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量子力学3 6/11 授業まとめ

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補足

$$\tfrac{1}{2} \otimes \tfrac{1}{2} = 1 \oplus 0$$

$$|11\rangle = |\uparrow\uparrow\rangle$$
$$|10\rangle = \tfrac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle\right)$$
$$|1\,{-1}\rangle = |\downarrow\downarrow\rangle$$

$J_z = 0$ の状態

$$(|\uparrow\downarrow\rangle, |\downarrow\uparrow\rangle)$$
$$\downarrow$$
$$(|10\rangle, |00\rangle)$$

$$|10\rangle = (|\uparrow\downarrow\rangle, |\downarrow\uparrow\rangle)\begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix}$$

$\langle 10|00\rangle = 0$
直交するように $\begin{pmatrix} * \\ * \end{pmatrix}$ を決める.

$$|00\rangle = (|\uparrow\downarrow\rangle, |\downarrow\uparrow\rangle)\begin{pmatrix} * \\ * \end{pmatrix}$$

$$\langle 10| = \left(1/\sqrt{2}, 1/\sqrt{2}\right)\begin{pmatrix} \langle\uparrow\downarrow| \\ \langle\downarrow\uparrow| \end{pmatrix}$$

$$\langle 10|00\rangle = \left(1/\sqrt{2}, 1/\sqrt{2}\right)\begin{pmatrix} \langle\uparrow\downarrow| \\ \langle\downarrow\uparrow| \end{pmatrix}(|\uparrow\downarrow\rangle, |\downarrow\uparrow\rangle)\begin{pmatrix} * \\ * \end{pmatrix}$$
$$= \left(1/\sqrt{2}, 1/\sqrt{2}\right)\begin{pmatrix} \langle\uparrow\downarrow|\uparrow\downarrow\rangle^{\fallingdotseq 1} & \langle\uparrow\downarrow|\downarrow\uparrow\rangle^{\fallingdotseq 0} \\ \underbrace{\langle\downarrow\uparrow|\uparrow\downarrow\rangle}_{\fallingdotseq 0} & \underbrace{\langle\downarrow\uparrow|\downarrow\uparrow\rangle}_{\fallingdotseq 1} \end{pmatrix}\begin{pmatrix} * \\ * \end{pmatrix}$$

$$\left(\langle\uparrow_1|\langle\downarrow_2||\downarrow_1\rangle|\uparrow_2\rangle = \underbrace{\langle\uparrow_1|\downarrow_1\rangle}_{\fallingdotseq 0}\underbrace{\langle\downarrow_2|\uparrow_2\rangle}_{\fallingdotseq 0}\right)$$

$$\langle 00|00\rangle = 1 = (*^* \ *^*)\begin{pmatrix} * \\ * \end{pmatrix}$$

$$\begin{pmatrix} * \\ * \end{pmatrix} = \begin{pmatrix} -1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix}$$
試行錯誤

$$\underset{\Psi_0}{(|10\rangle, |00\rangle)} = \underset{\psi_0}{(|\uparrow\downarrow\rangle, |\downarrow\uparrow\rangle)}\underbrace{\begin{pmatrix} * & * \\ * & * \end{pmatrix}}_{M}$$

$J_z = 0$ の2つの状態

$$\Psi_0 = \psi_0 M$$
$$\Psi_0 = (|10\rangle, |00\rangle)$$
$$\psi_0 = (|\uparrow\downarrow\rangle, |\downarrow\uparrow\rangle)$$

$$\psi_0^\dagger \psi_0 = E_2$$
$$\Psi_0^\dagger \Psi_0 = \begin{pmatrix} \langle 10| \\ \langle 00| \end{pmatrix}(|10\rangle, |00\rangle)$$
$$= \begin{pmatrix} \langle 10|10\rangle & \langle 10|00\rangle \\ \langle 00|10\rangle & \langle 00|00\rangle \end{pmatrix} = E_2$$

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$\Psi_0^\dagger = M^\dagger \psi_0^\dagger$

$\Psi_0^\dagger \Psi_0 = M^\dagger \underbrace{\psi_0^\dagger \psi_0}_{E_2} M = M^\dagger M = E_2$ $\qquad E_2 = MM^{-1} = MM^\dagger$

$\qquad M^\dagger = M^{-1}$: unitary

$M = \begin{pmatrix} 1/\sqrt{2} & * \\ 1/\sqrt{2} & * \end{pmatrix} = (\psi_c, \psi)$, $\psi_c = \begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix}$, $\psi = \begin{pmatrix} * \\ * \end{pmatrix}$

↑ 決めたい

$MM^\dagger = E_2 = (\psi_c \ \ \psi)\begin{pmatrix} \psi_c^\dagger \\ \psi^\dagger \end{pmatrix} = \underbrace{\psi_c \psi_c^\dagger}_{P} + \psi\psi^\dagger$

$E_2 = P + \psi\psi^\dagger$, $P = \psi_c \psi_c^\dagger$

$= \begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix} (1/\sqrt{2}, 1/\sqrt{2}) = \begin{pmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{pmatrix}$

