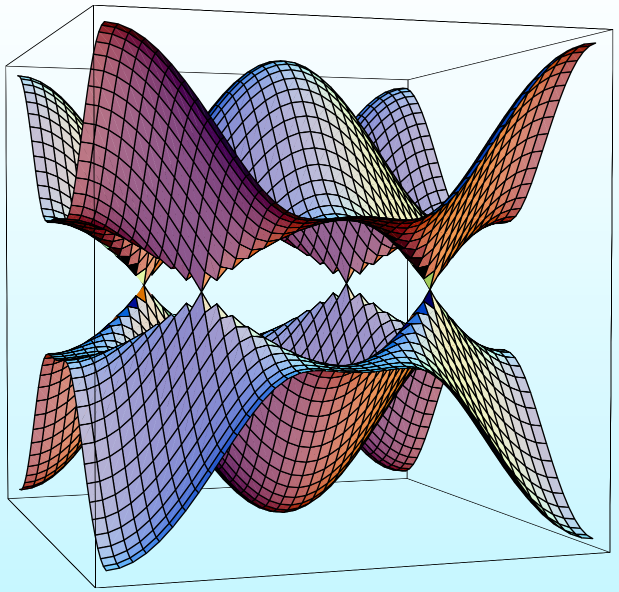
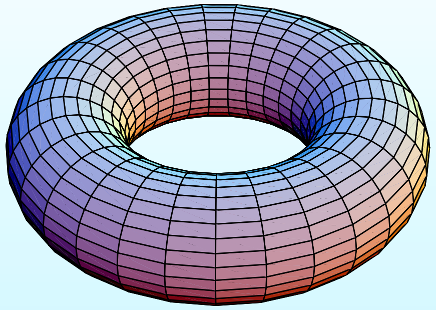
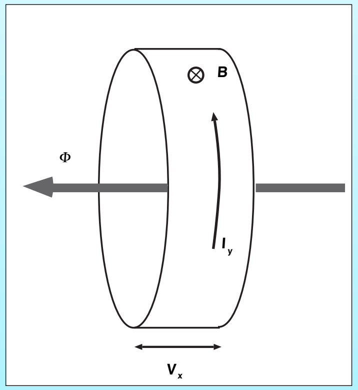
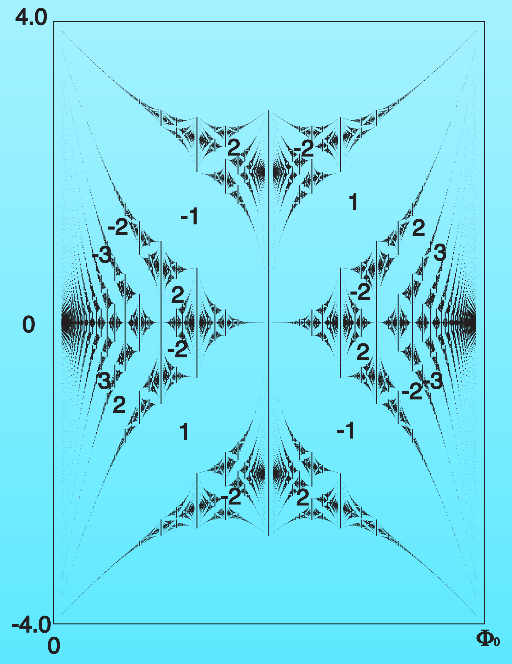




Explicit Gauge Fixing for Degenerate Multiplets



Generic Setup for Topological Orders



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Ref. Y. Hatsugai, cond-mat/0405551, to appear in JPSJ Lett.

Plan of today's Talk

- Topological Order
  - Universal ?   Useful?
  - Need Physical Quantities without Any Symmetry Breaking
- Chern Numbers
  - (Spin) Hall Conductance
  - Gauge Independent Quantities
  - Need Explicit Gauge Fixing
- Novel Generic Explicit Gauge Fixing
  - Allow Degeneracies
    - ex. Unitary Superconductors
  - Not Necessarily for Hermit Operators:
    - Normal Operators

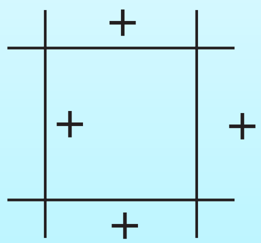
Topological Orders: Useful !

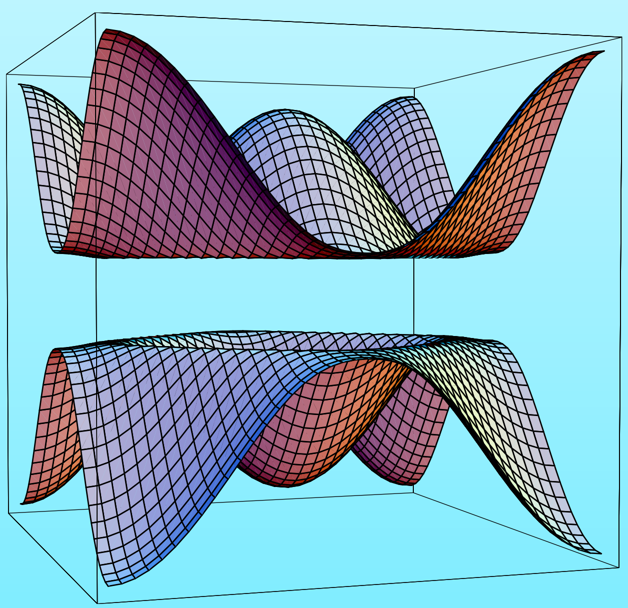
- Quantum Hall Effects (Integer & Fractional)
  - Bulk v.s. Edge
- Haldane Spin Chain with Integral Spin
  - Energy Gap, Localized Kennedy Triplets as Edge States
- Polyacetylene (SSH model)
- High-Tc (Chiral Spin State, Anyon-superconductivity)
- Chirality Order in Itinerant Magnetism
- Polarization of Insulators, KSV formula
- Anisotropic Superconductivity & Superfluidity
  - Zero Bias Peak and Boundary Time Reversal Symmetry Breaking as a Pierles Instabilities of 1D Zero modes Bands
  - Topological Origin of Gap Nodes of Superconductor
- Carbon Nano-Structures
  - Boundary Local Moments as Edge States, Pierles Instabilities

Various Superconductors: Symmetry of the Order Parameter is Enough?

