量子力学3 まとめレポート（4/17 授業分）

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In general,

$$\langle O \rangle_{\psi(t)} \equiv O^{obs}(t)$$

$$= \langle \psi(t) | O | \psi(t) \rangle$$

$$= \langle \psi(0) | e^{iHt/\hbar} O e^{-iHt/\hbar} | \psi(0) \rangle$$

if $[G, H] = 0$ commute with each other ($\Leftrightarrow [G, e^{iHt/\hbar}] = 0$)

$$\langle G \rangle_{\psi(t)} = G^{obs}(t) = \langle \psi(0) | e^{iHt/\hbar} G^{obs}(t) e^{-iHt/\hbar} | \psi(0) \rangle$$

$$= \langle \psi(0) | 1 \cdot G | \psi(0) \rangle$$

$G^{obs}(t)$: t independent $\longleftrightarrow$ conserved quantity.

$$\delta H = 0 \longrightarrow \frac{d}{dt} G^{obs}(t) = 0.$$

conservation law.

$$U_{\delta\lambda} = e^{i\delta\lambda G/\hbar}$$

transformation
Symmetry operation

ex) in 3 dim,

$$U_{\delta\vec{a}} = e^{-i\delta\vec{a}\cdot\vec{p}/\hbar} = e^{-\delta\vec{a}\cdot\nabla} \qquad \vec{r} \longmapsto \vec{r}' = \vec{r} + \delta\vec{a}$$

free particle,

$$H = \frac{\vec{P}^2}{2m} = \frac{1}{2m}(P_x^2 + P_y^2 + P_z^2)$$

$$[P_i, P_j] = 0 \longrightarrow [P_i, H] = 0.$$

$$\Rightarrow [\delta\vec{a}\cdot\vec{p}, H] = 0$$

$\longrightarrow \vec{P}^{obs}(t)$: conserved.

~ time translation（時間推進）~

in 1D space translation, $x \longmapsto x' = x + a$.

$$\psi(\underset{x'-a}{\underset{\shortparallel}{x}}) = \psi'(x'), \qquad \psi'(x) = \psi(x-a) = e^{-a d_x}\psi$$

時間についても同様の考え方を適用させていく。

$t \mapsto t' = t + \tau$ $\quad\quad \psi(t, x) = \psi'(t', x)$

$\psi'(t, x) = \psi(t - \tau, x) = e^{-\tau\partial_t}\psi$

$\therefore\ \psi \mapsto \psi' = \underbrace{e^{-\tau\partial_t}}_{U_\tau}\psi$ $\quad\quad U_\tau = e^{-\tau\partial_t}$

$= e^{i \cdot i\tau\hbar\partial_t/\hbar} = e^{i\tau H/\hbar}$

H: time independent

$U_{\delta\lambda} = e^{i\delta\lambda G}$ $\quad\quad G = H$ generator of the time translation

$[G, H] = [H, H] = 0 \quad \rightarrow \quad \delta H = 0.$

$H^{obs}_{(t)} = \underbrace{E(t)}_{t \text{ independent}}$ energy at time t

translation		conserved law
spacial	$\longrightarrow$	momentum
time	$\longrightarrow$	energy

☆ Rotation (回転)

↳ angular momentum (角運動量)

↳ spin

$\vec{r} \mapsto \vec{r}' = R\vec{r}$ $\quad$ R = real 3×3 matrix.

Q $\underline{\underline{|\vec{r}'| = |\vec{r}|}}$

$\tilde{\vec{r}} = {}^t\vec{r} = {}^t\begin{pmatrix} x \\ y \\ z \end{pmatrix} = (x, y, z)$

$|\vec{r}|^2 = \vec{r} \cdot \vec{r} = (x, y, z)\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \tilde{\vec{r}}\vec{r}$

$\vec{r}' = R\vec{r}$

$|\vec{r}'|^2 = \vec{r}' \cdot \vec{r}' = (x', y', z')\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \tilde{\vec{r}}'\vec{r}'$

$= (\widetilde{R\vec{r}})(R\vec{r})$

$= \tilde{\vec{r}}\underbrace{\tilde{R}R}_{E_3}\vec{r}$ $\quad\quad$ (※ $\widetilde{AB} = \tilde{B}\tilde{A}$)

3×3 unit matrix.

$= \vec{r}^2 = \tilde{\vec{r}}\vec{r}$

$\tilde{R}R = E_3$. $\tilde{R} = R^{\dagger}$ $RR^{-1} = E_3$.

R : orthogonal matrix 直交行列. $R \in O(3)$

$$\det R\tilde{R} = \det E_3 = 1$$
$$= \det R \det \tilde{R}$$
$$= (\det R)^2 \longrightarrow \det R = \pm 1,$$

if $\det R = +1$, special orthogonal matrix $R \in SO(3)$

if $\vec{r} \mapsto \vec{r}' = \vec{r} = E_3\vec{r}$ identity transformation

$R = E_3 \in SO(3)$ R continiously deformed to E_3.

and if $\det R = -1$,

$R = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$ space invarision in odd dim.

$\det R = (-1)^3 = -1$.