P : projection (射影)

$P^2 = (\psi_c \psi_c^\dagger)(\psi_c \psi_c^\dagger) = \psi_c \underbrace{\psi_c^\dagger \psi_c}_{E_2} \psi_c^\dagger = \psi_c \psi_c^\dagger = P$

$E_2 = P + \psi\psi^\dagger$

$E_2 - P = \psi\psi^\dagger$

↖ これも projection

$(\psi\psi^\dagger)^2 = \psi\underbrace{\psi^\dagger\psi}_{E_2}\psi^\dagger = \psi\psi^\dagger$

ϕ : 任意のベクトル

$\phi = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$(E_2 - P)\phi = \psi\psi^\dagger\phi$

$= \psi(* \ *)\begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$= \alpha\psi$

$(E_2 - P)^2 = E_2 - 2E_2P + P^2 = E_2 - 2P + P$

$= E_2 - P$

$(E_2 - P) = \begin{pmatrix} 1/2 & -1/2 \\ -1/2 & 1/2 \end{pmatrix}$, $(E_2 - P) = \begin{pmatrix} -1/2 \\ 1/2 \end{pmatrix} = \psi'$

$\|\psi\|^2 = 1/4 + 1/4 = 1/2$, $\|\psi'\| = 1/\sqrt{2}$

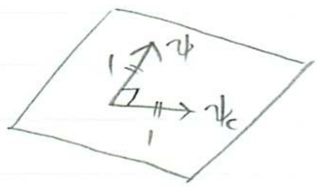
$\psi'/\|\psi'\| = \begin{pmatrix} -1/2 \\ 1/2 \end{pmatrix}\sqrt{2} = \begin{pmatrix} -1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix} = \psi$

$M = \begin{pmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}$, $\Psi_0 = \psi_0 M$

P : known , ϕ : given

$(E_2 - P)\phi = \psi' \propto \psi$, $\psi = \psi'/\|\psi'\|$

ψ_c, ψ : 2次元の基底ベクトル



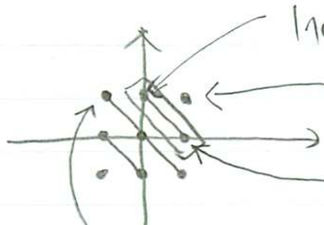
$\psi' = \phi - P\phi' = (E_2 - P)\phi$

$\psi = \psi'/\|\psi'\|$

$\|\psi_c\| = 1$

$P\phi = \psi_c \psi_c^\dagger \phi$

$1 \otimes 1 = 2 \oplus 1 \oplus 0$ (J_z : 一定の線)



$|10\rangle \otimes |11\rangle = |1011\rangle$

$|11\rangle \otimes |11\rangle = |1111\rangle = |22\rangle$

$|11\rangle \otimes |10\rangle = |1110\rangle$

$|1-1\rangle \otimes |11\rangle = |1-111\rangle$

$|j_1 m_1 j_2 m_2\rangle = |j_1 m_1\rangle \otimes |j_2 m_2\rangle = |1 m_1\rangle \otimes |1 m_2\rangle$

$J_z |22\rangle = \hbar 2 |22\rangle$

$J_+ |22\rangle = 0$

$J_z = 1$

$(|10\rangle \otimes |11\rangle, |11\rangle \otimes |10\rangle) = (|1011\rangle, |1110\rangle) = (|21\rangle, |11\rangle)$

$(|21\rangle, |11\rangle) = (|1011\rangle, |1110\rangle) M$

$$|21\rangle = \frac{1}{\hbar\sqrt{4\cdot 1}} J_- |22\rangle \qquad \left(|j\, m-1\rangle = \frac{1}{\hbar\sqrt{(j+m)(j-m+1)}} J_- |jm\rangle \right)$$

$$= \frac{1}{2\hbar} J_- |22\rangle = \frac{1}{2\hbar}(J_{1-} + J_{2-})(|11\rangle \otimes |11\rangle)$$

$$= \frac{1}{2\hbar}(\underbrace{J_{1-}|11\rangle}_{\hbar\sqrt{2}|10\rangle} \otimes |11\rangle + |11\rangle \otimes \underbrace{J_{2-}|11\rangle}_{\hbar\sqrt{2}|10\rangle})$$

$J_{1-}|11\rangle \otimes |11\rangle = \hbar\sqrt{2}\, |10\rangle \otimes |11\rangle$ $\qquad$ $|11\rangle \otimes J_{2-}|11\rangle = \hbar\sqrt{2}\, |11\rangle \otimes |10\rangle$

$$= \frac{1}{\sqrt{2}}(|10\rangle \otimes |11\rangle + |11\rangle \otimes |10\rangle)$$

$$= (|1011\rangle, |1110\rangle)\begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix}$$

$$\boxed{\begin{aligned}&(|21\rangle, |11\rangle) \\ &= (|1011\rangle, |1110\rangle)\begin{pmatrix} 1/\sqrt{2} & * \\ 1/\sqrt{2} & * \end{pmatrix}\end{aligned}}$$

$$|20\rangle = \frac{1}{\hbar^2\sqrt{4\cdot3\cdot2\cdot1}} J_-^2 |22\rangle \qquad \left(|j\,m-2\rangle = \frac{1}{\hbar^2\sqrt{(j+m)(j+m-1)(j-m+1)(j-m+2)}} J_-^2 |jm\rangle\right)$$

$$= \frac{1}{\hbar^2\sqrt{24}}(J_{1-} + J_{2-})^2|22\rangle = \frac{1}{\hbar^2\sqrt{24}}(J_{1-}^2 + 2J_{1-}J_{2-} + J_{2-}^2)(|11\rangle \otimes |11\rangle)$$