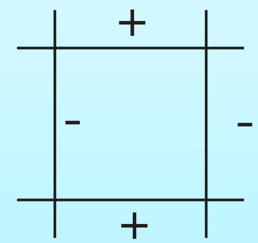
- 2D Examples (Singlet)

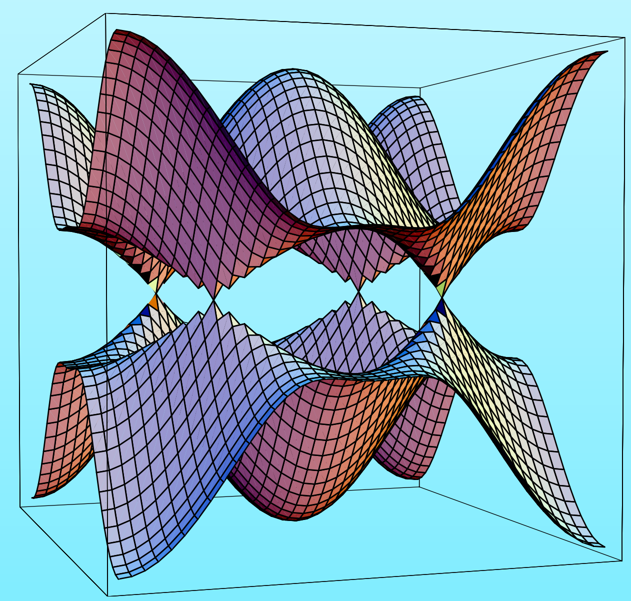
s-wave



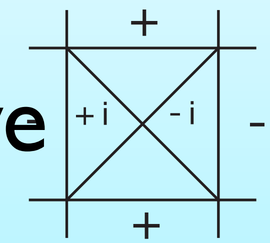


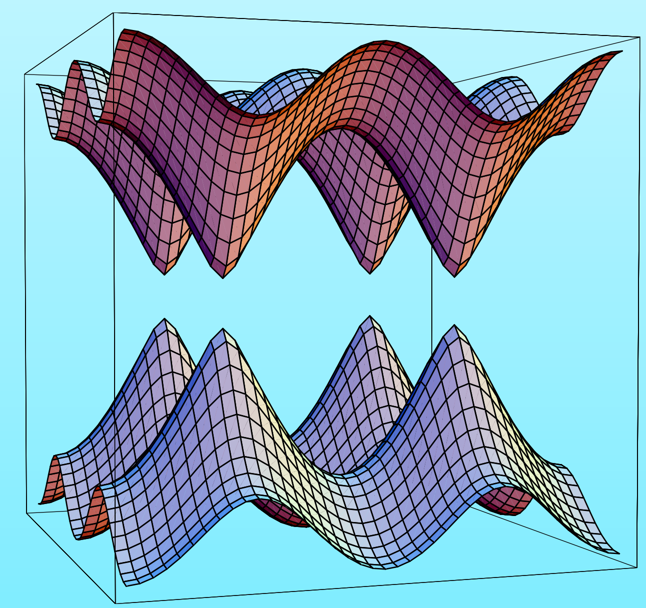
d-wave



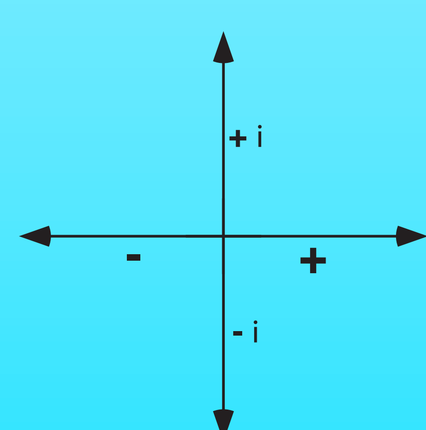


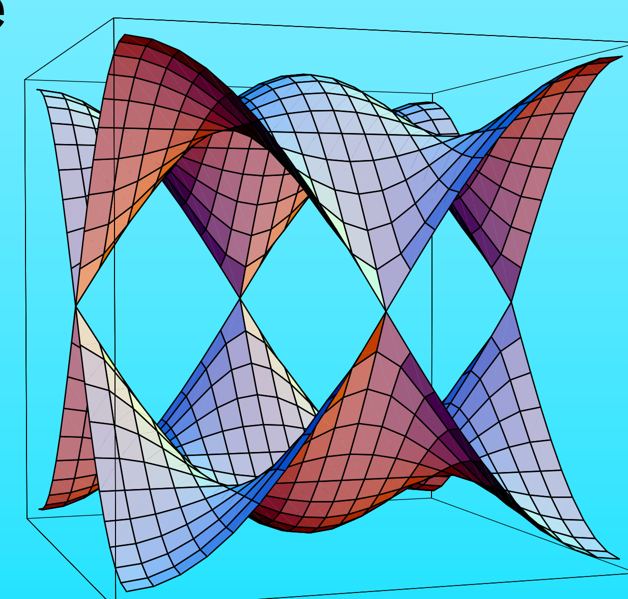
d+id-wave





- 2D Examples (Triplet) : Chiral p wave





What is a Topological Order?

