量子相転移としてのBEC-BCSクロスオーバー

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1. Introduction

Motivation

BEC-BCS crossover is realized experimentally.

(Regal et al. 2004, Bartenstein et al 2004, Zwierlein et al 2004)

Bose-Einstein condensation of real space molecules in strongly attractive system

↕

BCS superfluid in weakly coupled case

Two ground states are adiabatically connected.
(Leggett 1980)

For chiral symmetric case, BEC-BCS crossover can be distinguished as a quantum phase transition using Berry phase

Hamiltonian

$$\mathcal{H} = -t \sum_{\sigma=\uparrow,\downarrow} \sum_{<\mathbf{i},\mathbf{j}>} c^\dagger_{\mathbf{i},\sigma} c_{\mathbf{j},\sigma} - |U| \sum_{\mathbf{i}} n_{\mathbf{i},\uparrow} n_{\mathbf{i},\downarrow}$$

With mean field treatment:

$$\mathcal{H}_{\mathrm{MF}} = \sum_{\mathbf{i},\mathbf{j}} -t_{\mathbf{ij}} u^\dagger_{\mathbf{i}} u_{\mathbf{j}} + t_{\mathbf{ij}} d^\dagger_{\mathbf{i}} d_{\mathbf{j}} + \sum_{\mathbf{j}} \Delta \left(u^\dagger_{\mathbf{j}} d_{\mathbf{j}} + d^\dagger_{\mathbf{j}} u_{\mathbf{j}} \right)$$

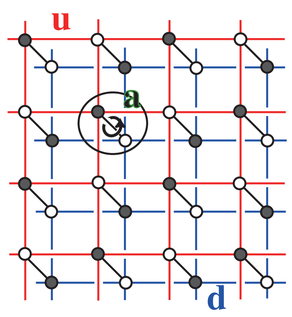
particle-hole transformation:

$$u_{\mathbf{i}} = d_{\mathbf{i},\uparrow} = c_{\mathbf{i},\uparrow}$$
$$d_{\mathbf{i}} = d_{\mathbf{i},\downarrow} = c^\dagger_{\mathbf{i},\downarrow}$$

$$\Delta = -|U| \langle c^\dagger_{\mathbf{i},\uparrow} c^\dagger_{\mathbf{i},\downarrow} \rangle = -|U| \langle u^\dagger_{\mathbf{i}} d_{\mathbf{i}} \rangle$$

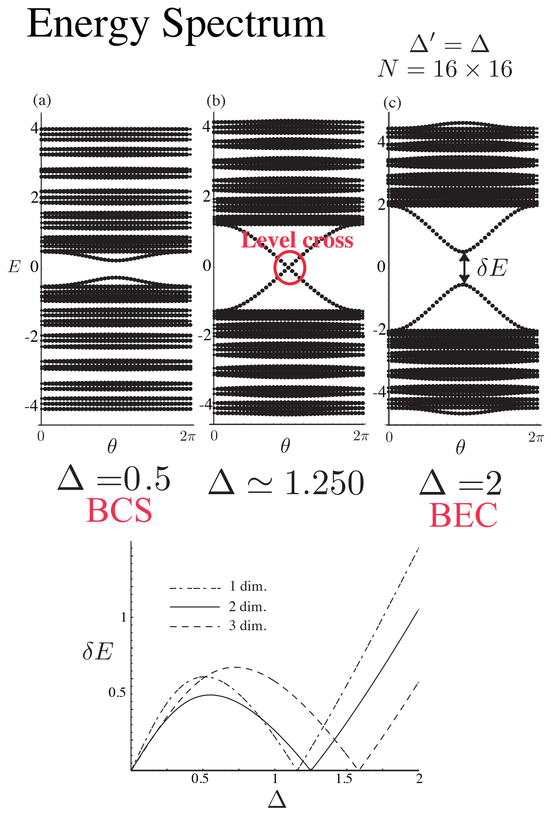
2. Results

U(1) Local Berry Phase



$$\Delta(u^\dagger_{\mathbf{a}} d_{\mathbf{a}} + d^\dagger_{\mathbf{a}} u_{\mathbf{a}}) \rightarrow \Delta'(e^{i\theta} u^\dagger_{\mathbf{a}} d_{\mathbf{a}} + e^{i\theta} d^\dagger_{\mathbf{a}} u_{\mathbf{a}})$$