$|2\,-1\rangle, |2\,-2\rangle$

$J_z = 0$ の状態 $(|20\rangle^\vee, |10\rangle^\vee, |00\rangle)$

$$|10\rangle = \frac{1}{\hbar\sqrt{2}} J_-|11\rangle = \frac{1}{\hbar\sqrt{2}}(J_{1-} + J_{2-})\frac{1}{\sqrt{2}}(|11\rangle \otimes |10\rangle + |10\rangle \otimes |11\rangle)$$

$$|20\rangle = (|1\,-1\rangle \otimes |11\rangle, |10\rangle \otimes |10\rangle, |11\rangle \otimes |1\,-1\rangle)\underbrace{\begin{pmatrix} * \\ * \\ * \end{pmatrix}}_{=\psi_{c_1}^\vee}$$

$$= (|1\,-111\rangle, |1010\rangle, |111\,-1\rangle)\psi_{c_1}^\vee$$

$$|10\rangle = (\,''\,)\psi_{c_2}^\vee$$

$$(|20\rangle, |10\rangle, |00\rangle) = (|1\,-111\rangle, |1010\rangle, |111\,-1\rangle) M$$

$$|1\,-1\rangle = \frac{1}{*} J_-^2|11\rangle \qquad M = (\psi_{c_1}^\vee, \psi_{c_2}^\vee, \psi^?) = (\psi_c, \psi)$$

"前の方法を適用"

☆既約テンソル演算

球面調和関数 Y_{lm}

$$L^2 Y_{lm} = \hbar^2 l(l+1) Y_{lm}$$

$$L_z Y_{lm} = \hbar m Y_{lm} \qquad L_\pm Y_{lm} = \hbar\sqrt{(l\mp m)(l\pm m+1)}\, Y_{l,m\pm1}$$

f：ある関数

$$L_z(Y_{lm}f) = (L_z Y_{lm})f + Y_{lm}L_z f = \hbar m Y_{lm} f + \underset{\sim\sim\sim\sim}{Y_{lm}L_z f}$$

$$L_z(Y_{lm}f) - Y_{lm}(L_z f) = [L_z, Y_{lm}]f = \hbar m Y_{lm} f$$

$$[L_z, Y_{lm}] = \hbar m Y_{lm}$$

$$[L_\pm, Y_{lm}] = \hbar\sqrt{(l\mp m)(l\pm m+1)}\, Y_{l,m\pm1}$$

これを参考に

$Y_{lm} \longrightarrow T^k_q$　　$[J_z, T^k_q] = \hbar m T^k_q$

$k \longleftrightarrow l$　　$[J_\pm, T^k_q] = \hbar\sqrt{(k\mp q)(k\pm q+1)}\, T^k_{q\pm1}$

$q \longleftrightarrow m$　　となる演算子 T^k_q：k次の既約テンソル演算子という。

$q: -k, -k+1, \cdots, k$

$2k+1$コ

$k=1$のとき Y_{lm}

$[J_z, T^1_q] = \hbar q T^1_q$　　$[J_\pm, T^1_q] = \hbar\sqrt{(1\mp q)(1\pm q+1)}\, T^1_{q\pm1}$

$[J_+, T^1_1] = 0$　　$[J_-, T^1_1] = \hbar\sqrt{2}\, T^1_0$

$[J_+, T^1_0] = \hbar\sqrt{2}\, T^1_{\pm1}$　　$[J_-, T^1_0] = \hbar\sqrt{2}\, T^1_{-1}$

$[J_+, T^1_{-1}] = \hbar\sqrt{2}\, T^1_0$　　$[J_-, T^1_{-1}] = 0$

$[J_+, J_+] = 0$　　$[J_+, J_z] = -\hbar J_+$

$[J_+, J_-] = 2\hbar J_-$

$$\left.\begin{aligned} T^1_1 &= -\frac{1}{\sqrt{2}}J_+ \\ T^1_0 &= J_z \\ T^1_{-1} &= \frac{1}{\sqrt{2}}J_- \end{aligned}\right)$$

とすると

T^1_q：1階既約テンソル

ベクトル演算子

$[J_i, V_j] = i\hbar\varepsilon_{ijk}V_k$

$\vec{V} = \vec{r}, \vec{p}, \vec{L}, \vec{S}$

$$T^1_1 = -\frac{1}{\sqrt{2}}V_+$$

$$T^1_0 = V_z$$

$$T^1_{-1} = \frac{1}{\sqrt{2}}V_-$$