

# *Band gap, dangling bond & spin*

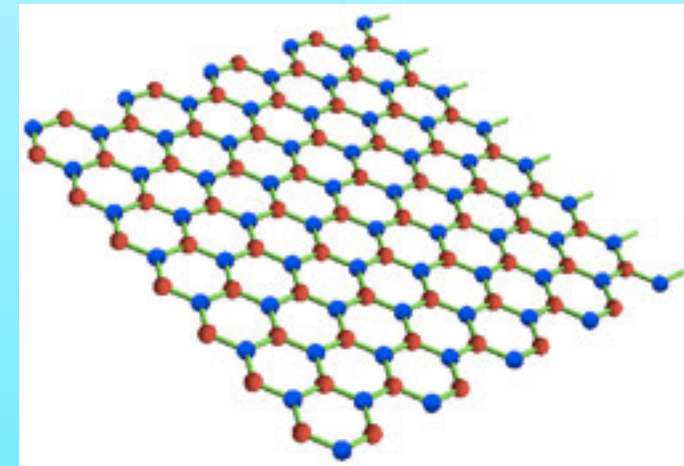
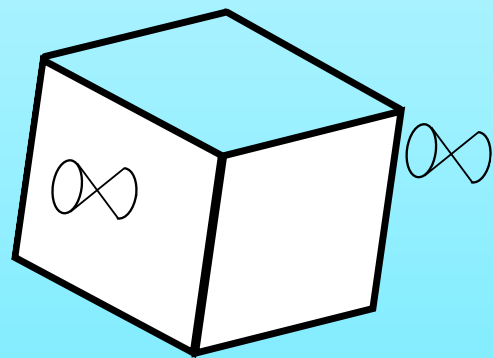
## *A physicist's viewpoint*

*From Newton to Dirac*

*Institute of Physics, TIMS  
University of Tsukuba  
JAPAN*

*Yasuhiro Hatsugai*

筑波大学・数理物質系・物理学域  
初貝 安弘





## The Nobel Prize in Physics 1956

William B. Shockley, John Bardeen, Walter H. Brattain

### The Nobel Prize in Physics 1956

William B. Shockley

John Bardeen

Walter H. Brattain



William Bradford  
Shockley



John Bardeen



Walter Houser  
Brattain

The Nobel Prize in Physics 1956 was awarded jointly to William Bradford Shockley, John Bardeen and Walter Houser Brattain *"for their researches on semiconductors and their discovery of the transistor effect"*.

Photos: Copyright © The Nobel Foundation

# Plan

★ *Metal, insulator & semiconductor*

★ *Bulk-edge correspondence :graphene, silicene and more*

★ *From Newton to Dirac for devices breakthrough*

*Band inversion*

*Graphene & silisene*

*Edge states*

*Topological insulators*

*Spin-orbit int.*

*Massless Dirac fermions*

*Majorana fermions*

# *Plan*

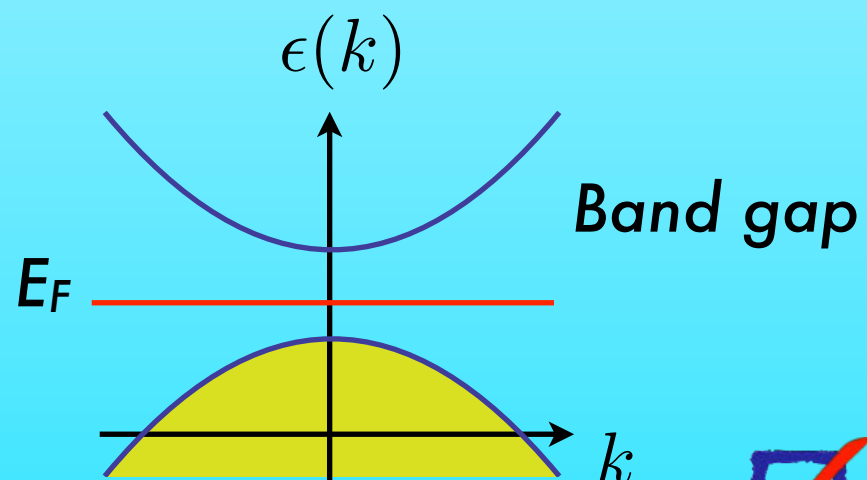
- ★ *Metal, insulator & semiconductor*
- ★ *Bulk-edge correspondence :graphene, silicene and more*
- ★ *From Newton to Dirac for devices breakthrough*

# Metal, insulator & semiconductor

Metal & Semiconductor (doped)



Insulator & Semiconductor (intrinsic)



☒ Useful !

carry current

☒ Interesting !

Response for small input

Lots of instabilities

magnetic ordering

superconductivity

...

Q: Boring ?

Yes, maybe  
(before 1980)

☒ No carrier

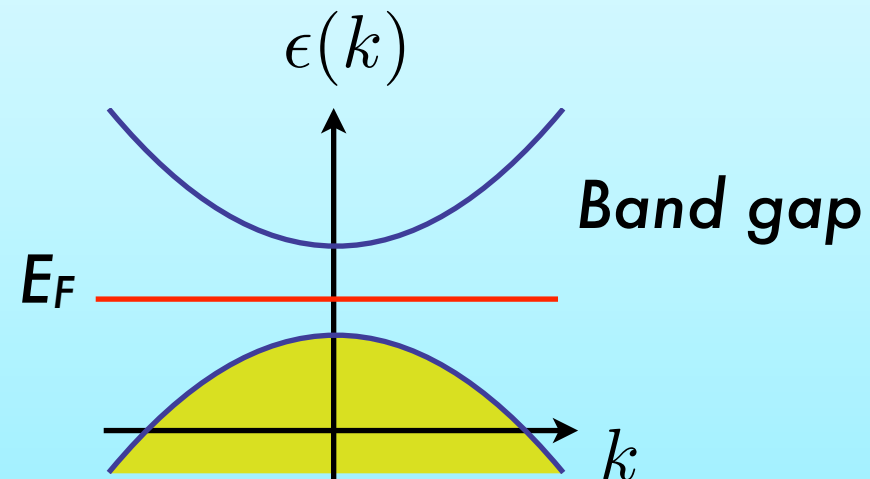
☒ No response for small perturbation



# Metal, insulator & semiconductor

Insulator & Semiconductor (intrinsic)

Q: Boring ?



No, it's a fun  
(Today:2013)

☑ Useful !

📌 Dissipationless current (?)

☑ Interesting !

📌 Lots of varieties

📌 Polarized phases

📌 (Magneto-electric) polarization

📌 With Spin-orbit with/without coulomb interaction

Non-trivial  
insulators  
(topological)

# Metal, insulator & semiconductor

*Topological*  $\approx$  *Insulator (semiconductor)*

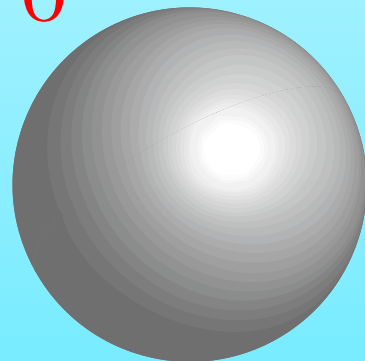
*Physics*

*Stable for small but finite perturbation*

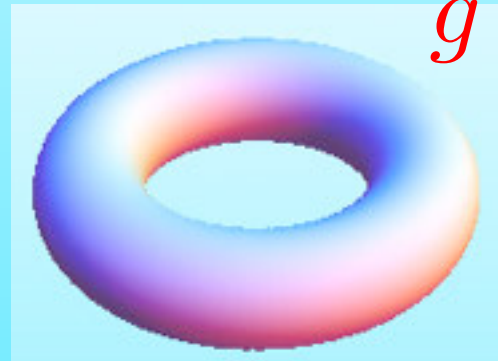
*math*

$g$ : # of holes

$g = 0$



$g = 1$



# Metal, insulator & semiconductor

*Topological*  $\approx$  *Insulator (semiconductor)*

*Physics*

*Stable for small but finite perturbation*

~~*math*~~

*Not joking: physicists are a bit more serious*

*Something  
to be observed & useful !*

*“EDGE STATES”*

*“Dangling bonds” exist ! when non trivial !*

*Bulk-edge correspondence*

Y. Hatsugai, Phys. Rev. Lett. 71, 3697 (1993)



# *Energy gap and its origin*

*Energy band (Bloch's theorem): energy region of extended states*

*Stable for small but finite perturbation*

*Not joking: physicists are a bit more serious*

*Something  
to be observed & useful !*

*"EDGE STATES"*

*"Dangling bonds" exist ! when non trivial !*

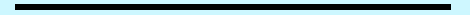
*Bulk-edge correspondence*

*Y. H., Phys. Rev. Lett. 71, 3697 (1993)*

# Energy band & gap : physicist & chemist ? *Sorry if I'm wrong*

*physicist*

*itinerant electrons*



# Energy band & gap : physicist & chemist ?

Sorry if I'm wrong

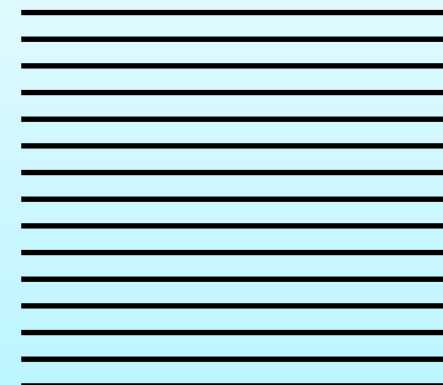
physicist

itinerant electrons



hopping

make energy band  
metal



# Energy band & gap : physicist & chemist ?

Sorry if I'm wrong

physicist

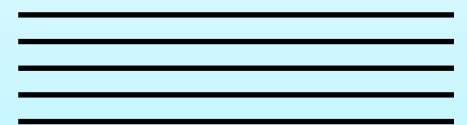
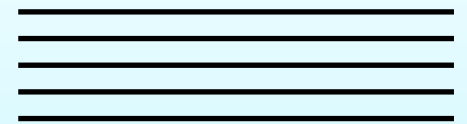
itinerant electrons



hopping

*Peierls instability*

Opening gap  
stabilize



# Energy band & gap : physicist & chemist ?

Sorry if I'm wrong

physicist

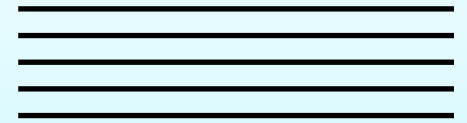
*itinerant electrons*



*hopping*

*Peierls instability*

Opening gap  
stabilize



chemist

*form molecules first*



# Energy band & gap : physicist & chemist ?

Sorry if I'm wrong

physicist

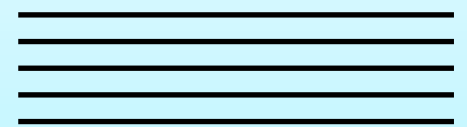
itinerant electrons



hopping

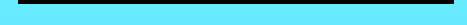
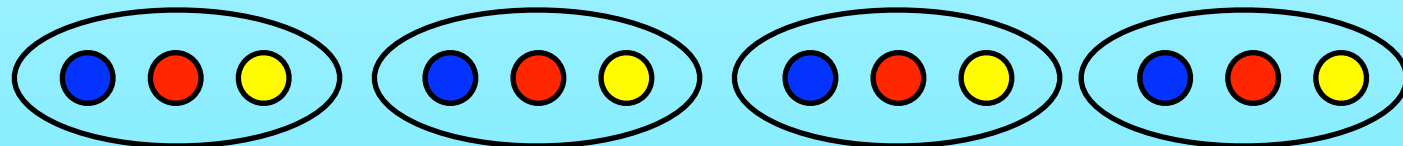
Peierls instability

Opening gap  
stabilize



chemist

form molecules first





# Energy band & gap : physicist & chemist ?

Sorry if I'm wrong

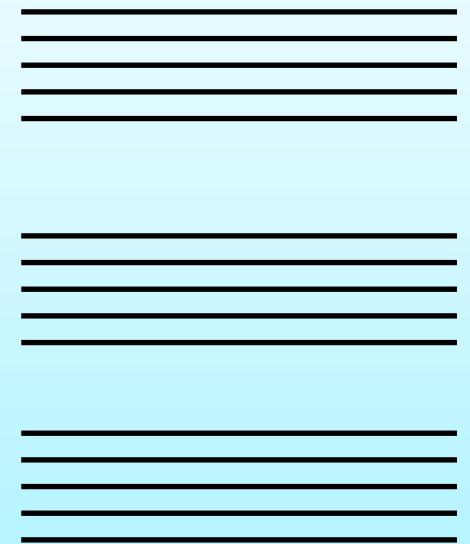
physicist

itinerant electrons



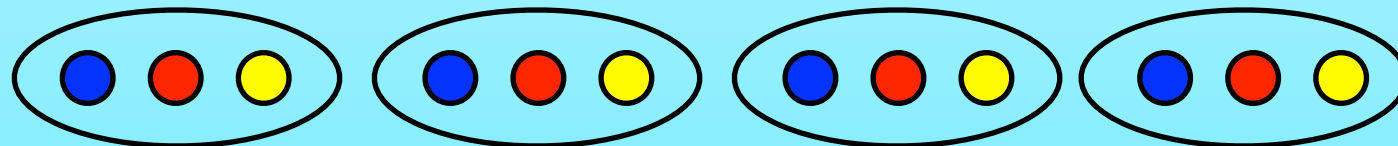
Peierls instability

stabilize



chemist

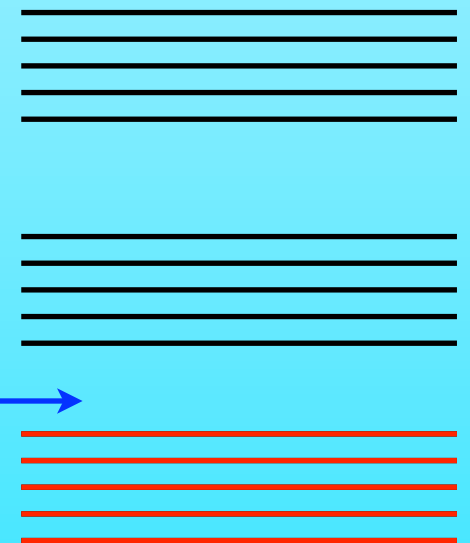
form molecules first



non orthogonality

short range entanglement

make bands of molecules



Dimer & Multimer

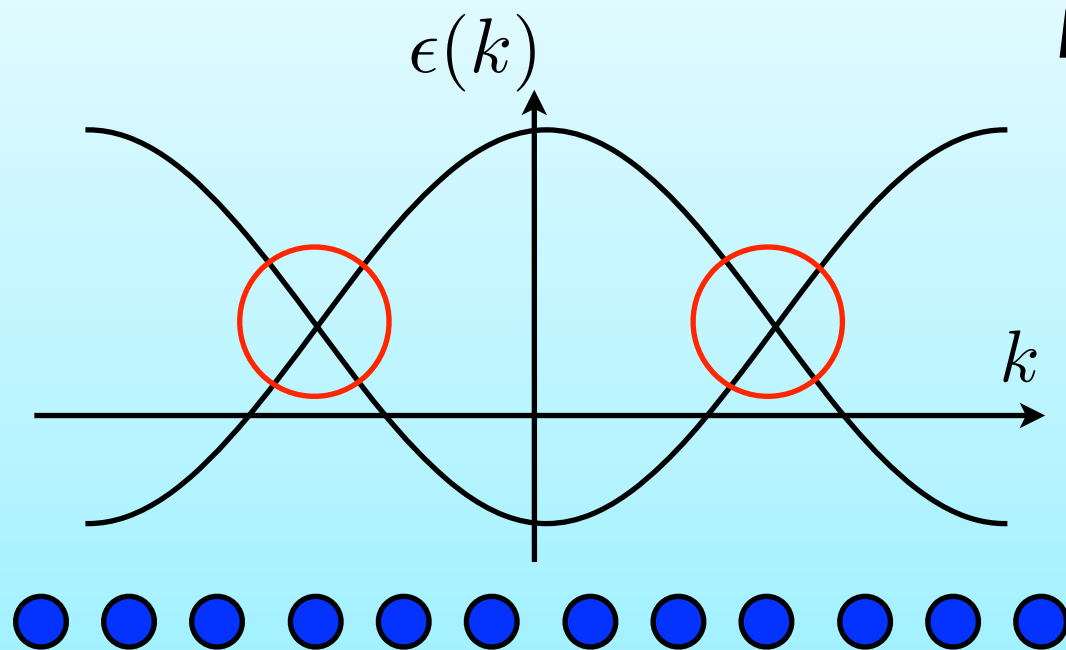
QUATUM Effects !