- Try to Characterize Quantum Mechanical states

  Without Any Symmetry Breaking
- Use Geometrical Phases
  - Gauge Structures naturally appear
- Need Physical Quantities : Topological Numbers
  - The (first) Chern Numbers
  - Gauge Invariants
- Need Explicit Gauge Fixing to Calculate
- Allow Degeneracies

Template: Quantum Hall Effects

- Hall Conductance as a topological Numbers

$$\sigma_{xy} = \frac{e^2}{h} N_j, \quad N_j = \sum_{\epsilon_j(k)<E_F} C_j$$

- Chern Number C_j

$$C_j = \frac{1}{2\pi i} \int d^2k\, \boldsymbol{\mathcal{B}}_z(\boldsymbol{k})$$

$$\boldsymbol{\mathcal{B}}(\boldsymbol{k}) = \boldsymbol{\nabla}_k \times \boldsymbol{\mathcal{A}}(\boldsymbol{k}), \quad \boldsymbol{\mathcal{A}}(\boldsymbol{k}) = \langle \boldsymbol{k} | \boldsymbol{\nabla}_k \boldsymbol{k} \rangle$$

- How to define a well defined Derivative ?

$$\boldsymbol{H}(\boldsymbol{k})\, \boldsymbol{\psi}(\boldsymbol{k}) = \epsilon(\boldsymbol{k})\, \boldsymbol{\psi}(\boldsymbol{k}) \qquad \boldsymbol{\psi}(\boldsymbol{k}) \to e^{i\theta}\, \boldsymbol{\psi}(\boldsymbol{k})$$

Phase is arbitrary

Kohmoto 85

Rule: Take the first component be real positive (Gauge fixing)

- When the states have a Degeneracy, we need More.

$$a(\boldsymbol{k})\psi_a(\boldsymbol{k}) + b(\boldsymbol{k})\psi_b(\boldsymbol{k})$$

Definitions:
Normal Operators and Multiplets

☆ Normal N dim. Matrix ($L^\dagger L = LL^\dagger$) : Diagonalizable by a Unitary Matirix

$$\boldsymbol{L}(x)\psi_i(x) = \epsilon_i(x)\psi_i(x) \quad x \in V : \text{Parameter space}$$

$$\text{ex. } \boldsymbol{L}(x) = H(k_x, k_y)$$

$$\boldsymbol{L}\boldsymbol{\mathcal{U}} = \boldsymbol{\mathcal{U}}\boldsymbol{\mathcal{E}}, \qquad \boldsymbol{\mathcal{E}} = \mathrm{diag}\left(\epsilon_1, \cdots, \epsilon_N\right)$$

☆ Multiplet: M dim. Linear space spanned by $\{\psi_1(x), \psi_2(x), \cdots, \psi_M(x)\}$

$$\begin{aligned} W &= W_1 \oplus W_2 \oplus \cdots \oplus W_M \\ &= \{c_1\psi_1 + c_2\psi_2 + \cdots + c_M\psi_M\}, \ c_j \in \mathbb{C} \end{aligned}$$

N×M Matrix

$$\boldsymbol{\Psi}(x) = (\psi_1, \psi_2, \cdots, \psi_M) = N \overset{M}{\boxed{\boldsymbol{\Psi}}} = \psi_1 \cdots \psi_M$$

$$\boldsymbol{\Psi}^\dagger \boldsymbol{\Psi} = \boldsymbol{I}_M \quad : \text{Orthonormalized } \psi_i^\dagger \psi_j = \delta_{ij}$$

Base Change of the Multiplet

- Base Change in W $$\Psi' = \Psi\omega, \quad \omega^\dagger\omega = \omega\omega^\dagger = \boldsymbol{I}_M$$
$$\Psi'^\dagger\Psi' = \boldsymbol{I}_M$$

- If L has a degenerate eigen space !

$$\boldsymbol{L}\psi_j = \epsilon\psi_j \rightarrow \boldsymbol{L}\psi'_j = \epsilon\psi'_j$$
$$\psi'_j = \omega_{j1}\psi_1 + \omega_{j2}\psi_2 + \cdots \omega_{jM}\psi_M$$

- We may allow this base change even without degeneracy

as for just a linear space

What do we need to have a well-defined multiplet ?

