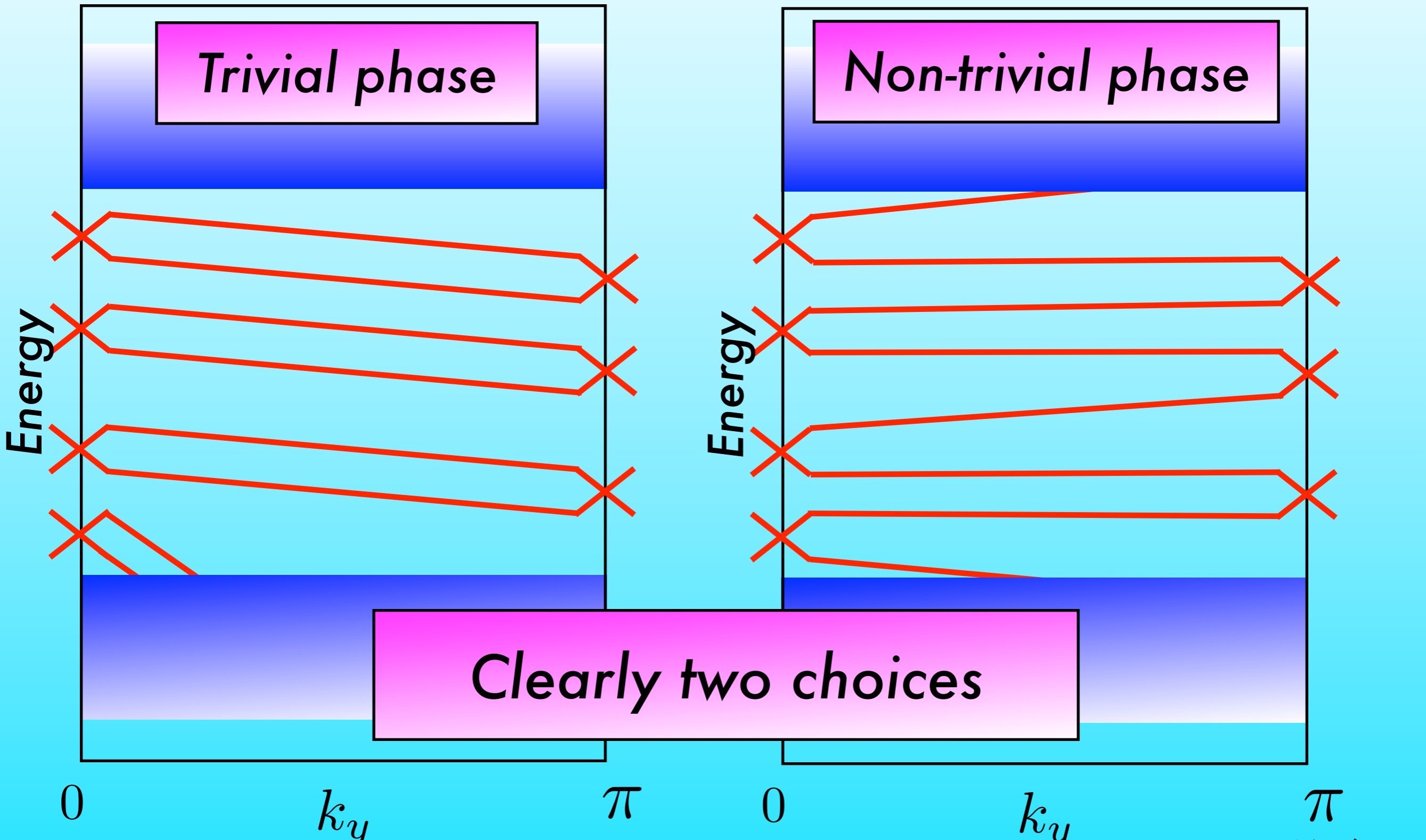
TR invariant Point

TR invariant Point

TR invariant Point

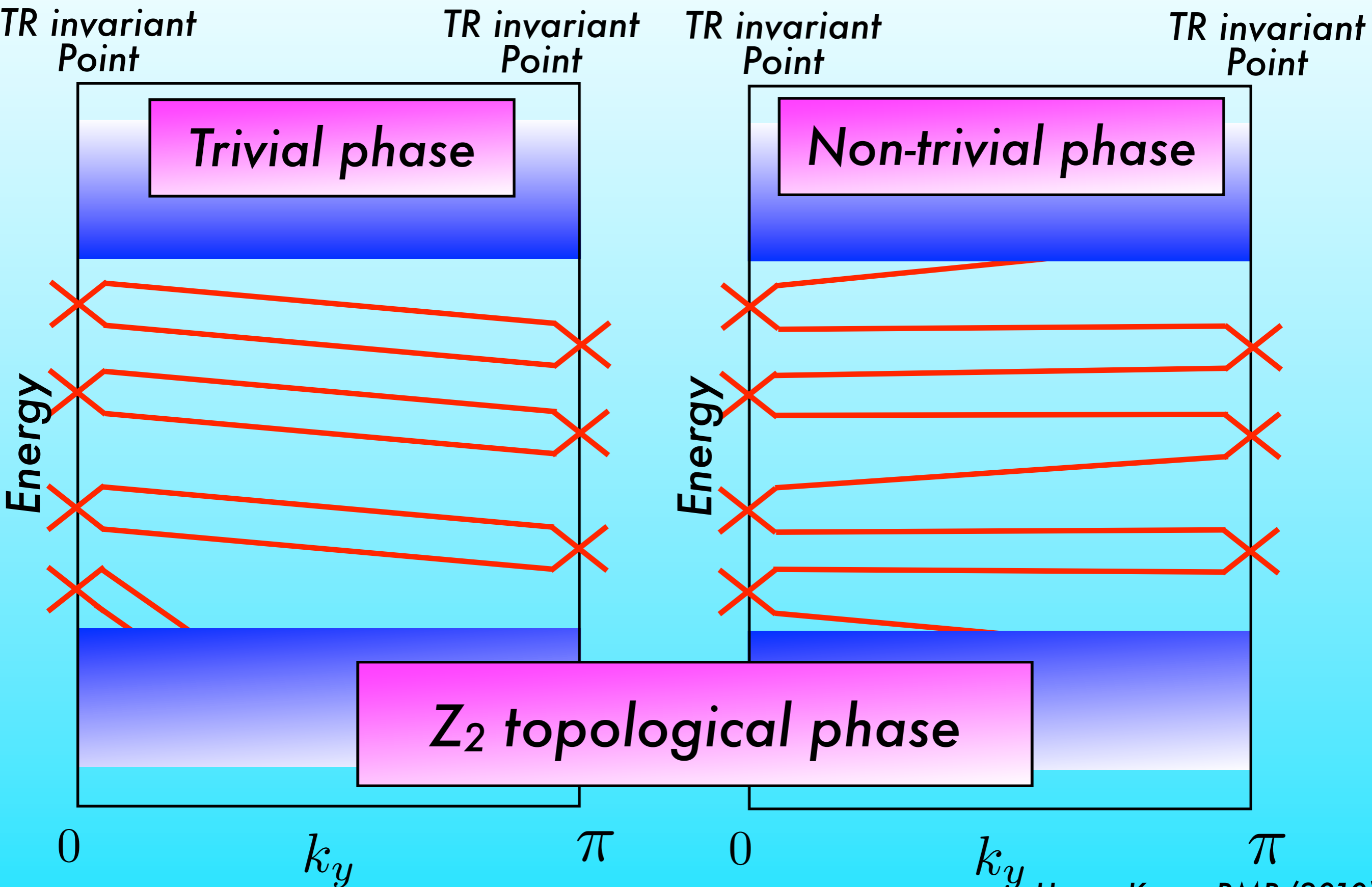
Trivial phase

Non-trivial phase



Clearly two choices