Adiabatic process

Insulator

# Energy gap and Dirac fermions

Extended Brillouin zone



Linear dispersion

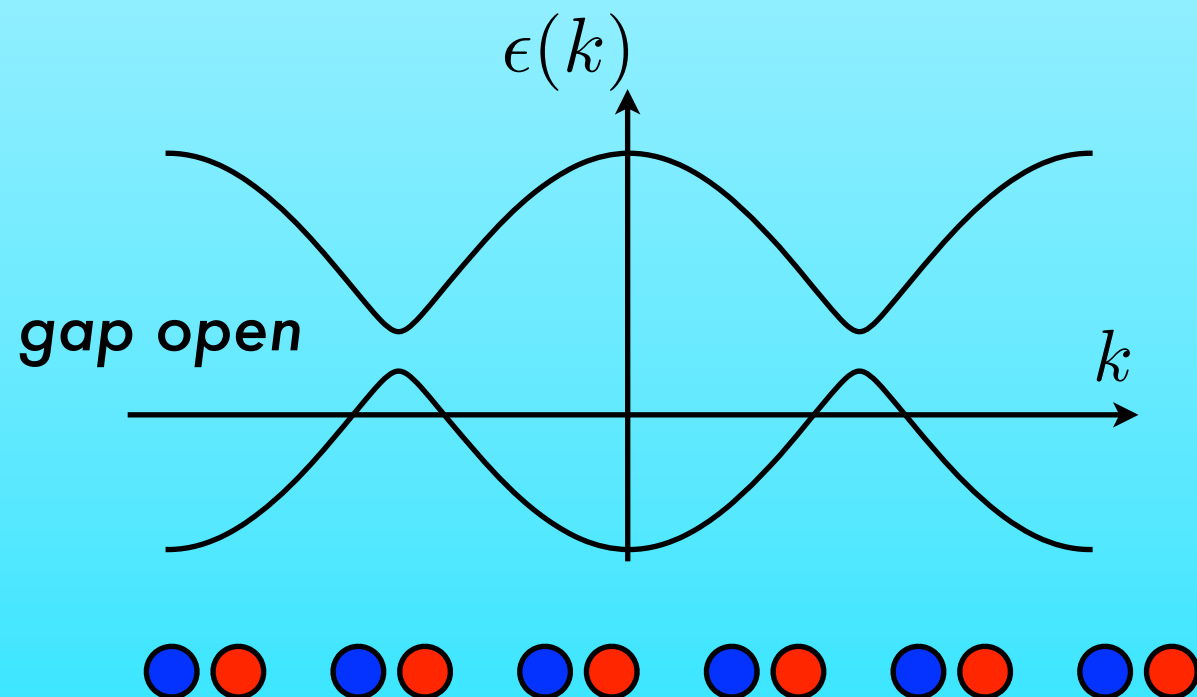
$$\epsilon(k) = \pm v_F |k|$$

1D graphene: polyacetylene

zero-gap semiconductor

Massless Dirac fermions

Fermion doubling  $\bigcirc \times 2$



dimerized

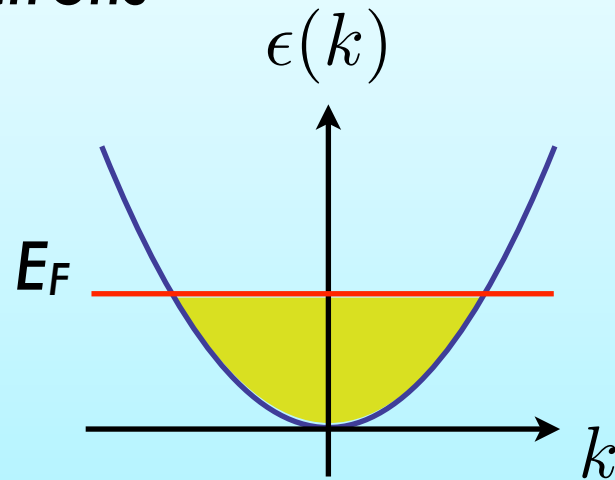
**Band gap!**

Massive Dirac fermions

$$\epsilon(k) = \pm v_F \sqrt{k^2 + m^2}$$

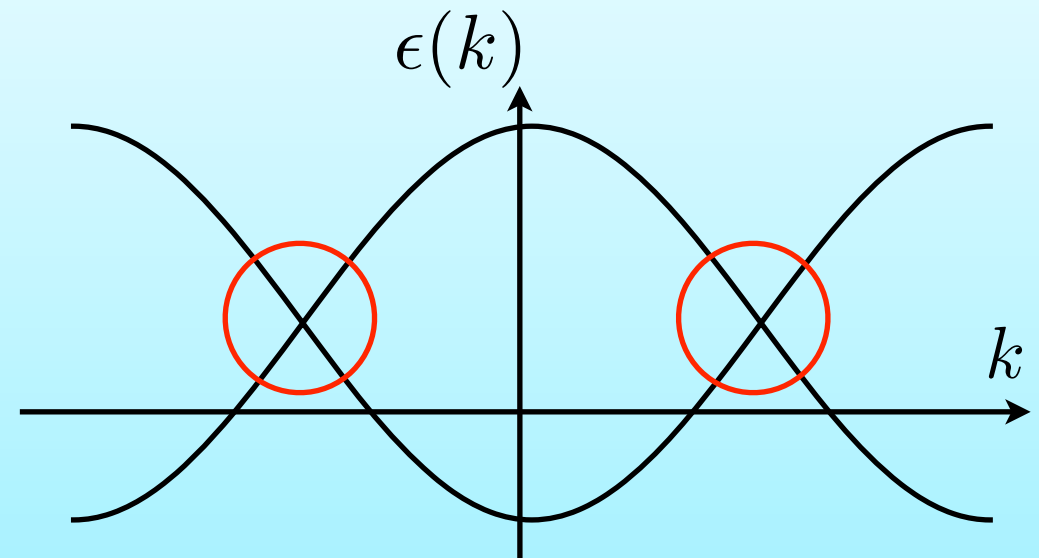
# Zero gap semiconductor: **half electrons**

Electrons



$$H_{ele} \propto k^2$$

Zero gap semiconductors



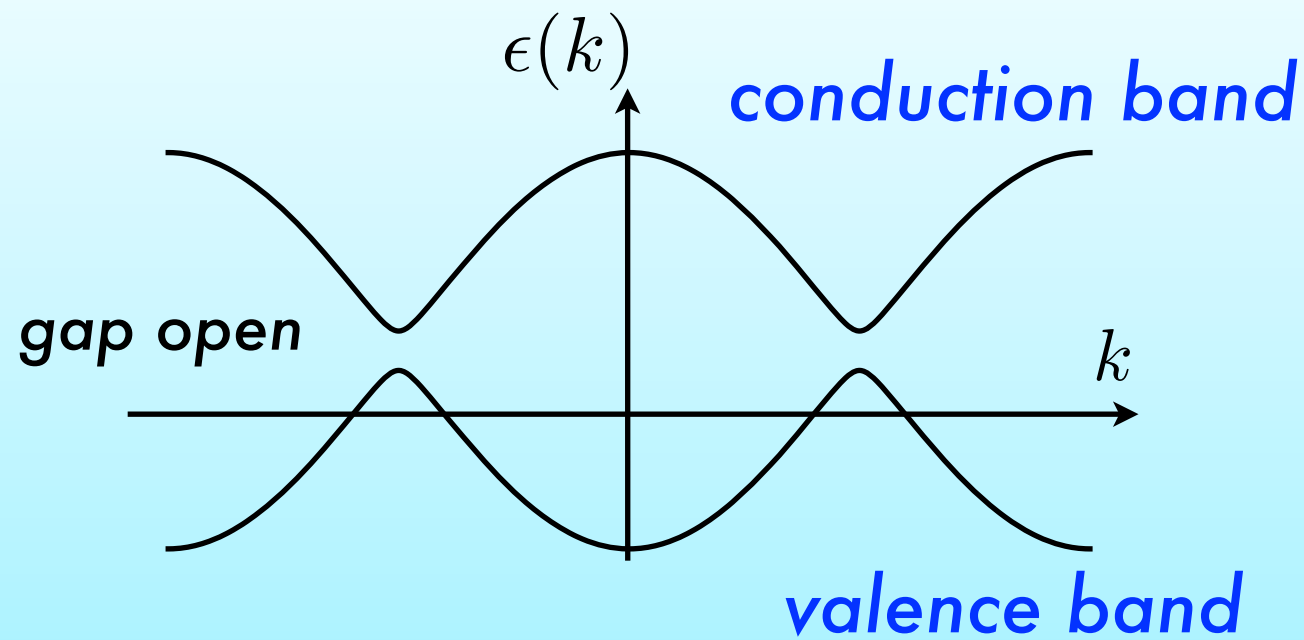
$$H_{zero} \propto |k|$$

$$\sqrt{H_{ele}} = H_{zero} : \text{half electrons}$$

$$H_{ele} = \sqrt{H_{ele}} \oplus \sqrt{H_{ele}}$$

# What's band inversion ? (2D topological insulator)

2D



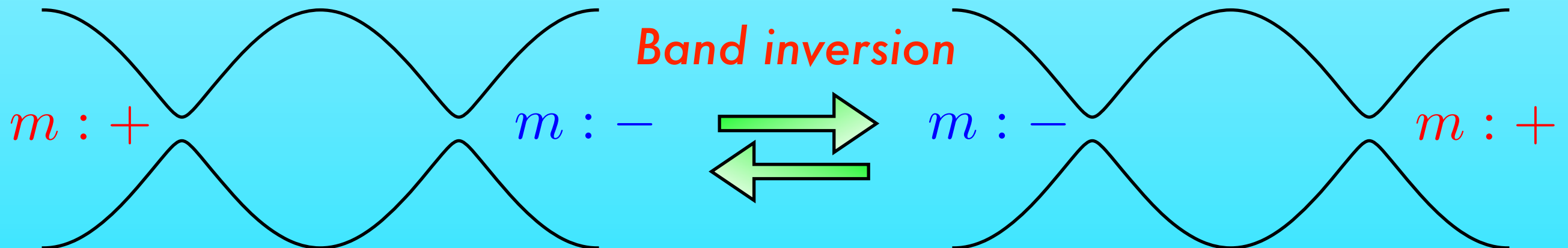
Massive Dirac fermions

$$\epsilon(k) = \pm \sqrt{k^2 + m^2}$$

eigen values :

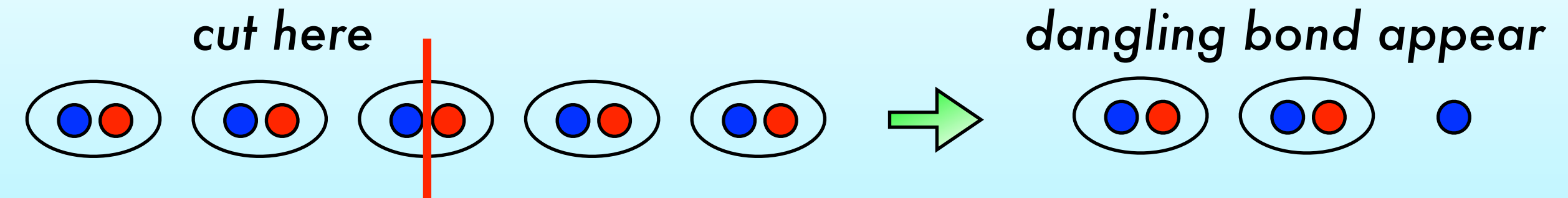
2 band:  $2 \times 2$  matrix

$$\begin{pmatrix} m & k_x - ik_y \\ k_x + ik_y & -m \end{pmatrix} \quad m \text{ can be negative !}$$



e.g. by changing width of the superlattice (HgTe-CdTe)

# Appearance of edge states

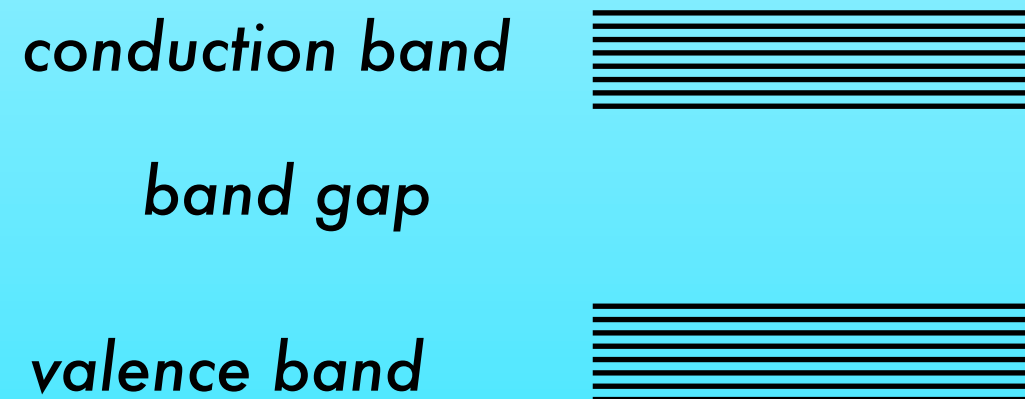


very long chain with boundaries

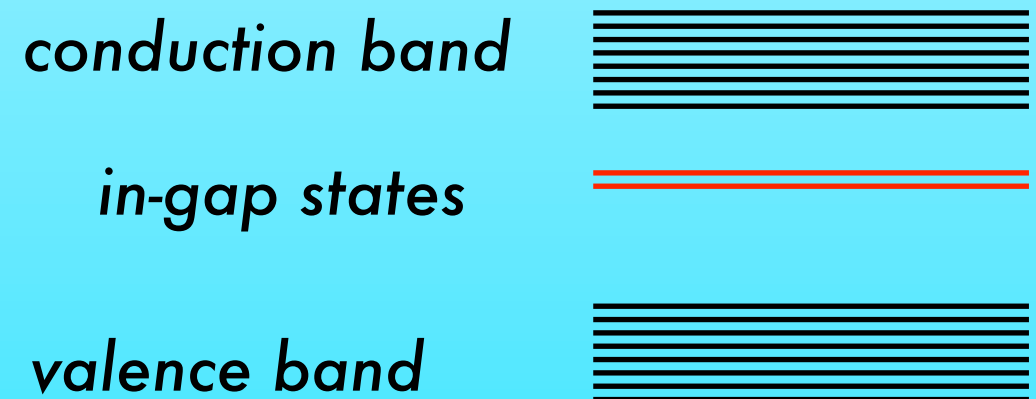
*in-gap states are induced*



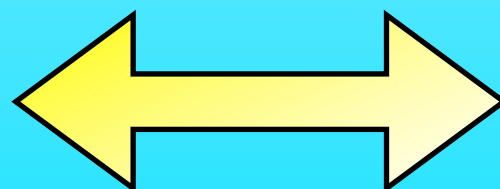
*without* boundaries



*with* boundaries



dimerization pattern




how the edge states appear

# Bulk-edge correspondence

*without* boundaries

conduction band 

band gap

valence band 

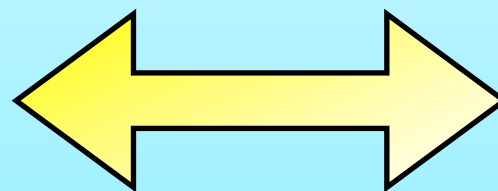
*with* boundaries

conduction band 

in-gap states 

valence band 

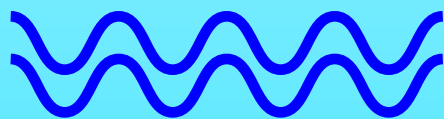
dimerization pattern



how the edge states appear

Bulk (before making boundaries) *determines* the edge states

The edge states *reflect* the dimerization pattern of the bulk



**Bulk state**

Control each other



**Edge state**



# Plan

- ★ *Metal, insulator & semiconductor*
- ★ *Bulk-edge correspondence :graphene, silicene and more*
- ★ *From Newton to Dirac for devices breakthrough*