Projection and Generic Gap

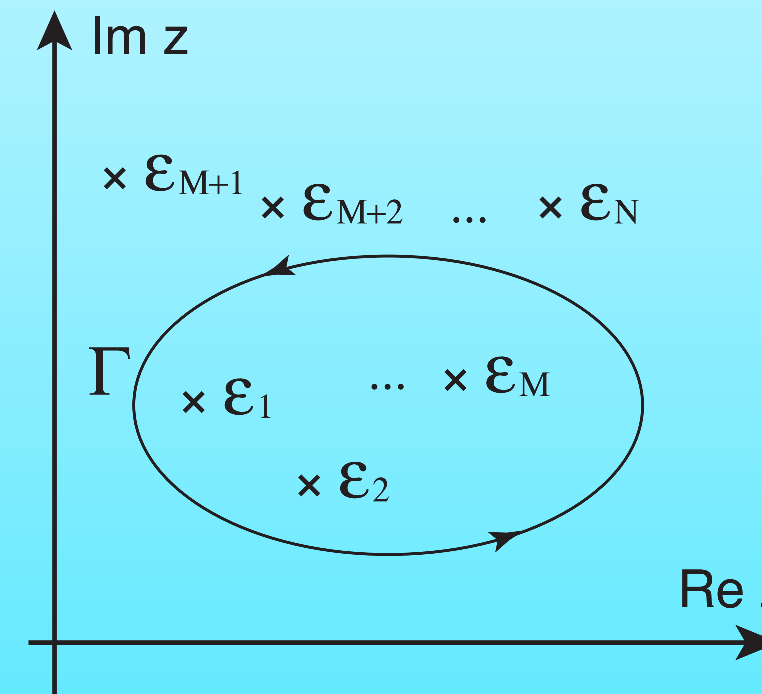
- Projection into W should be well-defined!

$$\boldsymbol{P} = \boldsymbol{\Psi}\boldsymbol{\Psi}^\dagger = \boldsymbol{\Psi}'\boldsymbol{\Psi}'^\dagger = \boxed{\boldsymbol{\Psi}}\ \boxed{\boldsymbol{\Psi}^\dagger} = \boxed{\boldsymbol{P}}$$

- Integral Expression of P (T. Kato)

$$\boldsymbol{P} = \sum_{i=1}^{M} \boldsymbol{P}_i = \frac{1}{2\pi i}\oint_\Gamma \boldsymbol{G}_T(z)$$

$$\boldsymbol{G}_T(z) = \frac{1}{z\boldsymbol{I} - \boldsymbol{L}} = \sum_i \frac{\boldsymbol{P}_i}{z - \epsilon_i}, \quad \boldsymbol{P}_i = \psi_i \psi_i^\dagger$$



- Condition for a Generic Gap

$$\epsilon_i(x) \neq \epsilon_j(x) \ {}^\forall x \in V$$

$$i \in I^{in} = \{1, \cdots, M\}, \ j \in I^{out} = \{M+1, \cdots, N\}$$

Connection and Gauge Transformation

☆ Connection (Non-Abelian)

$$\mathcal{A} = \mathbf{\Psi}^\dagger d\mathbf{\Psi} = \begin{array}{cc} M & \boxed{\quad \mathbf{\Psi}^\dagger \quad} \\ & N \end{array} \begin{array}{cc} & M \\ N & \boxed{\quad d\mathbf{\Psi} \quad} \end{array} = \begin{array}{cc} M & \\ \boxed{\quad \mathbf{\Psi}^\dagger d\mathbf{\Psi} \quad} & M \end{array}$$

$$= \mathbf{\Psi}^\dagger \frac{\partial \mathbf{\Psi}}{\partial x_i} dx_i$$

☆ Gauge Transformation (Base Change)

$$\mathbf{\Psi}' = \mathbf{\Psi}\boldsymbol{\omega}$$

$$\mathcal{A}' = \mathbf{\Psi}' d\mathbf{\Psi}'$$

$$= \boldsymbol{\omega}^{-1} \boldsymbol{\mathcal{A}} \boldsymbol{\omega} + d \log \boldsymbol{\omega}$$

☆ Field Strength

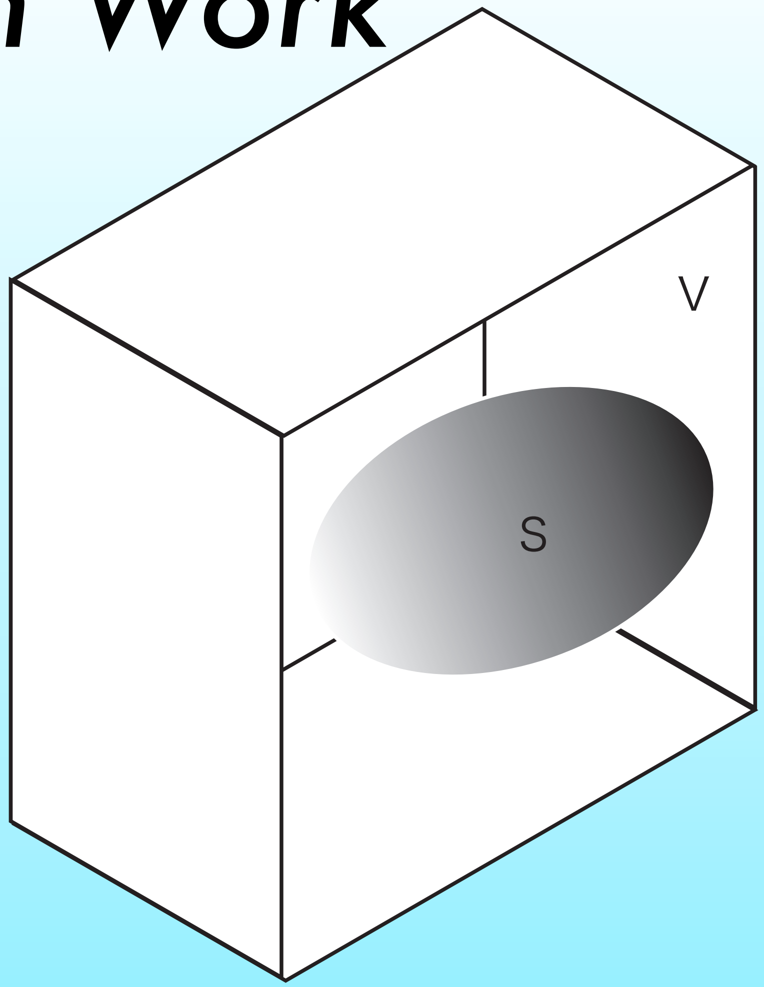
$$\mathcal{F} = d\boldsymbol{\mathcal{A}} + \boldsymbol{\mathcal{A}}\boldsymbol{\mathcal{A}}$$

$$\mathcal{F}' = \boldsymbol{\omega}^{-1} \mathcal{F} \boldsymbol{\omega}$$

$$\mathrm{Tr}_M \mathcal{F} = \mathrm{Tr}_M \mathcal{F}' = \mathrm{Tr}\ d\boldsymbol{\mathcal{A}}$$

cf. Wilczek & Zee (Berry Phase)