Energy Spectrum



cf. $\delta E = 2\Delta$ for $\theta = 0$

Chiral Symmety

After some unitary transformation:

$$\mathcal{H}_{\mathrm{MF}}(\theta) = \sum_{\mathbf{i},\mathbf{j}} u^\dagger_{\mathbf{i}} [D_N(\theta)]_{\mathbf{ij}} d_{\mathbf{j}} + \mathrm{H.c.}$$

At half-filling, Berry phase with chiral symmetry is quantized as

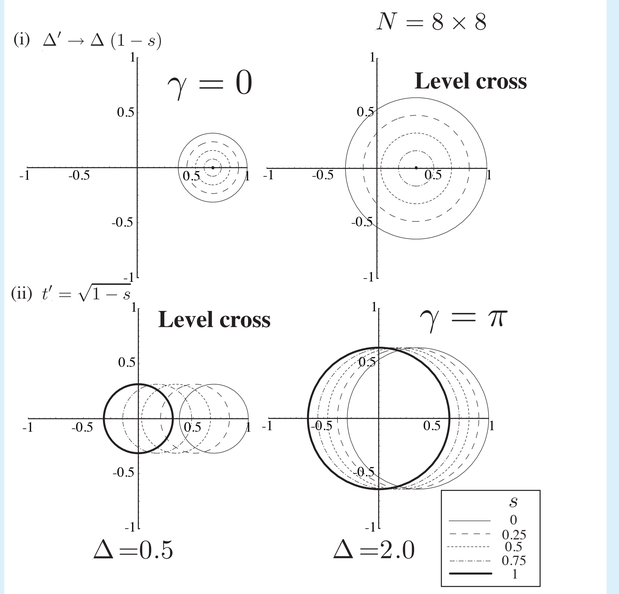
$$\gamma = 0, \pi \ (\mathrm{mod} 2\pi)$$

In general, at half-filling, Berry phase with chiral symmetry can be obtained as [1]

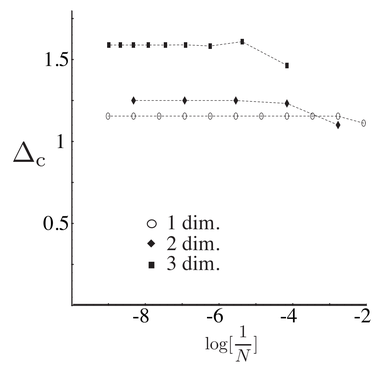
$$\gamma = \frac{1}{2} \int_0^{2\pi} d\theta \ \mathrm{Im} \log \det D_N(\theta)$$

θ-dependence

$\det D_N(\theta)_s / \det D_N(0)_{s=0}$



Size Dependence



In one-dimensional case we obtain analytically

$$\Delta_c = 2/\sqrt{3} \simeq 1.1547$$

3. Summary

1. In Hubbard Hamiltonian with onsite attraction, BEC-BCS crossover can be distinguished as a quantum phase transition using Berry phase at half-filling.
2. Bulk-edge correspondence as in Quantum Hall effect [2] (local twist: local perturbation)
3. In one-dimension, we can discuss the above argument analytically.

References

[1] Y.Hatsugai, unpublished.
[2] Y. Hatsugai, JPSJ 75 (2006) 123601.
[3] For details, M. Arikawa, I. Maruyama and Y. Hatsugai, preprint.

Adiabatic Deformation