# Zoo of Boundary (Edge) States in materials

From textbook examples to new discoveries

★ *Levinson's theorem to the Friedel's sum rule*

★ *Surface states of Semiconductors & polarization*

★ *Solitons in polyacetylene*

★ *Edge states in quantum Hall effects*

★ *Local moments in integer spin chains near the impurities*

★ *Zero bias conductance peaks of the d-wave superconductors*

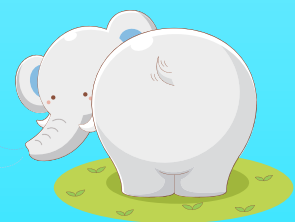
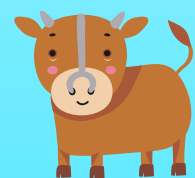
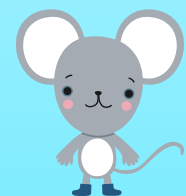
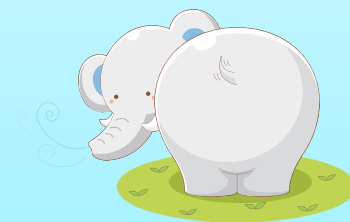
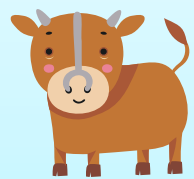
★ *Zero energy localized states of graphene*

★ *Zero energy localized states of silicene*

★ *Edge states in 2D cold atoms in optical lattice*

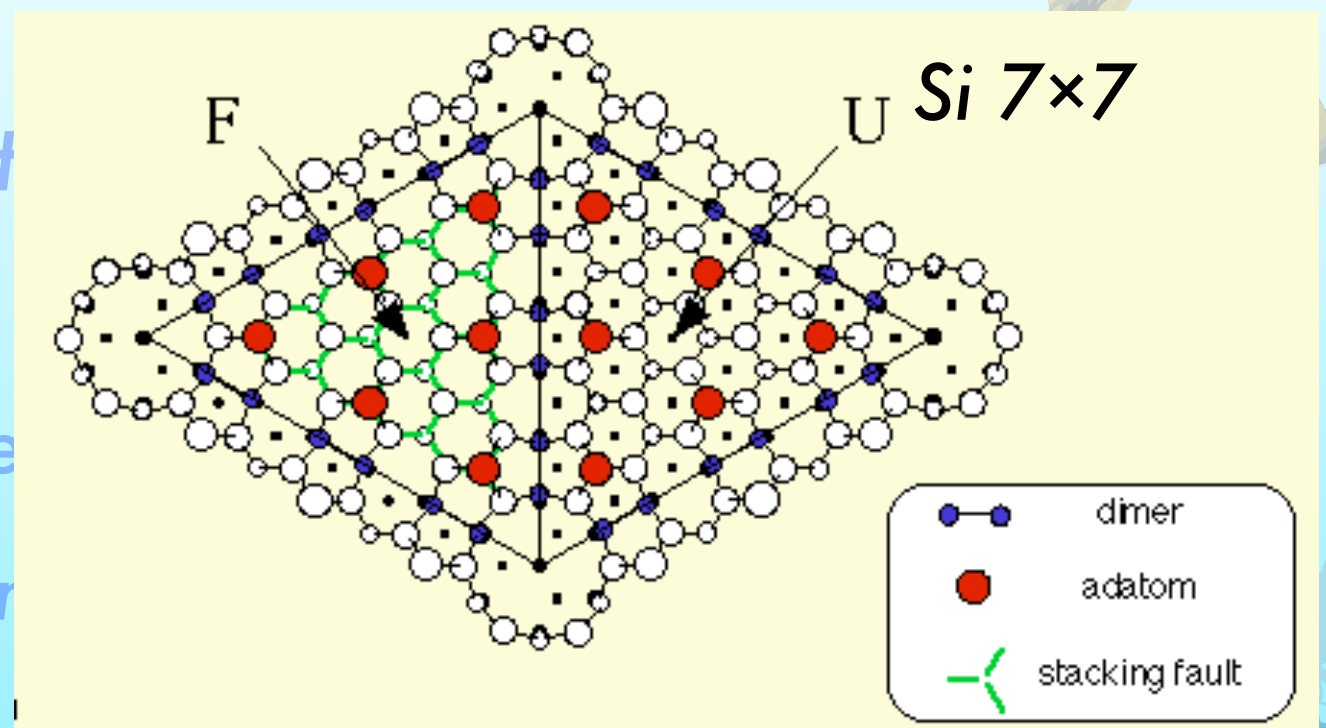
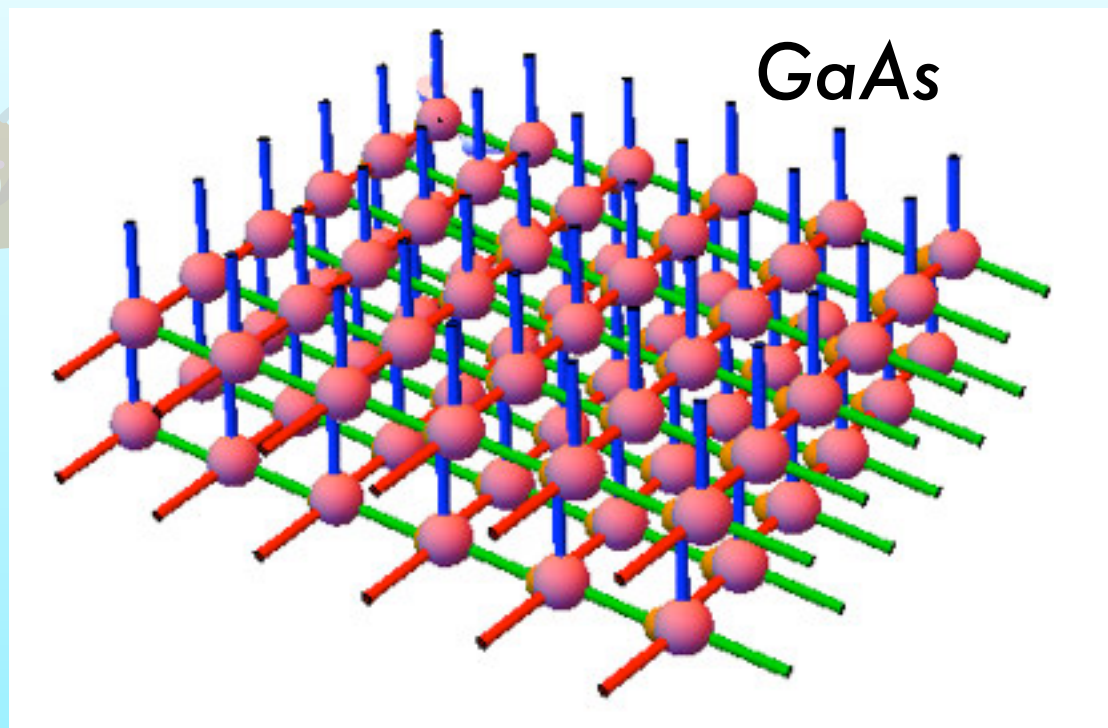
★ *One-way edge modes in gyromagnetic photonic crystals*

★ *Quantum Spin Hall Edge states*





# ★ Surface states of Semiconductors & polarization



PHYSICAL REVIEW B

VOLUME 47, NUMBER 3

15 JANUARY 1993-I

## Theory of polarization of crystalline solids

R. D. King-Smith and David Vanderbilt

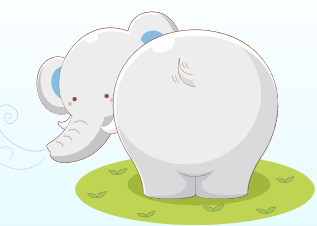
## Macroscopic polarization in crystalline dielectrics: the geometric phase approach

Raffaele Resta

Reviews of Modern Physics, Vol. 66, No. 3, July 1994

+ -+ -+ -+ -+ -+ -+ -+ -+ -+ -

lattice  
photonic crystals



# ★ Solitons in polyacetylene



VOLUME 42, NUMBER 25

PHYSICAL REVIEW LETTERS

18 JUNE 1979

## Solitons in Polyacetylene

W. P. Su, J. R. Schrieffer, and A. J. Heeger

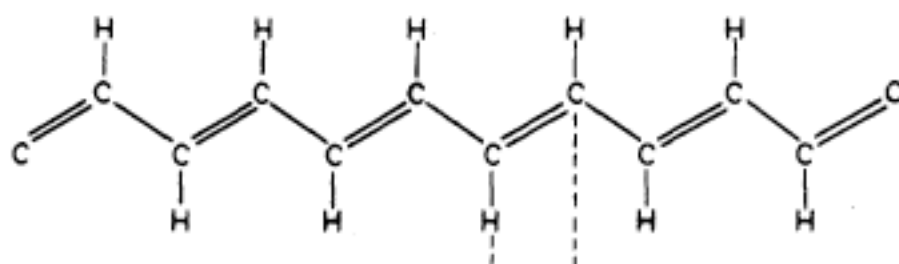
*Department of Physics, University of Pennsylvania, Philadelphia, Pennsylvania 19104*

(Received 15 March 1979)



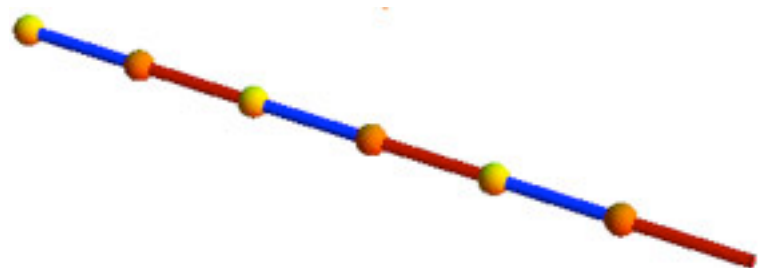
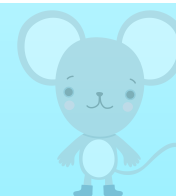
★ Edge states

★ Local modes

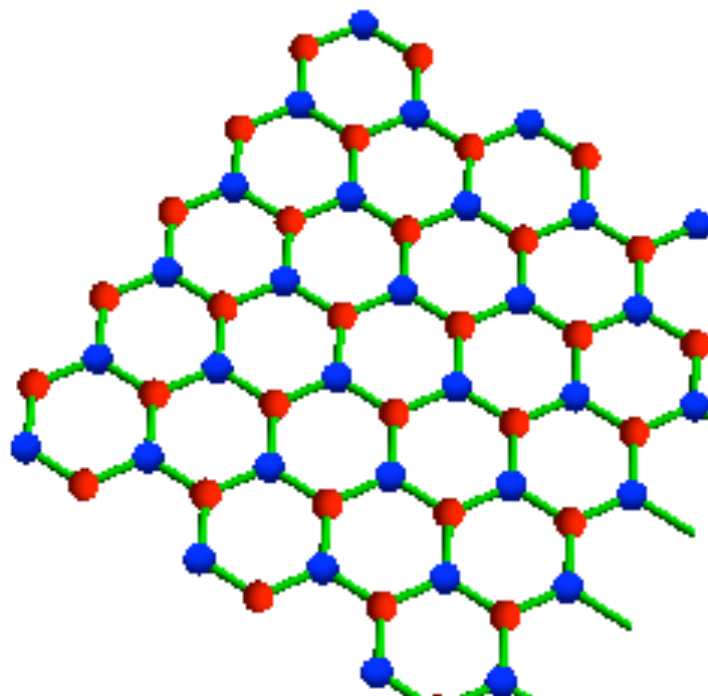


s

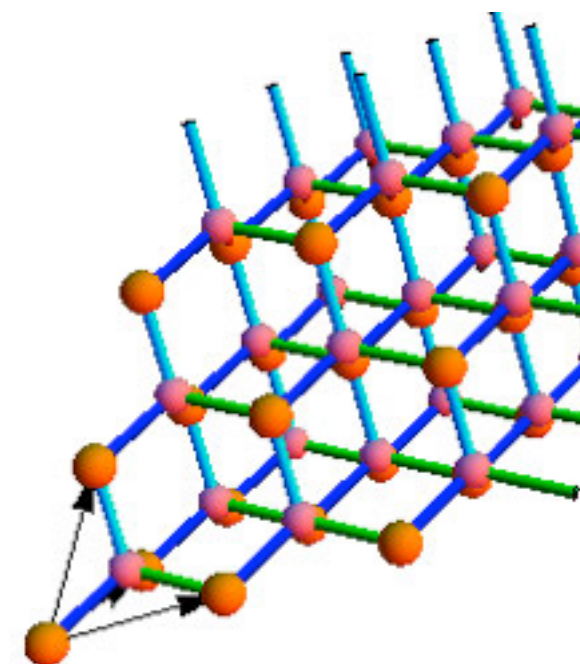
ins near the impurities



polyacetylene

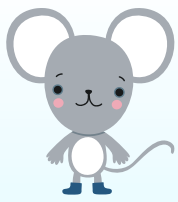


graphene



diamond





# ★ Edge states in quantum Hall effects

PHYSICAL REVIEW B

VOLUME 23, NUMBER 10

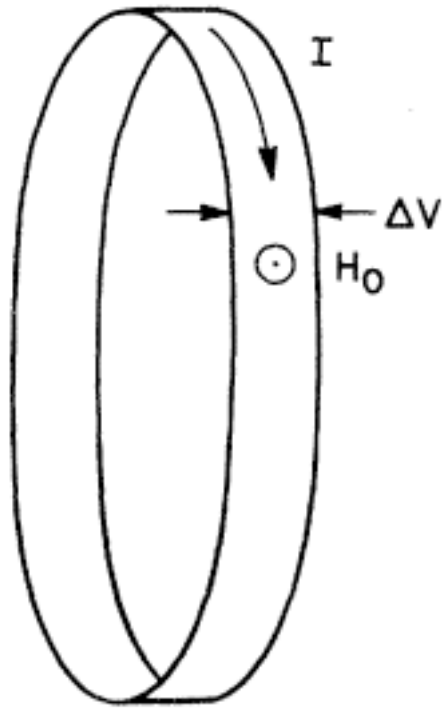
15 MAY 1981

## Quantized Hall conductivity in two dimensions

R. B. Laughlin

*Bell Laboratories, Murray Hill, New Jersey 07974*

(Received 20 January 1981)



**Everything started from here**

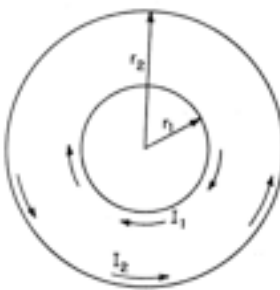
PHYSICAL REVIEW B

VOLUME 25, NUMBER 4

15 FEBRUARY 1982

## Quantized Hall conductance, current-carrying edge states, and the existence of extended states in a two-dimensional disordered potential

B. I. Halperin



PHYSICAL REVIEW B

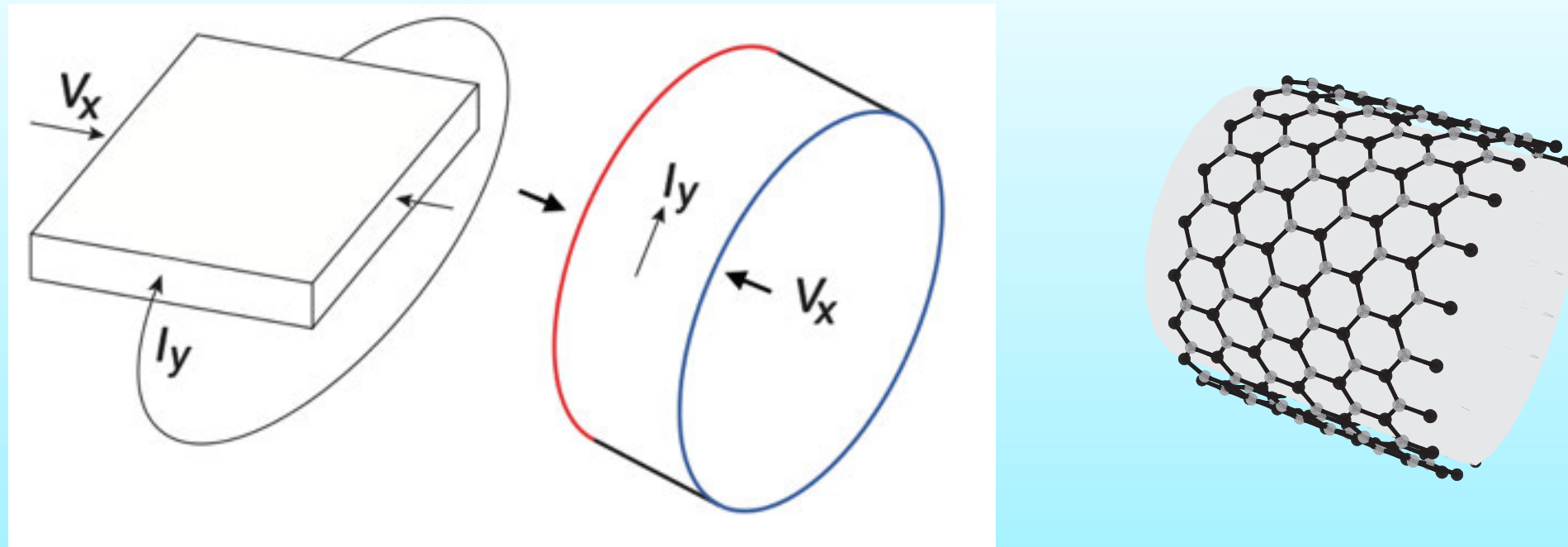
VOLUME 48, NUMBER 16

15 OCTOBER 1993-II

## Edge states in the integer quantum Hall effect and the Riemann surface of the Bloch function

Yasuhiro Hatsugai\*

# Hall Conductance has double Topological meanings



$$\sigma_{xy}^{\text{bulk}} = \sigma_{xy}^{\text{edge}}$$

Y. Hatsugai, Phys. Rev. Lett. 71, 3697 (1993)

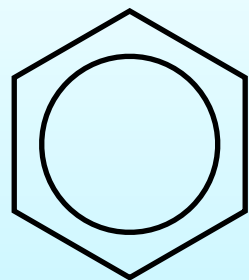
Y. Hatsugai, T.Fukui, H.Aoki, Phys. Rev. B. 74, 205414 (2006)