Chern Numbers and Patch Work



☆ Chern Numbers

$$C_S = \frac{1}{2\pi i}\int_S \mathrm{Tr}\,\boldsymbol{\mathcal{F}} = \frac{1}{2\pi i}\int_S \mathrm{Tr}\, d\boldsymbol{\mathcal{A}}$$

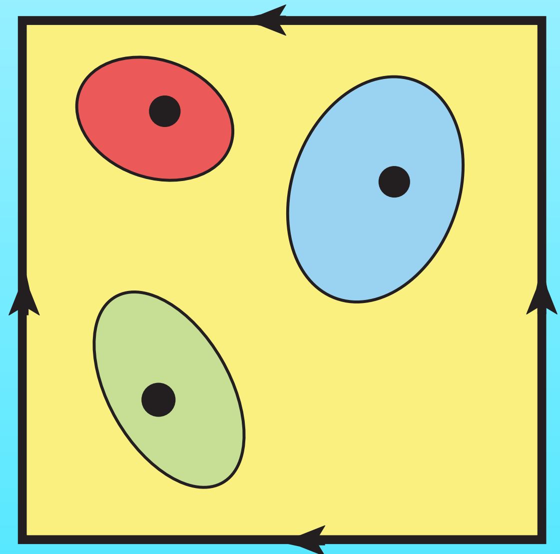
☆ Patch Work on S

$$C_S = \frac{1}{2\pi i}\sum_R \int_{S_i} d\mathrm{Tr}\,\boldsymbol{\mathcal{A}}_R = \frac{1}{2\pi i}\sum_R \int_{\partial S_R} \mathrm{Tr}\,\boldsymbol{\mathcal{A}}_R$$

$$= \frac{1}{2\pi i}\sum_{R\geq 1} \int_{\partial S_R} \mathrm{Tr}\,(\boldsymbol{\mathcal{A}}_R - \boldsymbol{\mathcal{A}}_0)$$

$$= \frac{1}{2\pi}\sum_{R\geq 1} \int_{\partial S_R} \mathrm{Im}\,\mathrm{Tr}\,\boldsymbol{\omega}_{0R}^{-1} d\boldsymbol{\omega}_{0R}$$

$$= \frac{1}{2\pi}\sum_{R\geq 1} \int_{\partial S_R} \mathrm{Im}\,\mathrm{Tr}\, d\log \boldsymbol{\omega}_{0R}$$



$$\boldsymbol{\mathcal{A}}(x) = \boldsymbol{\mathcal{A}}_R(x), \quad x \in S_R, \quad S = \cup S_R, \quad R = 0, 1, 2, \cdots$$

$$\boldsymbol{\Psi}_R = \boldsymbol{\Psi}_0 \boldsymbol{\omega}_{0R}$$

$$\boldsymbol{\mathcal{A}}_R = \boldsymbol{\omega}_{0R}^{-1} \boldsymbol{\mathcal{A}}_0 \boldsymbol{\omega}_{0R} + \boldsymbol{\omega}_{0R}^{-1} d\boldsymbol{\omega}_{0R}$$

Sum Rule and Quantum Phase Transition

- Direct Sum of the Linear Space $W = W_1 \oplus W_2$

$$\boldsymbol{\Psi} = (\boldsymbol{\Psi}_1, \boldsymbol{\Psi}_2) = \begin{bmatrix} \boldsymbol{\Psi}_1 & \boldsymbol{\Psi}_2 \end{bmatrix} \qquad \boldsymbol{\mathcal{A}} = \boldsymbol{\Psi}^\dagger d\boldsymbol{\Psi} = \begin{bmatrix} \boldsymbol{\mathcal{A}}_1 & \\ & \boldsymbol{\mathcal{A}}_2 \end{bmatrix}$$

$$\mathrm{Tr}\, \boldsymbol{\mathcal{A}} = \mathrm{Tr}\, \boldsymbol{\mathcal{A}}_1 + \mathrm{Tr}\, \boldsymbol{\mathcal{A}}_2$$

$$\boldsymbol{\mathcal{A}}_1 = \boldsymbol{\Psi}_1^\dagger d\boldsymbol{\Psi}_1, \quad \boldsymbol{\mathcal{A}}_2 = \boldsymbol{\Psi}_2^\dagger d\boldsymbol{\Psi}_2$$