# Topologically protected edge states



$Z_2$  edge states

# Topology *Edges characterize the featureless bulk states*

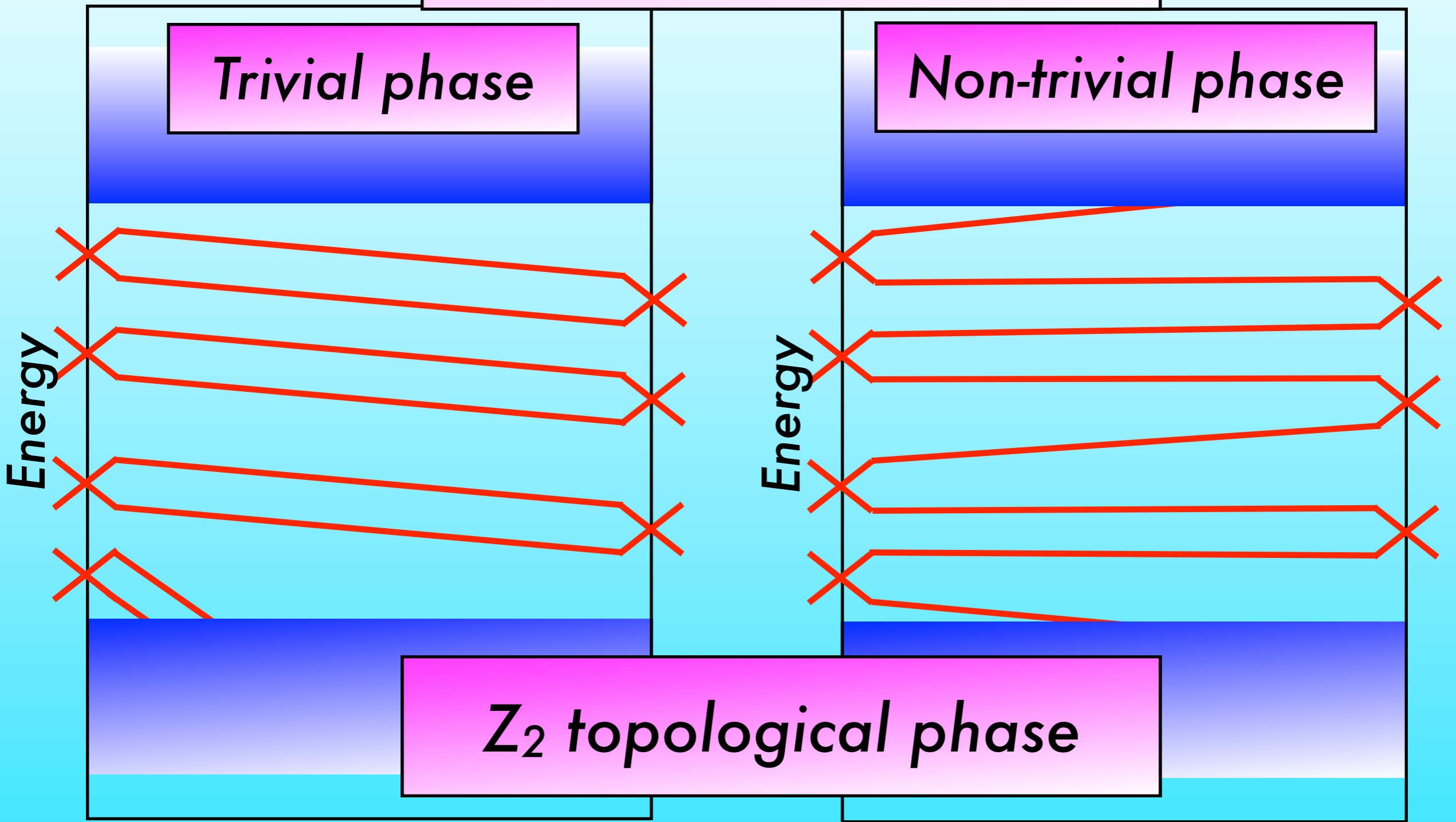
TR invariant Point