## Bulk-edge correspondence

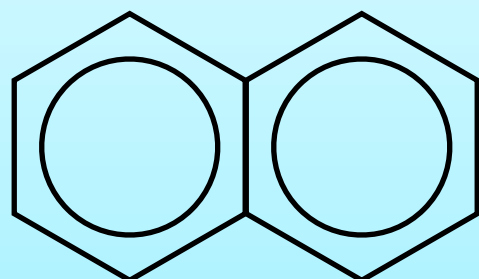


# Graphene??

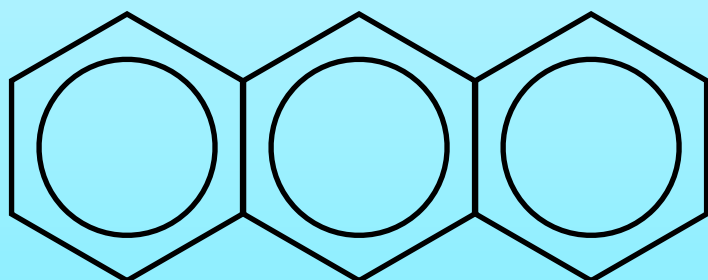
★  $\pi$ -electron systems



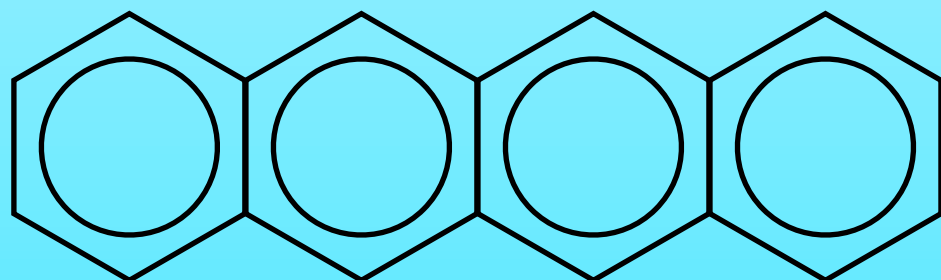
benzene



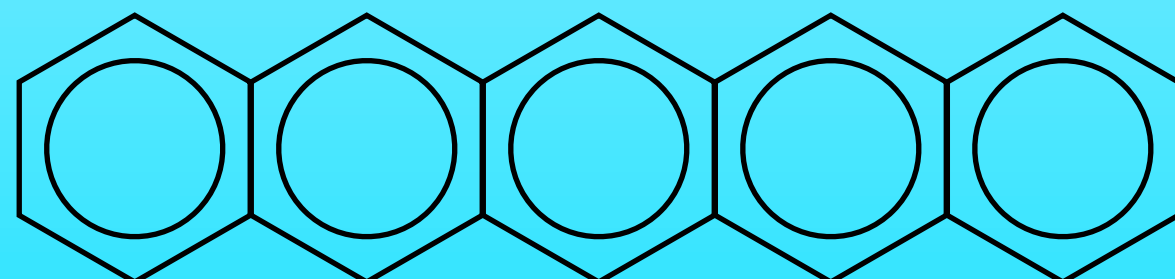
naphthalene



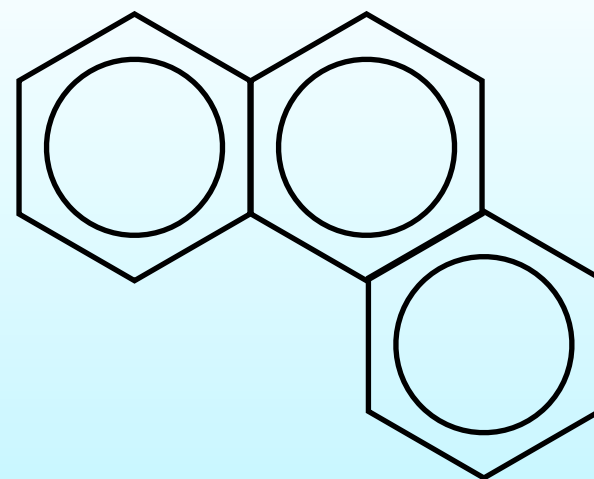
anthracene



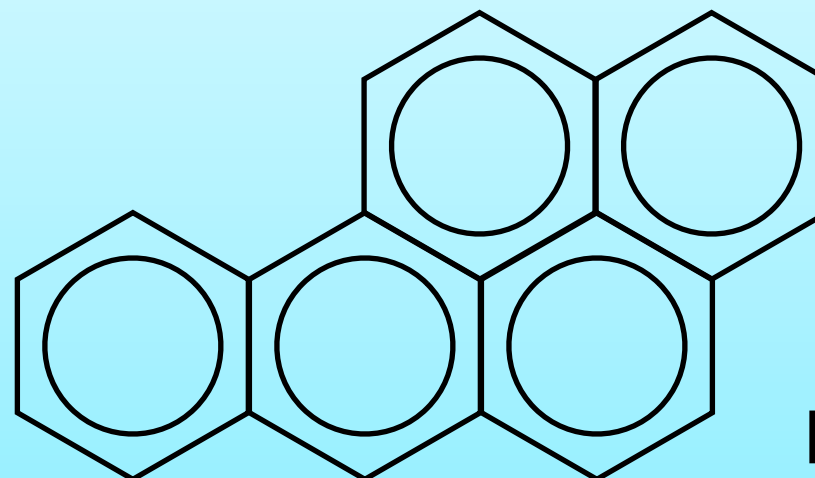
tetracene



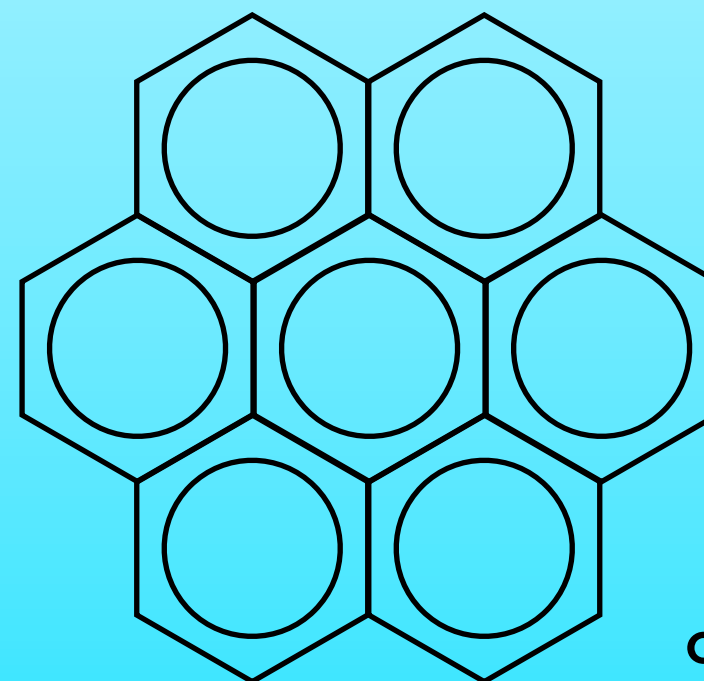
pentacene



phenanthrene



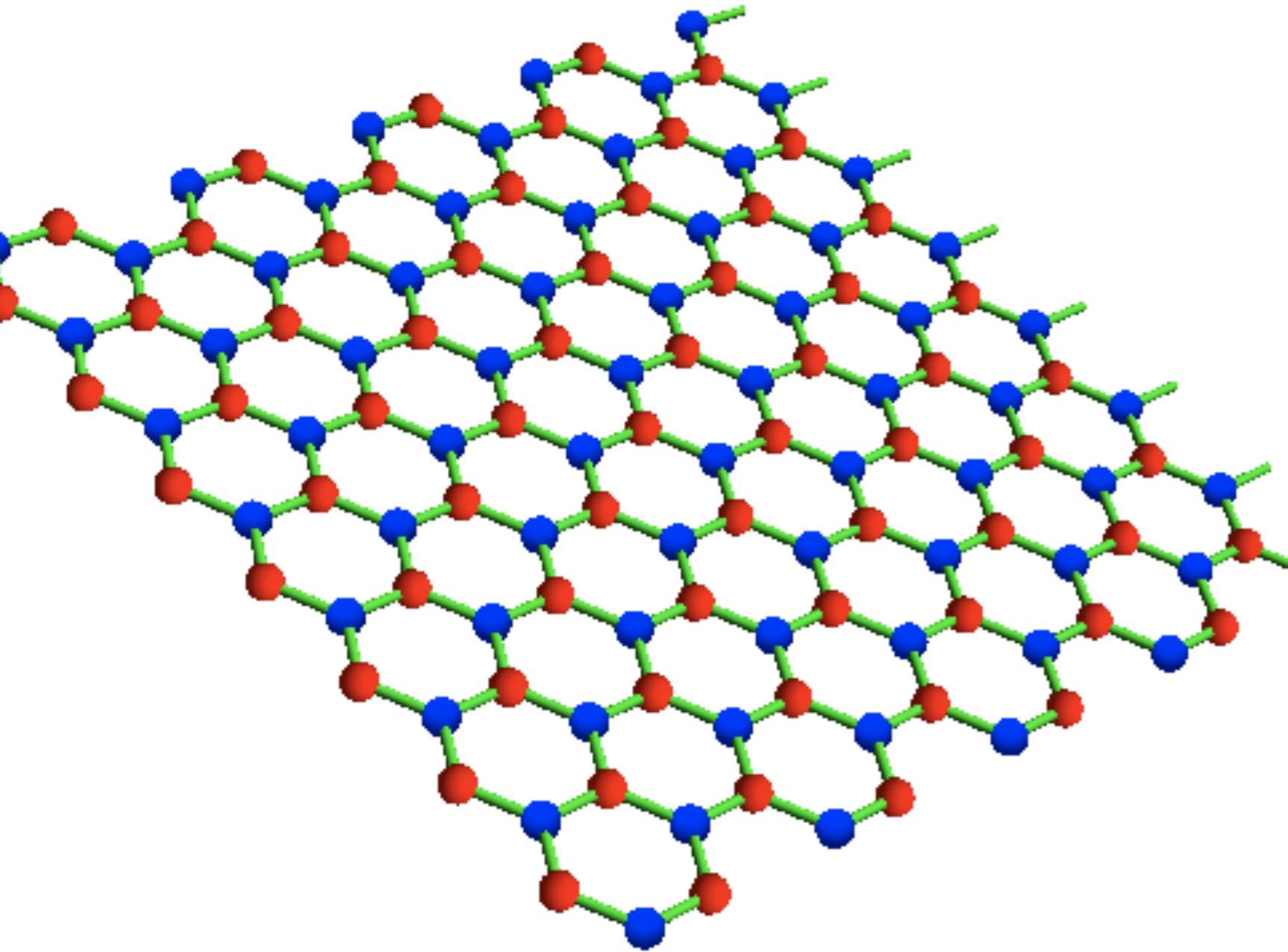
benzopyrene



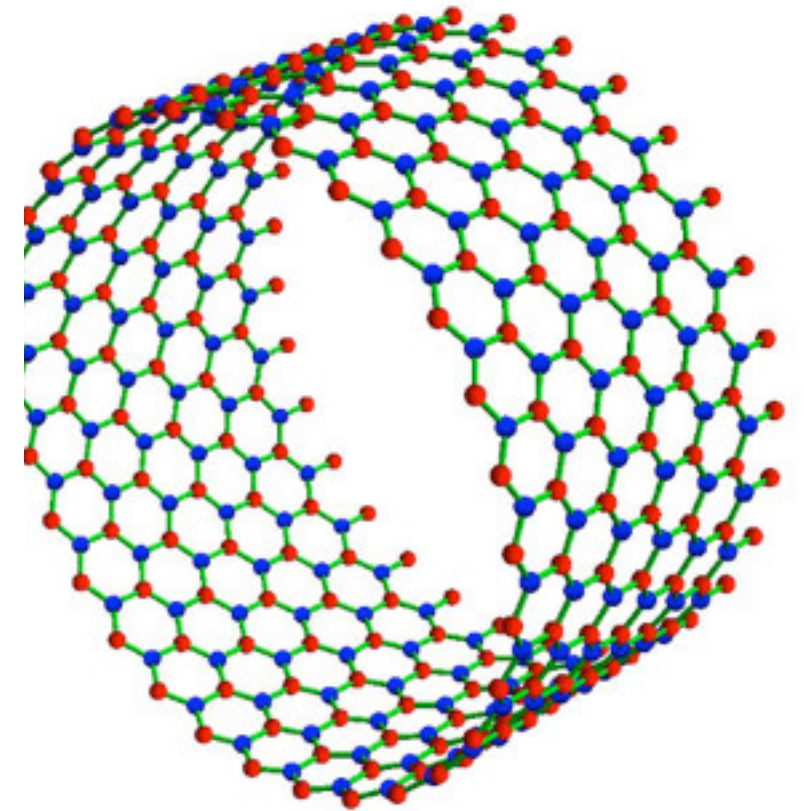
coronene

# Graphene??

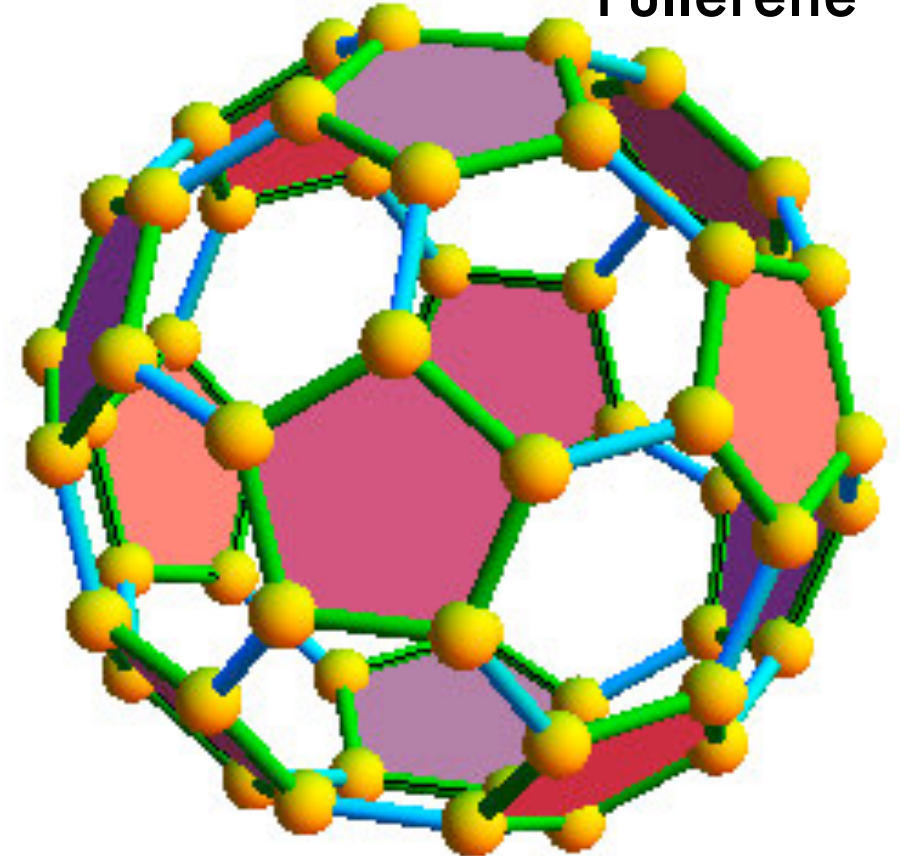
Graphene



Carbon Nano-Tube

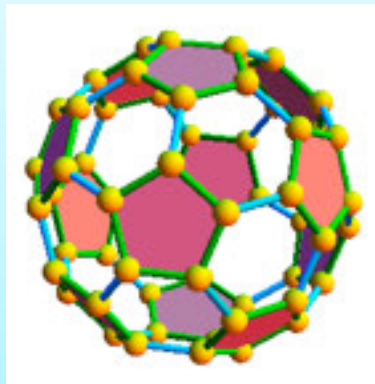


Fullerene



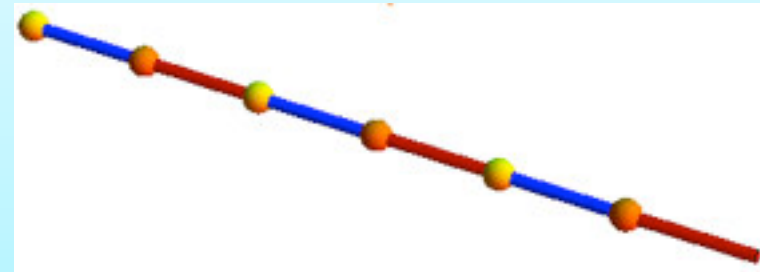
# Carbons in Dimensions 0,1,2,3,...