- Sum Rule of the Chern Number (ex. Multi Landau Levels)

$$C_S(W_1 \oplus W_2) = C_S(W_1) + C_S(W_2)$$

$$W_1 \oplus W_2 \leftrightarrows W_1 \oplus W_3$$

$$C_S(W_1) + C_S(W_2) \leftrightarrows C_S(W_1) + C_S(W_3)$$

Explicit Gauge Fixing

- Arbitrary Trial Multiplet : $\boldsymbol{\Phi} : \ \boldsymbol{\Phi}^\dagger \boldsymbol{\Phi} = \boldsymbol{I}_M$
- Unnormalized Multiplet by a Trial Staten $\boldsymbol{\Psi}_\Phi^U$

$$\boldsymbol{\Psi}_\Phi^U = \boldsymbol{P}\boldsymbol{\Phi} = \boldsymbol{\Psi}\boldsymbol{\Psi}^\dagger\boldsymbol{\Phi} = \boldsymbol{\Psi}\boldsymbol{\eta}_\Phi, \quad \boldsymbol{\eta}_\Phi = \boldsymbol{\Psi}^\dagger\boldsymbol{\Phi}$$

$$= \boxed{\boldsymbol{\Psi}}\ \boxed{\boldsymbol{\Psi}^\dagger}\ \boxed{\boldsymbol{\Phi}} = \boxed{\boldsymbol{\Psi}}\ \boxed{\boldsymbol{\eta}_\Phi}$$

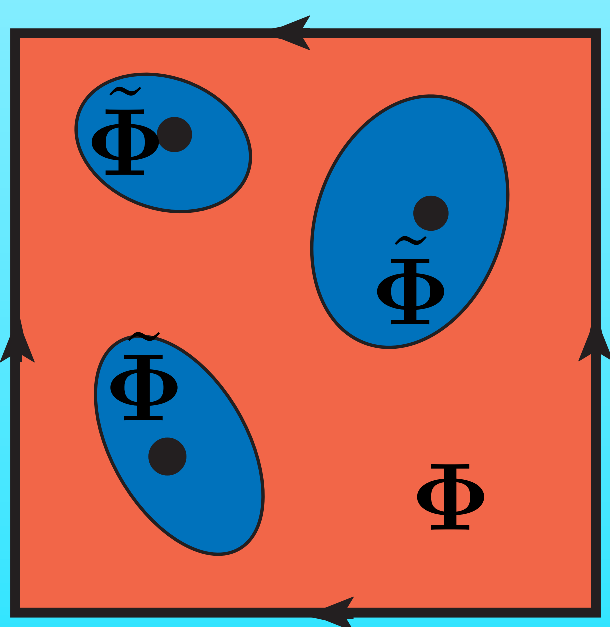
- Normalizable only if the Norm does not vanish: $\det \boldsymbol{O}_\Phi = \boldsymbol{\Psi}_\Phi^{U\dagger}\boldsymbol{\Psi}_\Phi^U$

$$= \det \boldsymbol{\eta}_\Phi^\dagger \boldsymbol{\eta}_\Phi \neq 0$$

$$\overline{\boldsymbol{\Psi}}_\Phi = \boldsymbol{\Psi}_\Phi^U \left(\boldsymbol{\Psi}_\Phi^{U\dagger} \boldsymbol{\Psi}_\Phi^U \right)^{-\frac{1}{2}}$$

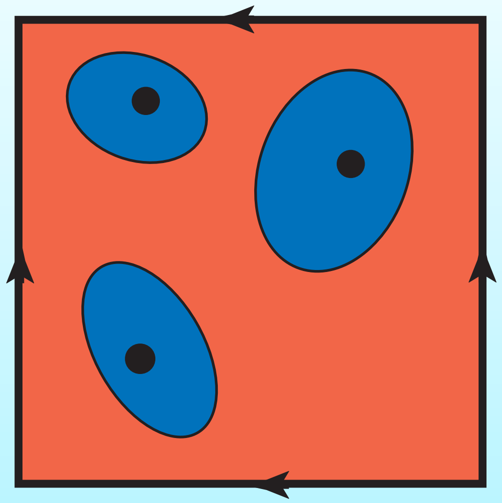
- When the Norm vanishes, we need another trial state
  - $\det \boldsymbol{O}_\Phi = |\det \boldsymbol{\eta}_\Phi|^2 = 0$: Define a Vortex

→ Different Gauge $\boldsymbol{\Phi}, \tilde{\boldsymbol{\Phi}}$



Gauge Transformation for the Multiplet

- Gauge Transformation between different Gauges



$$\overline{\boldsymbol{\Psi}}_{\tilde{\Phi}} = \overline{\boldsymbol{\Psi}}_{\Phi} \boldsymbol{\omega}_{\Phi\tilde{\Phi}}$$

$$\begin{aligned}\boldsymbol{\omega}_{\Phi\tilde{\Phi}} &= (\boldsymbol{\eta}_{\Phi}^{\dagger}\boldsymbol{\eta}_{\Phi})^{1/2}\boldsymbol{\eta}_{\Phi}^{-1}\boldsymbol{\eta}_{\tilde{\Phi}}(\boldsymbol{\eta}_{\tilde{\Phi}}^{\dagger}\boldsymbol{\eta}_{\tilde{\Phi}})^{-1/2} \\ &= \boldsymbol{o}_{\Phi}\boldsymbol{\eta}_{\Phi}^{-1}\boldsymbol{\eta}_{\tilde{\Phi}}\boldsymbol{o}_{\tilde{\Phi}}^{-1}\end{aligned}$$

$\boldsymbol{o}_{\Phi}, \boldsymbol{o}_{\tilde{\Phi}}$: positive definite

$$\mathrm{Im}\,\mathrm{Tr}\,\log \boldsymbol{\omega}_{\Phi\tilde{\Phi}} = -\,\mathrm{Im}\,\mathrm{Tr}\,\log(\boldsymbol{\eta}_{\tilde{\Phi}}^{\dagger}\boldsymbol{\eta}_{\Phi}) = -\,\mathrm{Im}\,\mathrm{Tr}\,\log \tilde{\boldsymbol{\Phi}}^{\dagger}\boldsymbol{P}\boldsymbol{\Phi}$$