TR invariant Point

*Edges characterize the featureless bulk*

*Trivial phase*

*Non-trivial phase*



$Z_2$  topological phase

$Z_2$  edge states

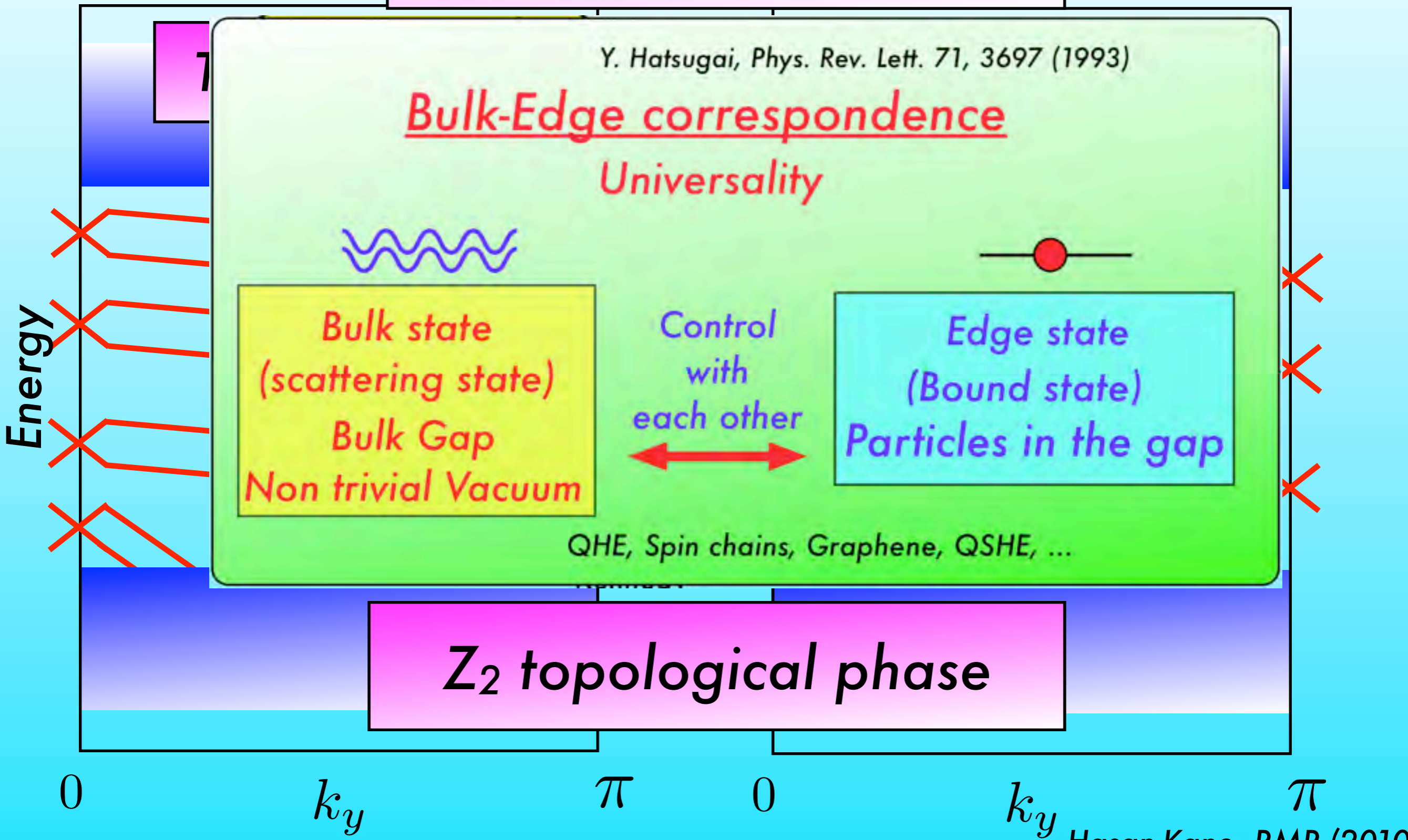
Topology