fullerene  $D=0$



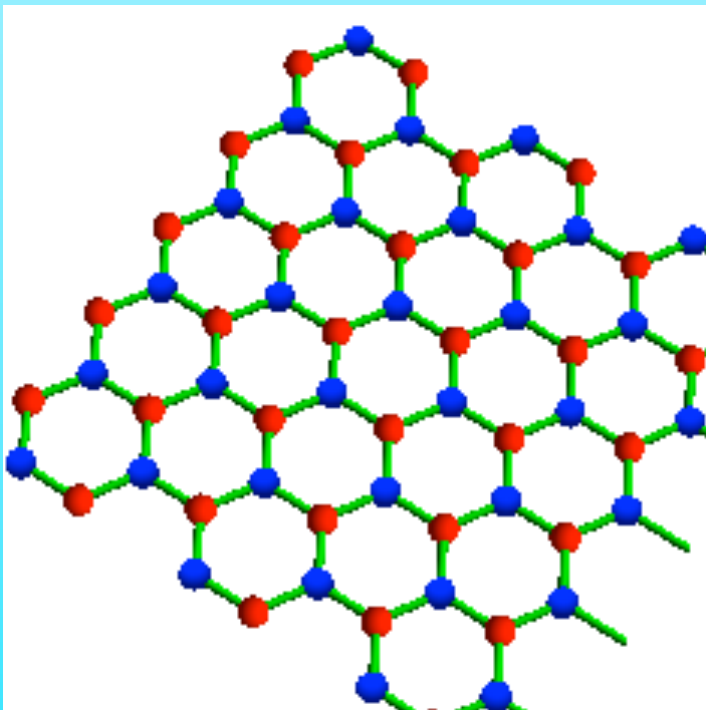
 The Nobel Prize in Chemistry 1996  
Robert F. Curl Jr., Sir Harold Kroto, Richard E. Smalley

polyacetylene  $D=1$



 The Nobel Prize in Chemistry 2000  
Alan Heeger, Alan G. MacDiarmid, Hideki Shirakawa

graphene



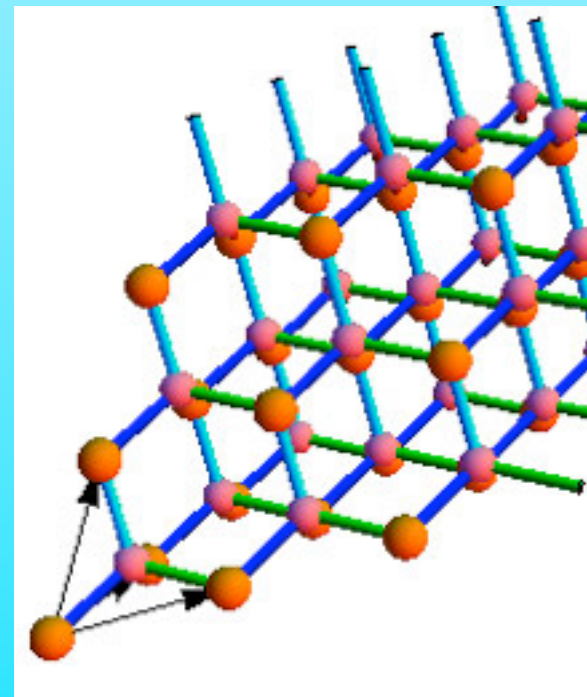
$D=2$  : tricky & lucky  
dim. for physicists

diamond  $D=3$

4  $D$  graphene  
for lattice  
gauge theory

M. Creutz

JHEP04(2008)017

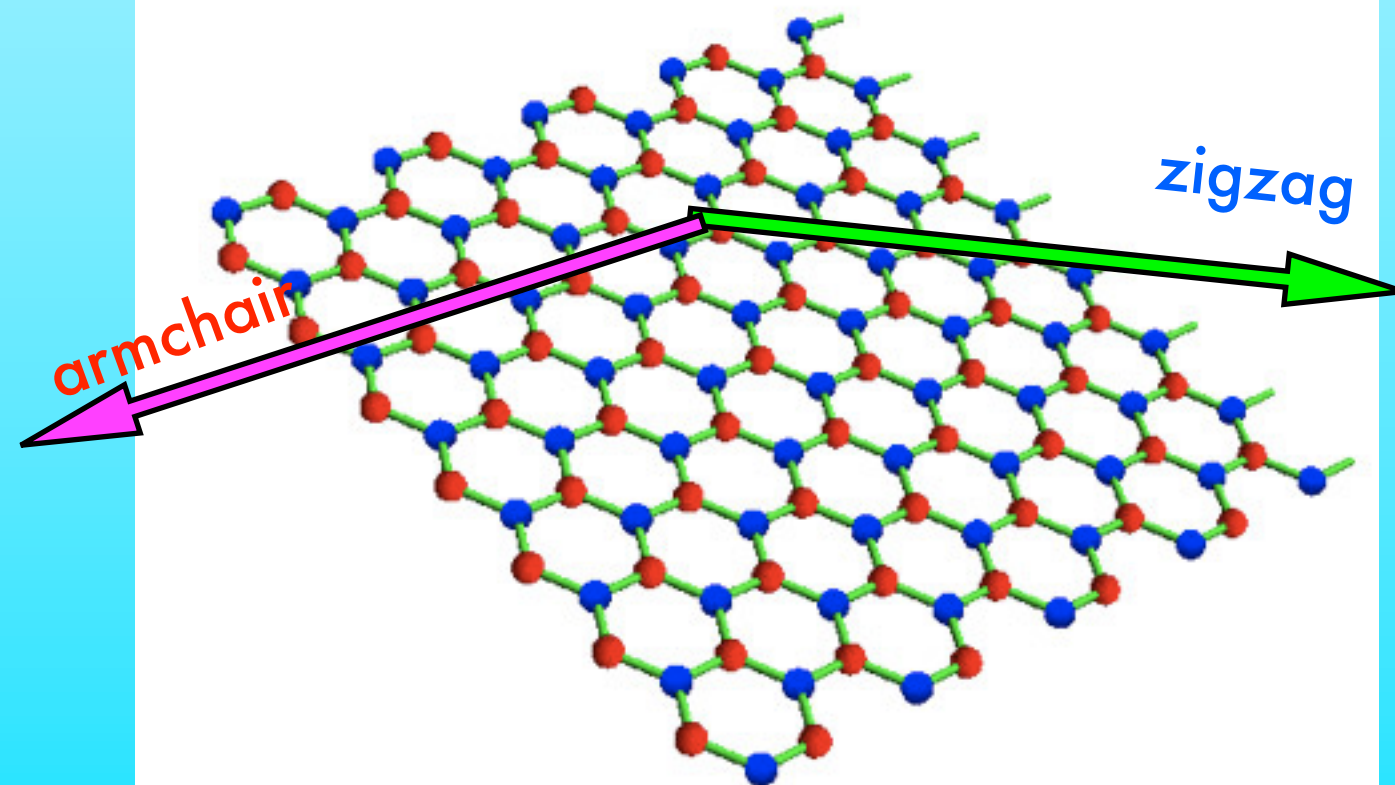
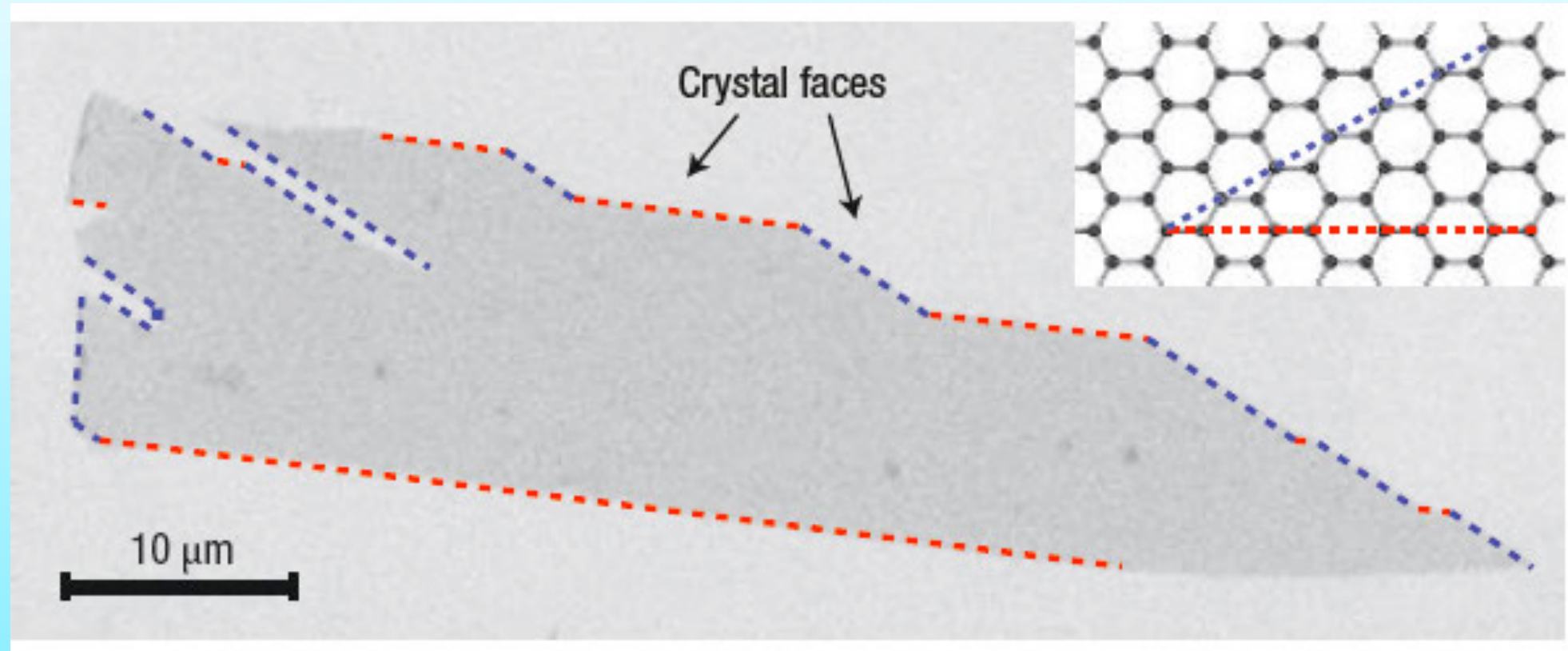


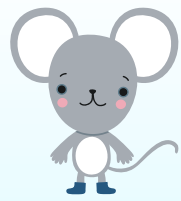
 The Nobel Prize in Physics 2010  
Andre Geim, Konstantin Novoselov