- Chern Number by the present Transformation Matrix $\boldsymbol{\omega}_{\Phi\tilde{\Phi}}$

$$C_S = \frac{1}{2\pi}\oint_{\partial S_0^{\Phi}} d\Omega = N_{\Omega}(S_0^{\Phi})$$

$$\begin{aligned}\Omega &= \mathrm{Im}\,\mathrm{Tr}\,\log \tilde{\boldsymbol{\Phi}}^{\dagger}\boldsymbol{P}\boldsymbol{\Phi} \\ &= \mathrm{Arg}\,\det \tilde{\boldsymbol{\Phi}}^{\dagger}\boldsymbol{P}\boldsymbol{\Phi}\end{aligned}$$

Example I: One Dimensional Case

- The Multiplet is one-dimensional: Classic situation

$$\boldsymbol{\Psi} = \psi = \boxed{\psi}, \quad \boldsymbol{P} = \boxed{\psi}\,\boxed{\psi^\dagger}$$

- Take

$$\boldsymbol{\Phi} = \begin{pmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix}, \quad \tilde{\boldsymbol{\Phi}} = \begin{pmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{pmatrix}.$$

- Then

$$\begin{aligned} \tilde{\boldsymbol{\Phi}} \boldsymbol{P} \boldsymbol{\Phi} &= (\boldsymbol{P})_{N1} = \psi_N^* \psi_1 \\ C_S &= \frac{1}{2\pi} \oint_{\partial S_0^\Phi} \mathrm{Im}\, d \log \frac{\psi_1}{\psi_N} \\ \boldsymbol{O}_\Phi &= \boldsymbol{\eta}_{\tilde{\Phi}}^\dagger \boldsymbol{\eta}_\Phi = |\psi_1|^2 \end{aligned}$$

Classic Expression

Kohmoto

Example II: Dirac Monople and Strings

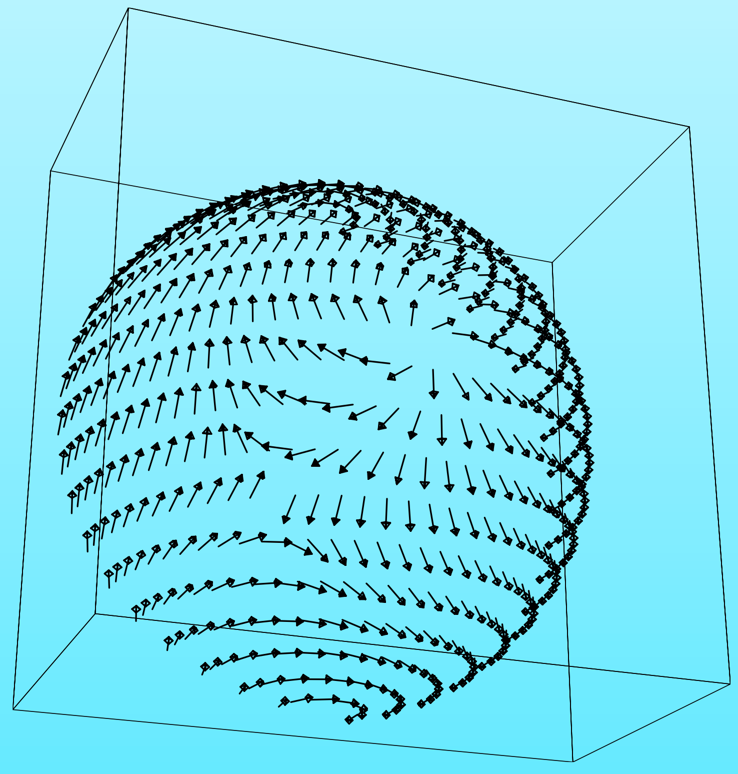
- Move Strings To Anywhere: It does not have any physical Meaning

$$\boldsymbol{H}(x) = \boldsymbol{R}(x) \cdot \boldsymbol{\sigma}$$

$$\boldsymbol{\Psi}(x) = \begin{pmatrix} -\sin\frac{\theta(x)}{2} \\ e^{i\phi(x)}\cos\frac{\theta(x)}{2} \end{pmatrix}, \quad \boldsymbol{P} = \boldsymbol{\Psi}\boldsymbol{\Psi}^\dagger = \begin{pmatrix} \sin^2\frac{\theta}{2} & -e^{-i\phi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} \\ -e^{i\phi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} & \cos^2\frac{\theta}{2} \end{pmatrix}$$

$$\boldsymbol{\Phi} = \begin{pmatrix} \cos\frac{\chi}{2} \\ e^{i\xi}\sin\frac{\chi}{2} \end{pmatrix}, \quad \tilde{\boldsymbol{\Phi}} = \begin{pmatrix} \cos\frac{\tilde{\chi}}{2} \\ e^{i\tilde{\xi}}\sin\frac{\tilde{\chi}}{2} \end{pmatrix},$$

$$\begin{aligned}\Omega &= \mathrm{Im}\,\mathrm{Tr}\,\log \tilde{\boldsymbol{\Phi}}^\dagger \boldsymbol{P} \boldsymbol{\Phi} \\ &= const. + \mathrm{Arg}\,\left(-\sin\frac{\theta}{2}\cos\frac{\chi}{2} + e^{-i(\phi-\xi)}\cos\frac{\theta}{2}\sin\frac{\chi}{2}\right)\end{aligned}$$