Edges characterize the featureless bulk

states

TR invariant Point

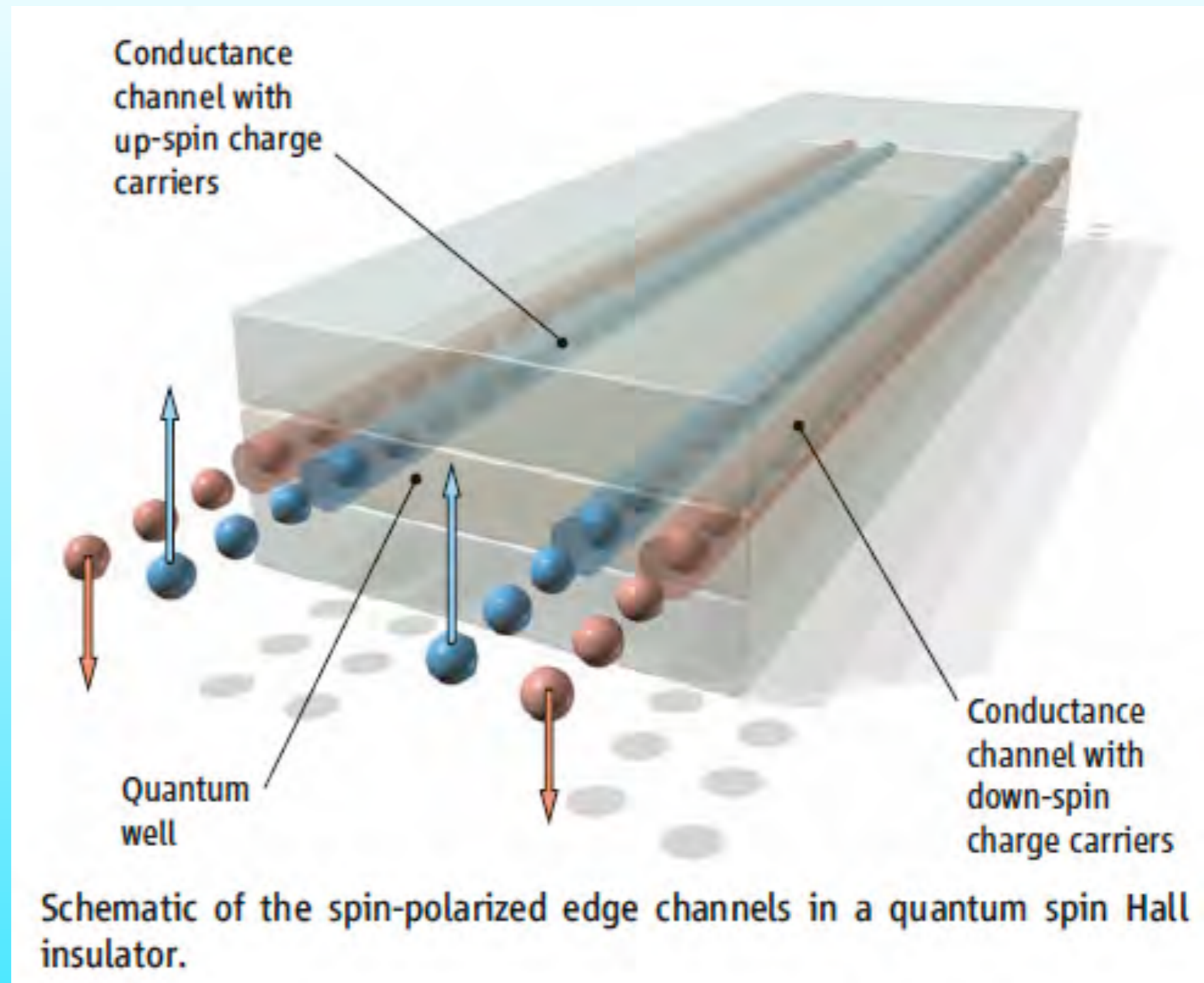
TR invariant Point



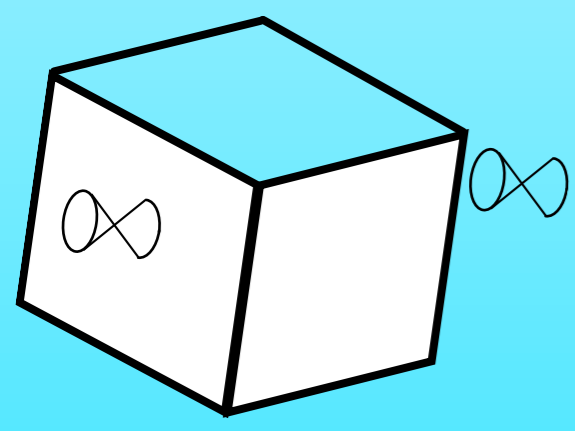
$Z_2$  topological phase

# Spin Hall edge states

2D



3D



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Konig, Wiedmann, Brüne, Roth, Hartmut Buhmann, Molenkamp, Qi and Zhang, Science 318, 776 (2007)