# Graphene crystal

A. Geim & K. Novoselov, *Nat. Mat.* 6, 183 (2007)



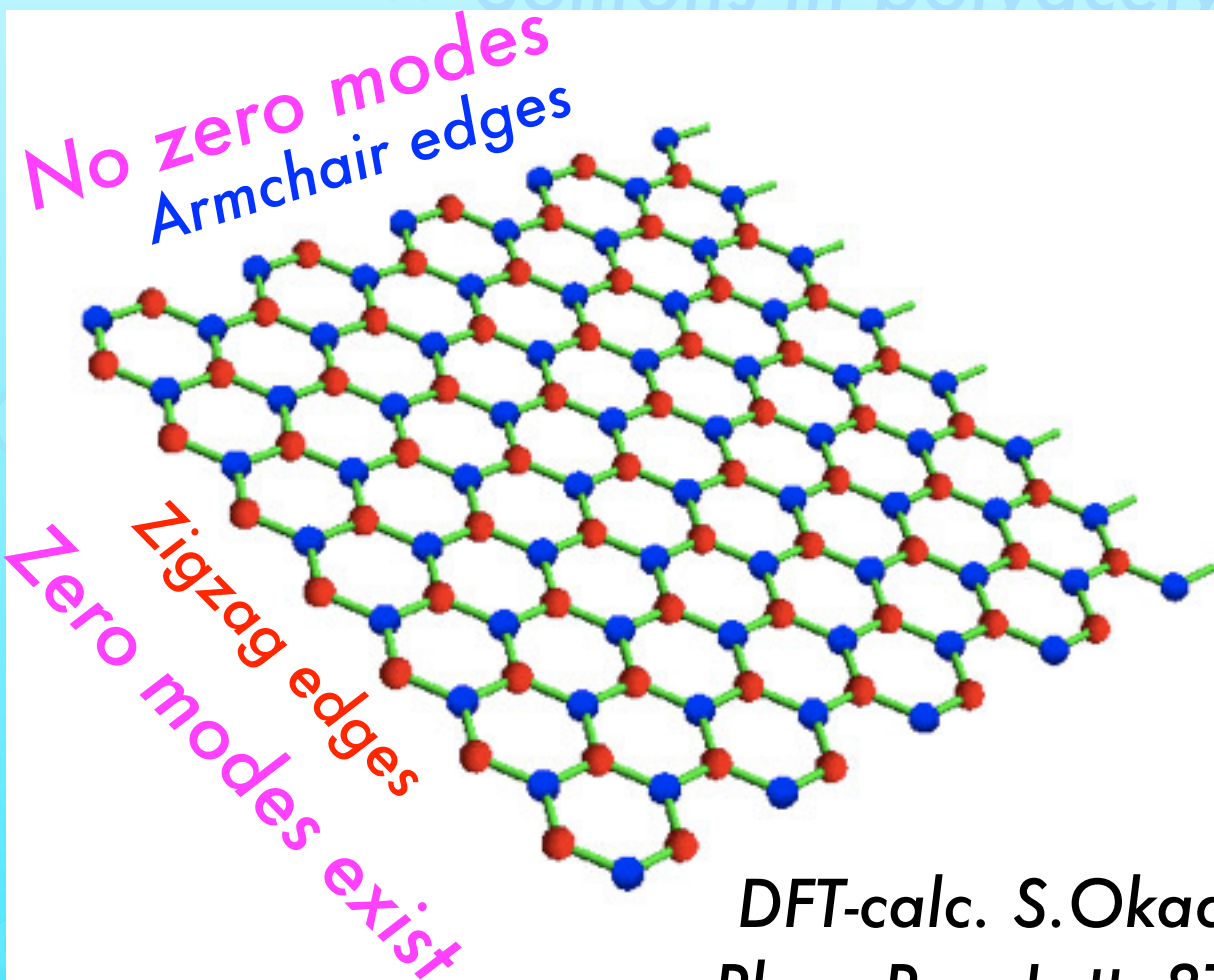


# ★ Zero energy localized states of graphene

## Peculiar Localized State at Zigzag Graphite Edge

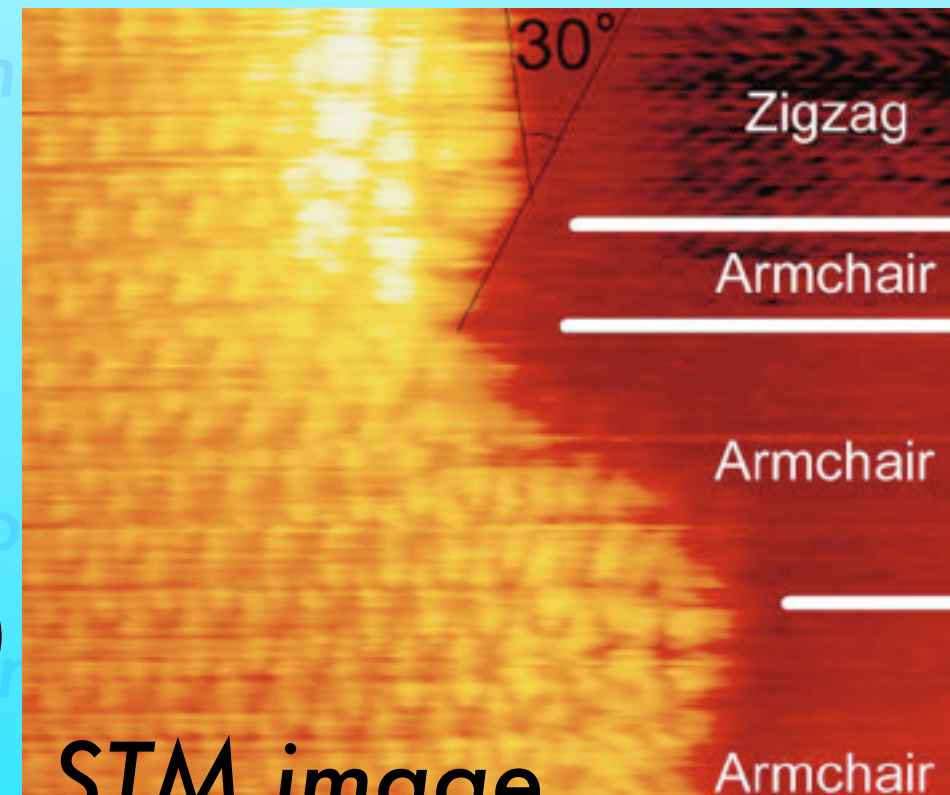
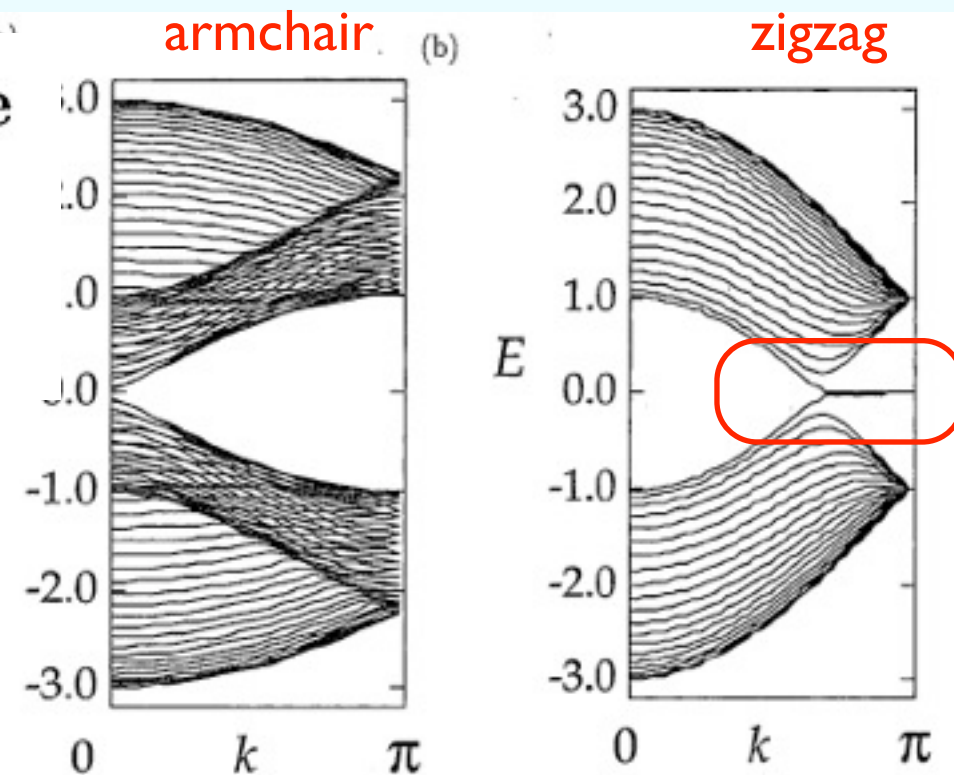
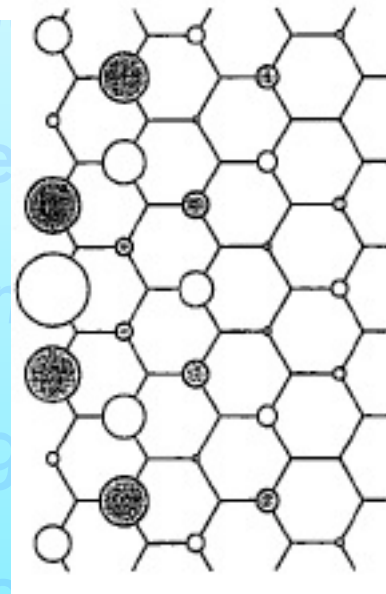
Mitsutaka FUJITA, Katsunori WAKABAYASHI, Kyoko NAKADA  
and Koichi KUSAKABE<sup>1</sup>

Journal of the Physical Society of Japan  
Vol. 65, No. 7, July, 1996, pp. 1920-1923



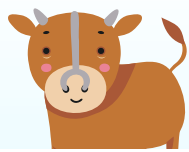
DFT-calc. S.Okada, A.Oshiyama,  
Phys. Rev. Lett. 87, 146803 (2001)

Kobayashi et al,  
Phys. Rev. B71, 193406 (2005)



STM image





# Zero bias conductance peaks of the d-wave superconductors

VOLUME 72, NUMBER 10

PHYSICAL REVIEW LETTERS

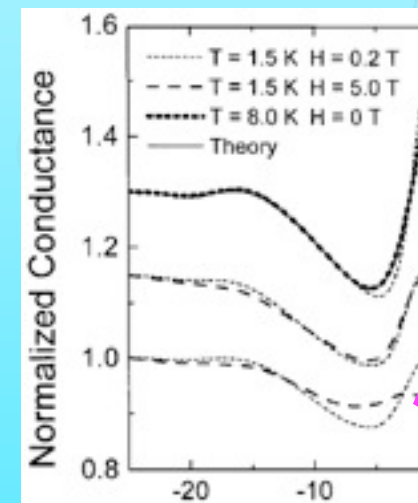
7 MARCH 1994

## Midgap Surface States as a Novel Signature for $d_{x_a^2-x_b^2}$ -Wave Superconductivity

Chia-Ren Hu

VOLUME 74, NU

24 APRIL 1995



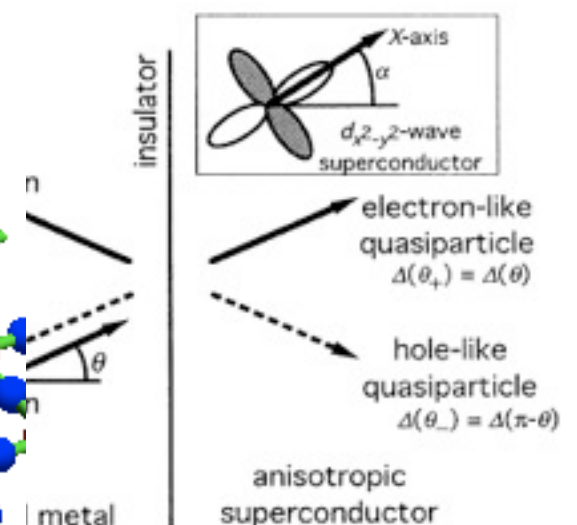
VOLUME 83, NU

Zero modes exist  
(110) boundary

Andreev  
bound states

(100) boundary  
No zero modes

ors

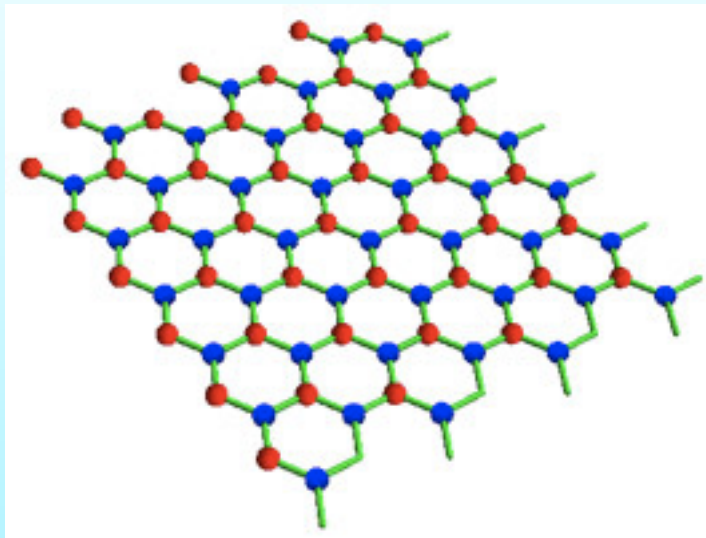


29 NOVEMBER 1999

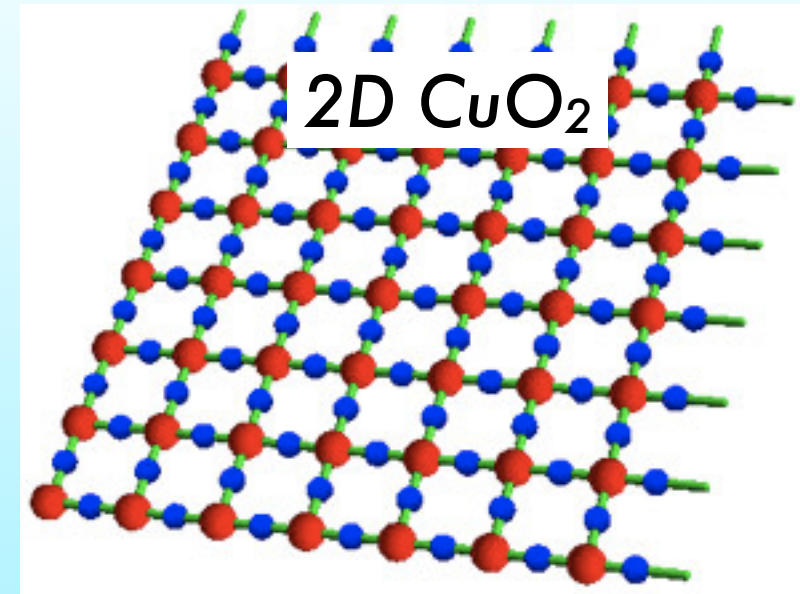
rface



# Universality in the zero modes of Dirac Fermions



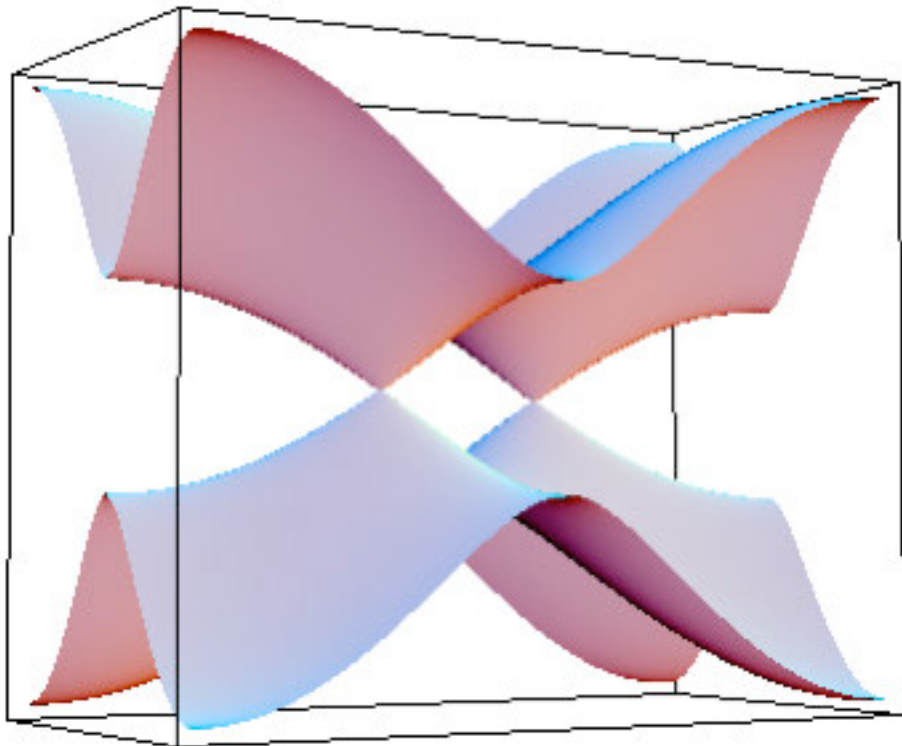
**Bulk**  $\longleftrightarrow$  **Edge**



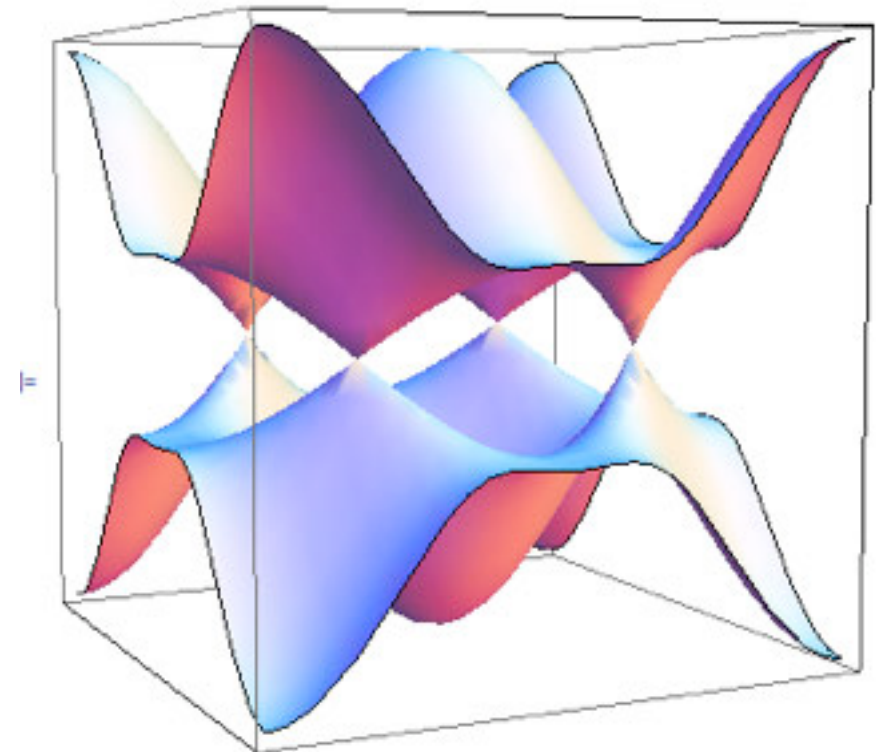
S.Ryu & Y.H., Phys. Rev. Lett. 89, 077002 (2002)

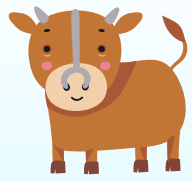
Y.H., Solid.State Comm. 149, 1061 (2009)

Graphene



d-wave superconductor





# Quantum Spin Hall Edge states

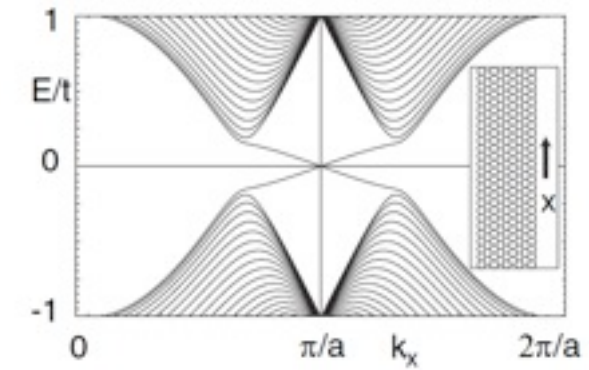
PRL **95**, 226801 (2005)