- Only the sum of the singularities is an observable

Example III: Unitary Superconductors

- Eigen states are Doubly degenerate

Parameter x: Momentum k

$$\boldsymbol{H}\psi = E\psi, \qquad \boldsymbol{H} = \begin{pmatrix} \epsilon \boldsymbol{I}_2 & \boldsymbol{\Delta} \\ \boldsymbol{\Delta}^\dagger & -\epsilon \boldsymbol{I}_2 \end{pmatrix}$$:4×4 Matrix

$$\boldsymbol{\Delta} \equiv |\Delta| \boldsymbol{\Delta}_0, \quad |\Delta| \geq 0, \quad \boldsymbol{\Delta}\boldsymbol{\Delta}^\dagger = |\Delta|^2 \boldsymbol{I}_2$$

$$\boldsymbol{\Delta}_0 : 2 \times 2 \text{ unitary matrix},$$

$$E = -R, R = \sqrt{\epsilon + |\Delta|^2}$$

$$\psi(\boldsymbol{w}) = \begin{pmatrix} -\sin\frac{\theta}{2}\,\boldsymbol{w} \\ \cos\frac{\theta}{2}\,\boldsymbol{\Delta}_0^\dagger \boldsymbol{w} \end{pmatrix}$$

$$\boldsymbol{w} = \begin{pmatrix} * \\ * \end{pmatrix}, \boldsymbol{w}^\dagger \boldsymbol{w} = 1$$ Arbitrary 2 dim. vector

Ex. III: Unitary Superconductors (cont.)

- Doubly degenerate Multiplet : M=2

$$\mathbf{\Psi} = (\psi(\boldsymbol{w}_1), \psi(\boldsymbol{w}_2))$$

$$\boldsymbol{w}_i^\dagger \boldsymbol{w}_j = \delta_{ij}, \quad \boldsymbol{w}_1 \boldsymbol{w}_1^\dagger + \boldsymbol{w}_2 \boldsymbol{w}_2^\dagger = \boldsymbol{I}_2$$

- Projection into the ground state Multiplet

$$\boldsymbol{P} = \mathbf{\Psi}\mathbf{\Psi}^\dagger = (\psi(\boldsymbol{w}_1), \psi(\boldsymbol{w}_2)) \begin{pmatrix} \psi^\dagger(\boldsymbol{w}_1) \\ \psi^\dagger(\boldsymbol{w}_2) \end{pmatrix}$$

$$= \begin{pmatrix} -\sin\frac{\theta}{2}\,\boldsymbol{w}_1 & -\sin\frac{\theta}{2}\,\boldsymbol{w}_2 \\ \cos\frac{\theta}{2}\,\mathbf{\Delta}_0^\dagger \boldsymbol{w}_1 & \cos\frac{\theta}{2}\,\mathbf{\Delta}_0^\dagger \boldsymbol{w}_2 \end{pmatrix} \begin{pmatrix} -\sin\frac{\theta}{2}\,\boldsymbol{w}_1^\dagger & \cos\frac{\theta}{2}\,\mathbf{\Delta}_0 \boldsymbol{w}_1^\dagger \\ -\sin\frac{\theta}{2}\,\boldsymbol{w}_2^\dagger & \cos\frac{\theta}{2}\,\mathbf{\Delta}_0 \boldsymbol{w}_2^\dagger \end{pmatrix}$$

$$= \begin{pmatrix} \boldsymbol{I}_2 \sin^2\frac{\theta}{2} & -\mathbf{\Delta}_0 \sin\frac{\theta}{2}\cos\frac{\theta}{2} \\ -\mathbf{\Delta}_0^\dagger \sin\frac{\theta}{2}\cos\frac{\theta}{2} & \boldsymbol{I}_2 \cos^2\frac{\theta}{2} \end{pmatrix}$$

Ex. III: Unitary Superconductors (cont.2)

$$\boldsymbol{\Phi} = \begin{pmatrix} \mathbf{0} \\ \boldsymbol{I}_2 \end{pmatrix}, \quad \tilde{\boldsymbol{\Phi}} = \begin{pmatrix} \boldsymbol{I}_2 \\ \mathbf{0} \end{pmatrix}$$

$$\det \boldsymbol{O}_\Phi = \det \boldsymbol{\Phi}^\dagger P \boldsymbol{\Phi} = \cos^4 \frac{\theta}{2}$$ =0 : South Pole

$$\begin{aligned} \Omega =& \mathrm{Arg} \det \tilde{\boldsymbol{\Phi}} \boldsymbol{P} \boldsymbol{\Phi} \\ =& \mathrm{Arg} \sin^2 \frac{\theta}{2} \cos^2 \frac{\theta}{2} \det \boldsymbol{\Delta}_0 = \mathrm{Arg} \det \boldsymbol{\Delta}_0 \end{aligned}$$

$$\begin{aligned} C_S =& \frac{1}{\pi} \oint_{\text{south pole}} d\Theta \\ \boldsymbol{\Delta}_0 =& e^{i\Theta} e^{i\hat{n}\cdot\vec{\sigma}}, \ |\hat{n}| = 1 \end{aligned}$$

Summary

- Topological Order
- Chern Numbers
  - Gauge Independent Quantities
  - Need Explicit Gauge Fixing
- Novel Generic Explicit Gauge Fixing
  - Allow Degeneracies
- Applications
  - One Dim.
  - Dirac Monopole
  - Unitary Superconductors

Ref. Y. Hatsugai, cond-mat/0405551, to appear in JPSJ Lett.

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