PHYSICAL REVIEW LETTERS

week ending  
25 NOVEMBER 2005

## Quantum Spin Hall Effect in Graphene

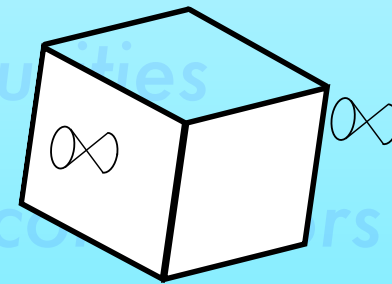
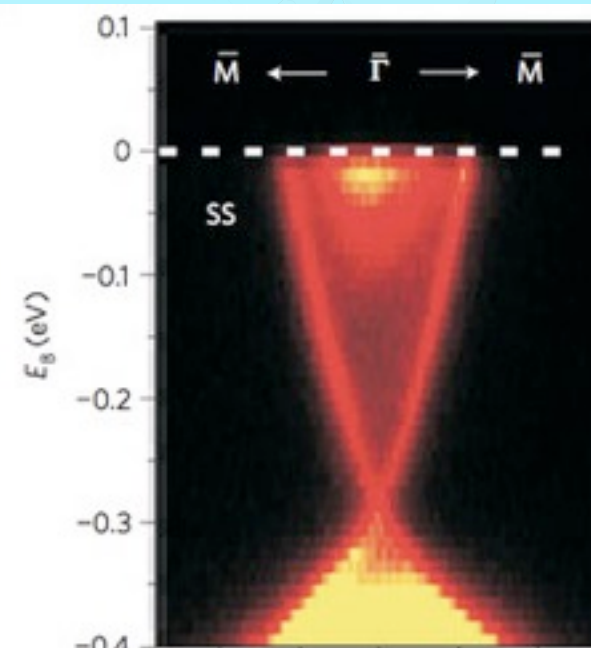
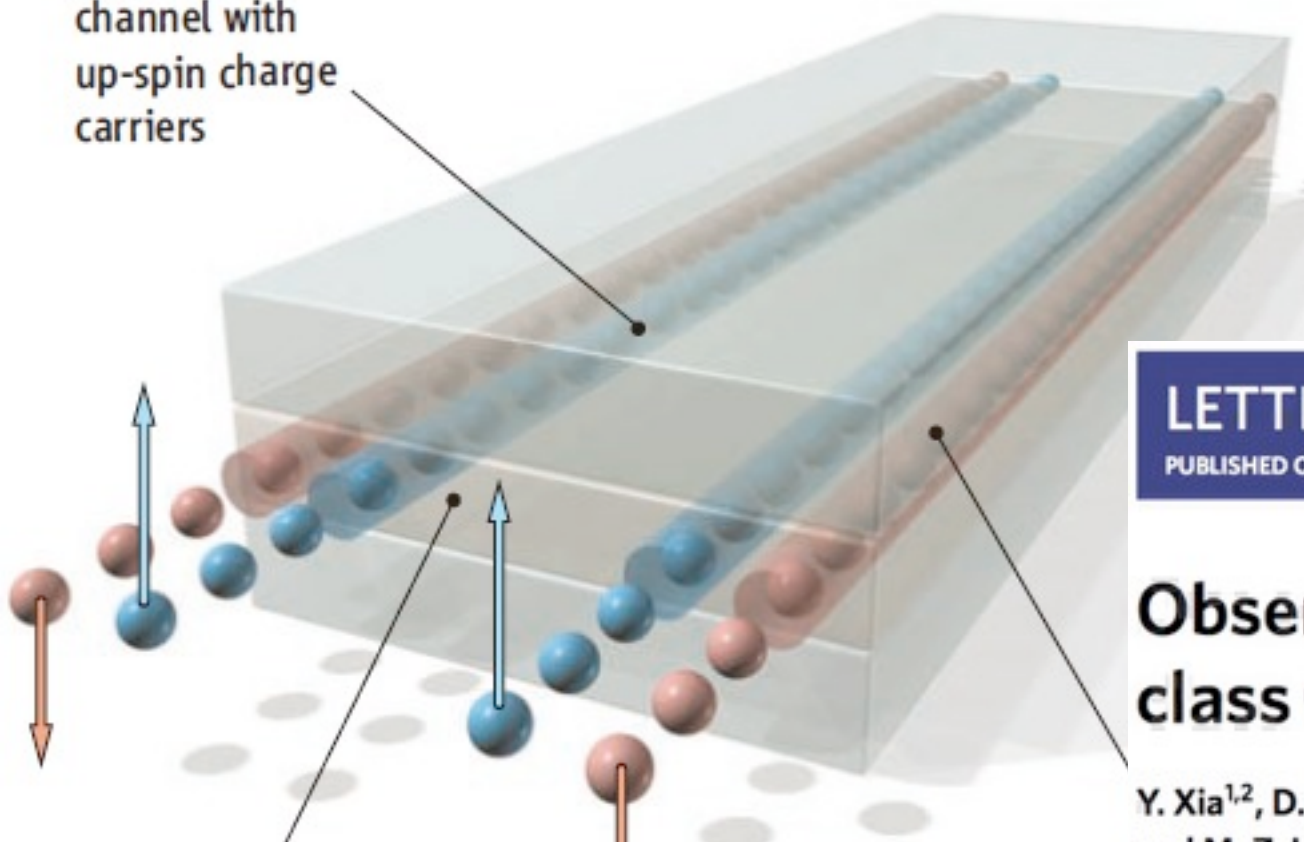
C. L. Kane and E. J. Mele



## Quantum Spin Hall Insulator State in HgTe Quantum Wells

Markus König,<sup>1</sup> Steffen Wiedmann,<sup>1</sup> Christoph Brüne,<sup>1</sup> Andreas Roth,<sup>1</sup> Hartmut Buhmann,<sup>1</sup> Laurens W. Molenkamp,<sup>1\*</sup> Xiao-Liang Qi,<sup>2</sup> Shou-Cheng Zhang<sup>2</sup>

channel with  
up-spin charge  
carriers



LETTERS

PUBLISHED ONLINE: 10 MAY 2009 | DOI: 10.1038/NPHYS1274

nature  
physics

## Observation of a large-gap topological-insulator class with a single Dirac cone on the surface

Y. Xia<sup>1,2</sup>, D. Qian<sup>1,3</sup>, D. Hsieh<sup>1,2</sup>, L. Wray<sup>1</sup>, A. Pal<sup>1</sup>, H. Lin<sup>4</sup>, A. Bansil<sup>4</sup>, D. Grauer<sup>5</sup>, Y. S. Hor<sup>5</sup>, R. J. Cava<sup>5</sup> and M. Z. Hasan<sup>1,2,6\*</sup>





# ★ Edge states in 2D cold atoms in optical lattice

PRL **108**, 255303 (2012)

PHYSICAL REVIEW LETTERS

week ending  
22 JUNE 2012

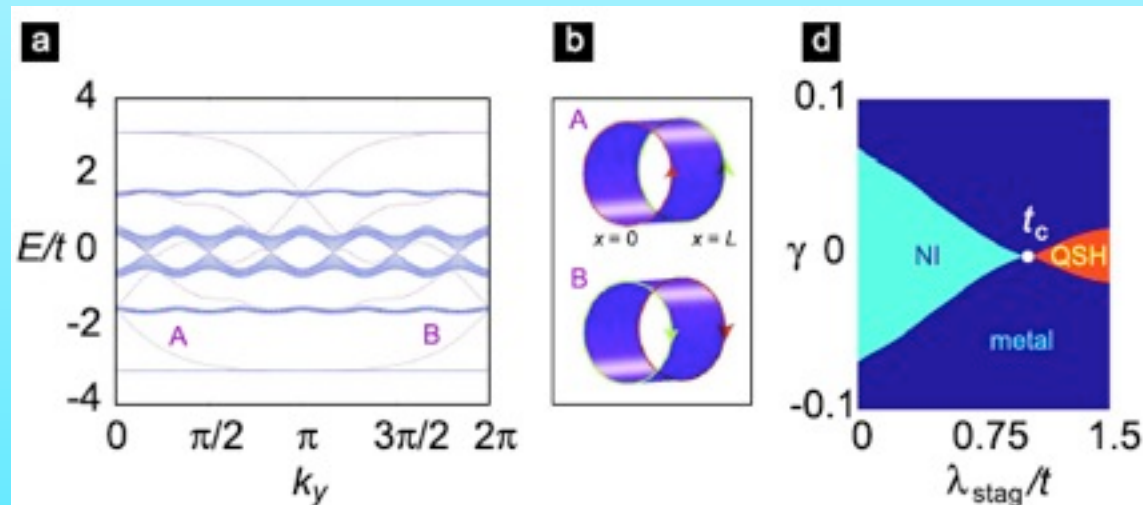
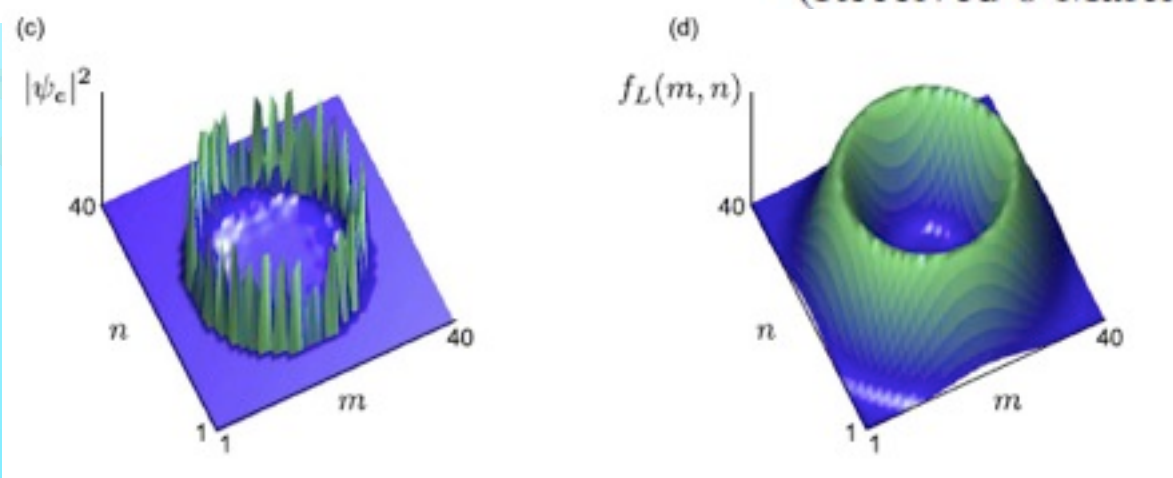
## Detecting Chiral Edge States in the Hofstadter Optical Lattice

Nathan Goldman,<sup>1,\*</sup> Jérôme Beugnon,<sup>2</sup> and Fabrice Gerbier<sup>2</sup>

<sup>1</sup>Center for Nonlinear Phenomena and Complex Systems-Université Libre de Bruxelles (U.L.B.), B-1050 Brussels, Belgium

<sup>2</sup>Laboratoire Kastler Brossel, CNRS, ENS, UPMC, 24 rue Lhomond, 75005 Paris

(Received 6 March 2012; published 19 June 2012)



PRL **105**, 255302 (2010)

PHYSICAL REVIEW LETTERS

week ending  
17 DECEMBER 2010

## Realistic Time-Reversal Invariant Topological Insulators with Neutral Atoms

N. Goldman,<sup>1</sup> I. Satija,<sup>2,3</sup> P. Nikolic,<sup>2,3</sup> A. Bermudez,<sup>4</sup> M. A. Martin-Delgado,<sup>4</sup> M. Lewenstein,<sup>5,6</sup> and I. B. Spielman<sup>7</sup>





# ★ One-way edge modes in gyromagnetic photonic crystals

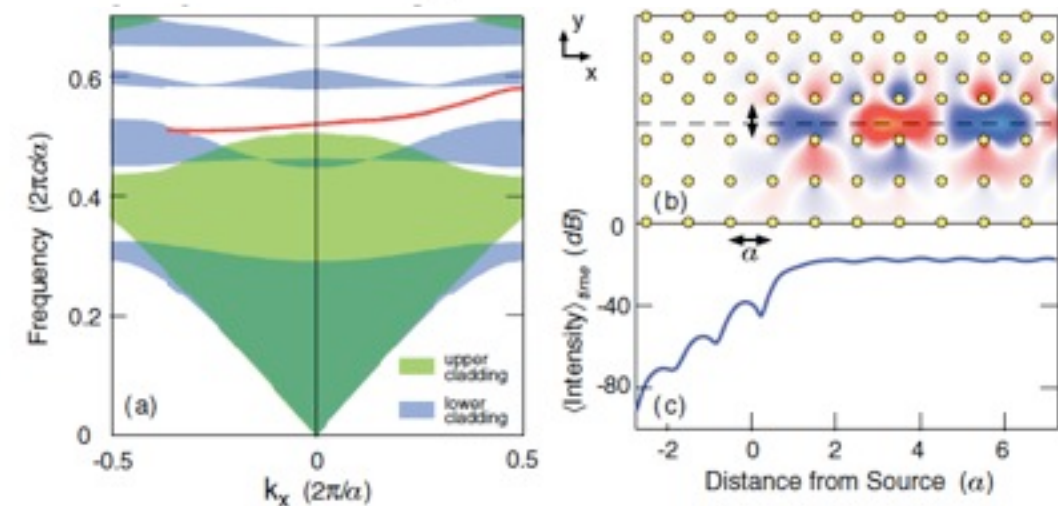
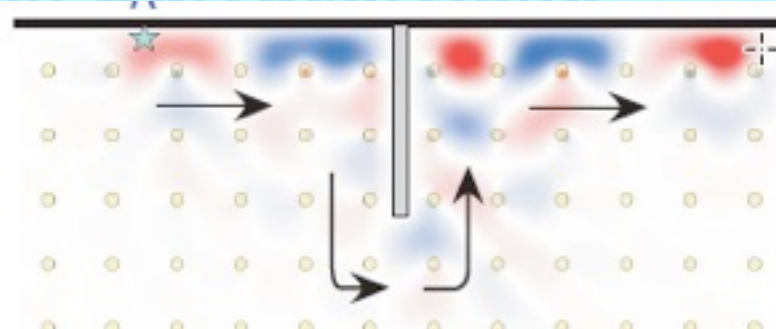
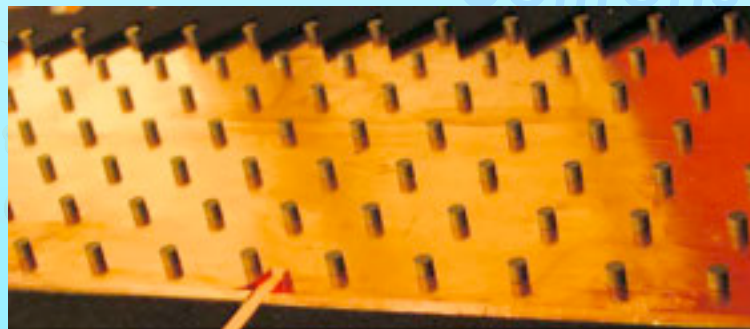
PRL 100, 013905 (2008)

PHYSICAL REVIEW LETTERS

week ending  
11 JANUARY 2008

## Reflection-Free One-Way Edge Modes in a Gyromagnetic Photonic Crystal

Zheng Wang, Y.D. Chong, John D. Joannopoulos, and Marin Soljačić



Vol 461 | 8 October 2009 | doi:10.1038/nature08293

## Observation of unidirectional backscattering-immune topological electromagnetic states

Zheng Wang<sup>1\*</sup>, Yidong Chong<sup>1†\*</sup>, J. D. Joannopoulos<sup>1</sup> & Marin Soljačić<sup>1</sup>

PRL 100, 013904 (2008)

PHYSICAL REVIEW LETTERS

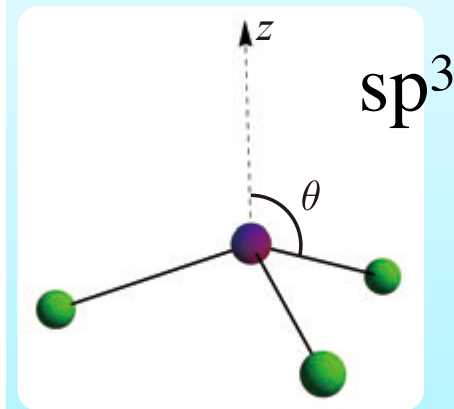
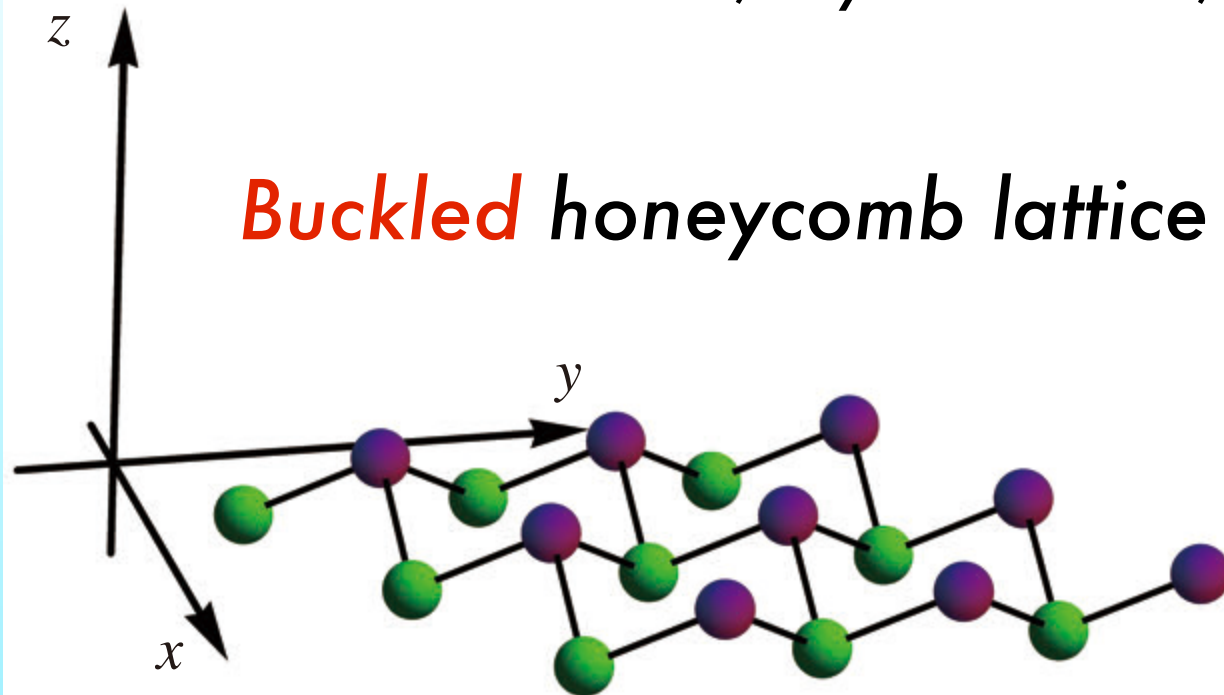
week ending  
11 JANUARY 2008

## Possible Realization of Directional Optical Waveguides in Photonic Crystals with Broken Time-Reversal Symmetry

F. D. M. Haldane and S. Raghu\*

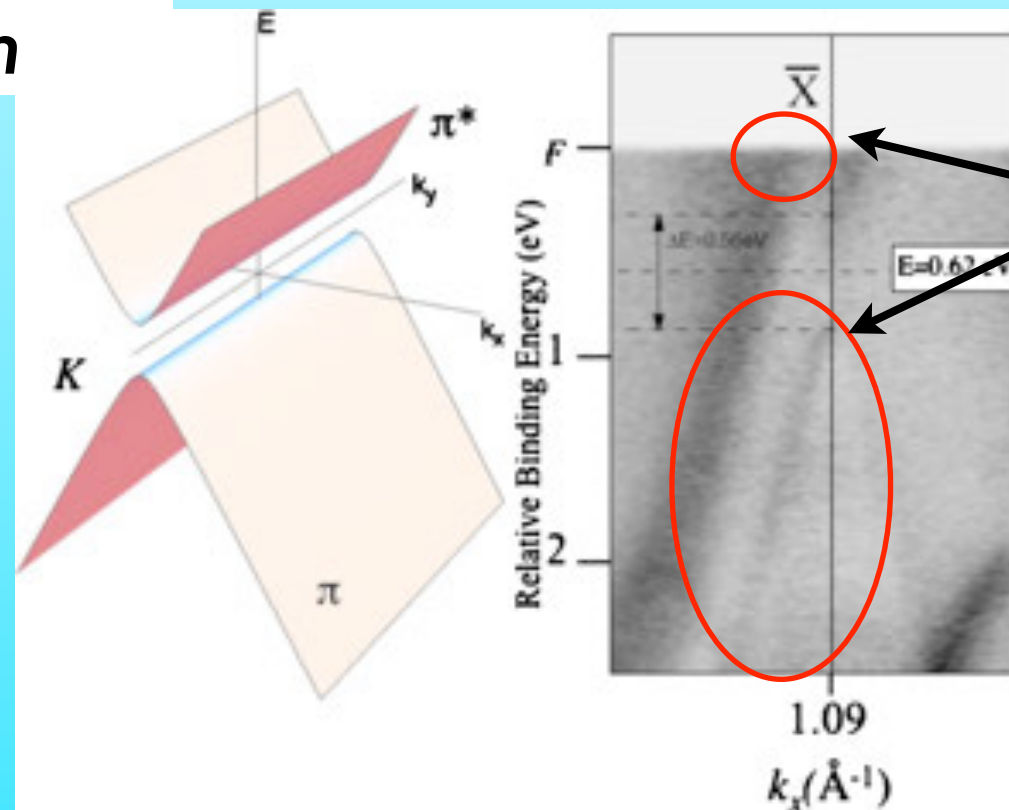
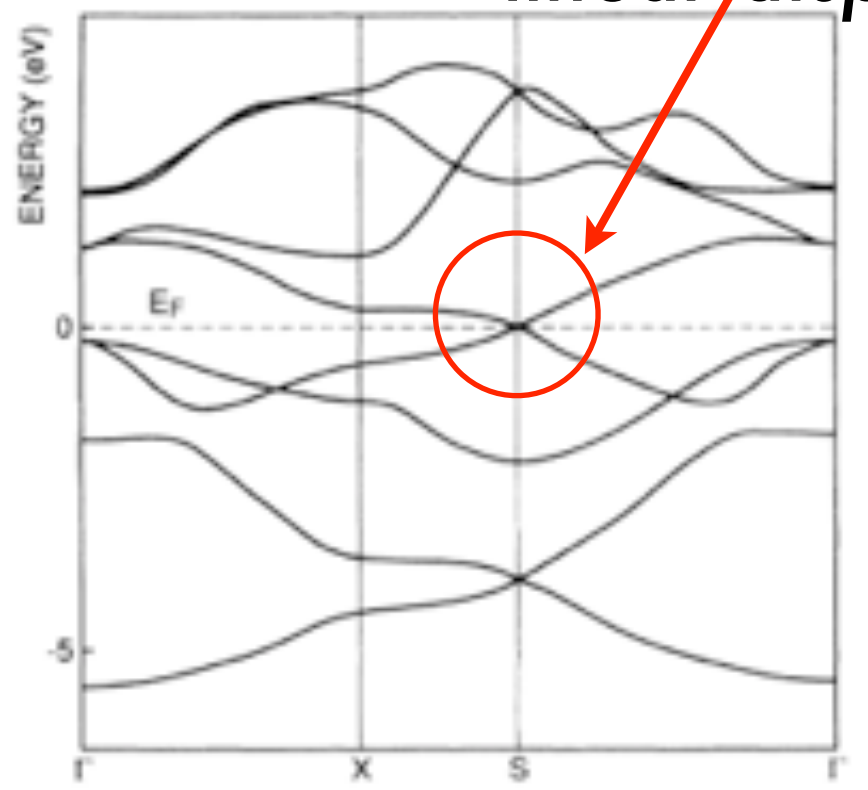
# Silicene : silicon analogue of graphene

K. Takeda and K. Shiraishi, Phys. Rev. B. 50, 14916 (1994)



**ARPES**

linear dispersion



Dirac cone

**Zero gap semiconductor**

P. D. Padova, et. al APL 96, (2010) 261905

# *Plan*

- ★ *Metal, insulator & semiconductor*
- ★ *Bulk-edge correspondence :graphene, silicene and more*
- ★ *From Newton to Dirac for devices breakthrough*

# From Newton to Dirac

★ Classical & Quantum

M. Planck ~1900



The Munich physics professor [Philipp von Jolly](#) advised Planck against going into physics, saying, "in this field, almost everything is already discovered, and all that remains is to fill a few holes."

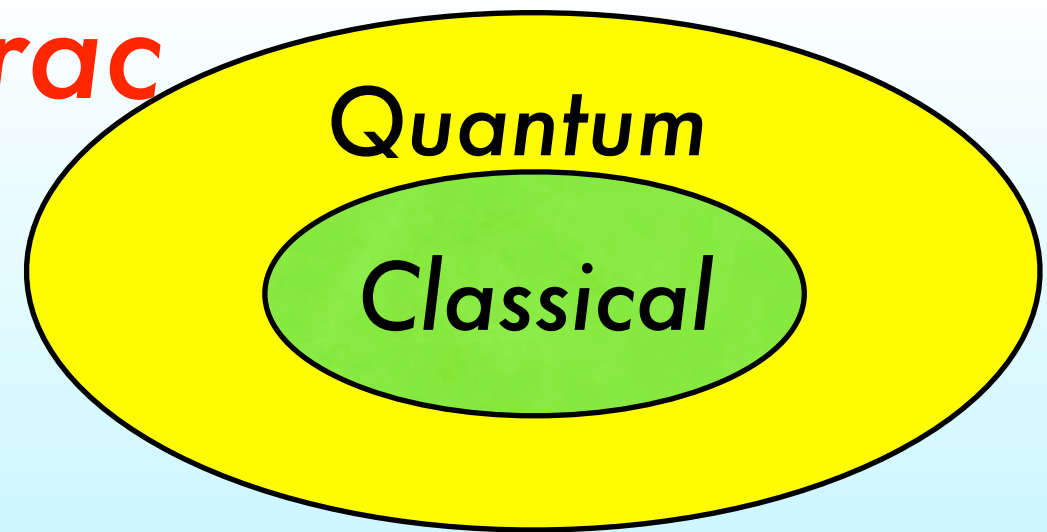
Wrong ! Quantum mechanics & Relativity

Newton (classical) → Dirac (quantum & relativistic)

Crossover : No clear boundaries

Future devices: Breakthrough by quantum effects

Quantum computer may be too hard





# From Newton to Dirac

## ★ Classical & Quantum

**Future devices: Breakthrough by quantum effects**

### ★ Planck定数：唯一の量子論固有の定数

$$\hbar = 6.626068 \times 10^{-34} [\text{J} \cdot \text{s}]$$

$$[\text{J} \cdot \text{s}] = (\text{エネルギー}) \times (\text{時間}) = (\text{運動量}) \times (\text{長さ}) = (\text{角運動量}) \\ = [\text{kg} \cdot \text{m/s}^2 \cdot \text{m} \cdot \text{s}] = [\text{kg} \cdot \text{m/s} \cdot \text{m}]$$

$$\left. \begin{array}{l} (\text{エネルギー}) \times (\text{時間}) \\ (\text{運動量}) \times (\text{長さ}) \\ (\text{角運動量}) \end{array} \right\} \begin{array}{l} \gg \hbar : \text{classical} \\ \approx \hbar : \text{quantum} \end{array}$$

$$\hbar/10[\text{nm}] = p = 6.6 \times 10^{-26} [\text{kg} \cdot \text{m/s}], \quad p^2/2m_e = 0.015[\text{eV}]$$

$$\hbar/10[\text{fs}] = E = 6.6 \times 10^{-20} [\text{J}] = 0.4[\text{eV}]$$

$$\text{electron spin} = \hbar/2 \quad : \text{purely quantum}$$



# From Newton to Dirac

What's spin ?      Non-relativistic limit of the Dirac hamiltonian

$H =$  **Spin**  **Quantum mechanics + Relativity**

**Spin: natural inner degree of freedom**

**relativistic correction**

$$H\psi = \psi E \quad \psi = \begin{bmatrix} \psi_{\uparrow} \\ \psi_{\downarrow} \end{bmatrix} \quad \text{spinor}$$

**charge**  $\rho = |\psi|^2, \quad j = -i\hbar\psi^{\dagger}\nabla\psi + h.c.$

$$\frac{\partial\rho}{\partial t} + \text{div } j = 0$$

**conserve**

**spin**  $\rho_S = \psi^{\dagger}\sigma\psi, \quad j_S = -i\hbar\psi^{\dagger}\sigma\nabla\psi + h.c.$

$$S^+ = \psi_{\uparrow}^{\dagger}\psi_{\downarrow} \quad S^- = \psi_{\downarrow}^{\dagger}\psi_{\uparrow}$$

$$\frac{\partial\rho_S}{\partial t} + \text{div } j_S \neq 0$$

**does not conserve**

**Time reversal**  $S \rightarrow -S$

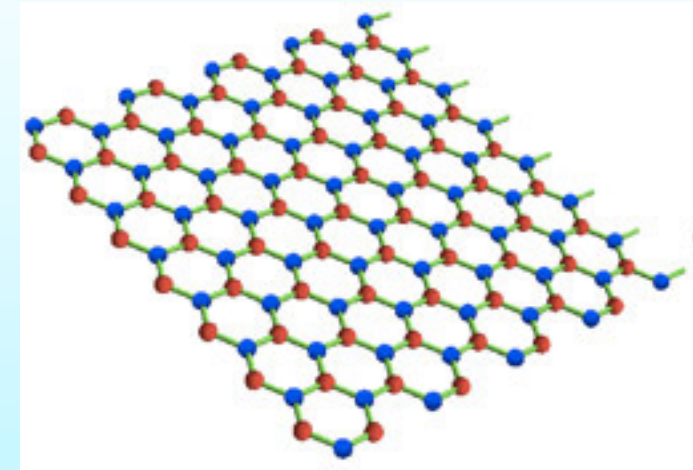
**spin breaks TR**

**spinor is OK : Kramers degeneracy:**

# One more half electrons

## 1. Zero gap semiconductors

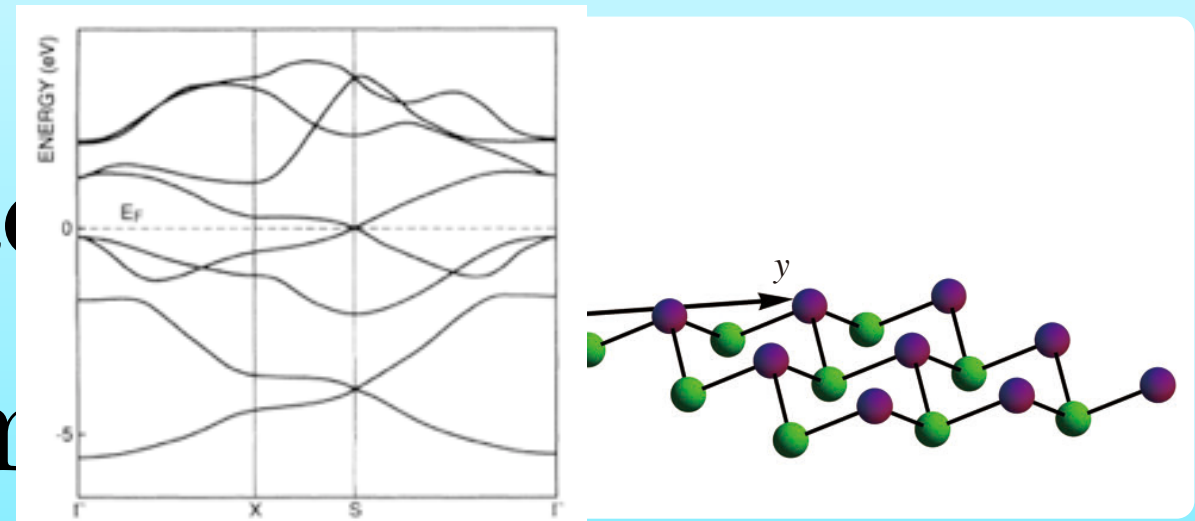
$$H_{zero} = \pm \sqrt{H_{ele}}$$



## 2. Majorana fermions

$$H_{Majo,R} = \text{Re}$$

$$H_{Majo,I} = \text{Im}$$



$$H_{ele} = H_{Majo,R} + iH_{Majo,I}$$

Breakthrough of the Year, 2012



Science 25 May 2012:  
Vol. 336 no. 6084 pp. 1003-1007

**Signatures of Majorana Fermions in Hybrid Superconductor-Semiconductor Nanowire Devices**

V. Mourik<sup>1,\*</sup>, K. Zuo<sup>1,\*</sup>, S. M. Frolov<sup>1</sup>, S. R. Plissard<sup>2</sup>, E. P. A. M. Bakkers<sup>1,2</sup>, L. P. Kouwenhoven<sup>1,†</sup>

# Conclusion

Quantum effects are everywhere  
in  
condensed matter physics

Use of the Quantum effects  
in quantum nano devices