201510900
和田 [illegible]

$$\mathbb{J} = \mathbb{J}_1 + \mathbb{J}_2 \qquad J^2 |j, m\rangle = \hbar^2 j(j+1) |j, m\rangle$$

$$[J^\alpha, J^\beta] = i\hbar\, \varepsilon^{\alpha\beta\gamma} J^\gamma \qquad J_z |j, m\rangle = \hbar m |j, m\rangle$$

$\mathbb{J}_1 + \mathbb{J}_2 = J_1 \otimes 1 + 1 \otimes J_2$ について．$|j, m\rangle$ を考える．

$$|j, m\rangle = \sum_{m_1 m_2} |\underbrace{j_1}, m_1\rangle \otimes |\underbrace{j_2}_{\&}, m_2\rangle\, C^{jm}_{m_1 m_2\, j_1 j_2} \qquad \textit{linear combination}$$

jは固定しておくとして….

$$= \sum_{m_1 m_2} |j, m\rangle \underbrace{\langle j, m | j_1, m_1, j_2, m_2\rangle} = C^{jm}_{m_1 m_2\, j_1 j_2} \qquad \textit{CG coef.}$$

- $\frac{1}{2} \otimes \frac{1}{2} = 0 \oplus 1$
  - Triplet $2j+1 = 2\cdot 1 + 1 = 3$
  - Singlet (角運動量 0) $2j+1 = 2\cdot 0 + 1 = 1$.

- $\frac{1}{2} \otimes 1 = \frac{3}{2} \oplus \frac{1}{2}$　　*state counting* (基底の数を数える)
  i.e. *dimension* を数える．

$$\begin{array}{lcll} \frac{1}{2} & \rightsquigarrow & \pm\frac{1}{2} & \sim 2コ \\ \otimes & & & \quad\times \\ 1 & \rightsquigarrow & 0, \pm 1 & \sim 3コ \\ & & & \quad\shortparallel \\ & & & \quad 6コ \end{array} \qquad \begin{array}{lcll} \frac{3}{2} & \rightsquigarrow & \pm\frac{3}{2}, \pm\frac{1}{2} & \sim 4コ \\ \oplus & & & \quad + \\ \frac{1}{2} & \rightsquigarrow & \pm\frac{1}{2} & \sim 2コ \\ & & & \quad\shortparallel \\ & & & \quad 6コ \end{array}$$

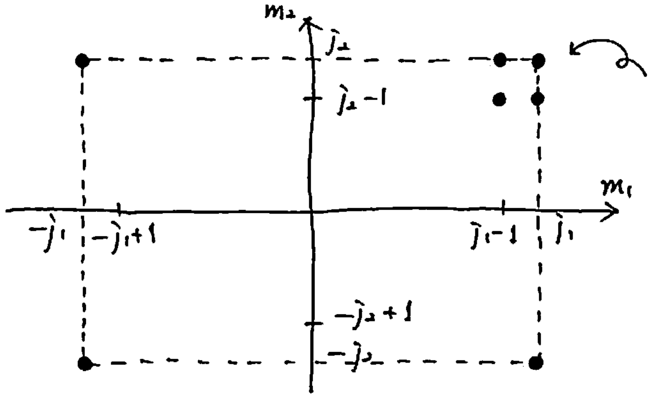
- $1 \otimes 1 = 2 \oplus 1 \oplus 0$

$$\begin{array}{lcll} 1 & \rightsquigarrow & 0, \pm 1 & \sim 3コ \\ \otimes & & & \quad\times \\ 1 & \sim\sim\sim & & \sim 3コ \\ & & & \quad\shortparallel \\ & & & \quad 9コ \end{array} \qquad \begin{array}{lcll} 2 & \rightsquigarrow & \pm 2, \pm 1, 0 & \sim 5コ \\ & & & \quad + \\ 1 & \rightsquigarrow & \pm 1, 0 & \sim 3コ \\ & & & \quad + \\ 0 & \rightsquigarrow & 0 & \sim 1コ \\ & & & \quad\shortparallel \\ & & & \quad 9コ \end{array}$$

- *generically*

$$j_1 \otimes j_2 = \underbrace{j_{max}}_{= j_1 + j_2} \oplus j_{max} - 1 \oplus \cdots \oplus j_{min} + 1 \oplus \underbrace{j_{min}}_{= |j_1 - j_2|}$$

どうしてこうなるのか…



i.e. 状態数は格子点数と一致する.

$J^+ = J_1^+ + J_2^+$

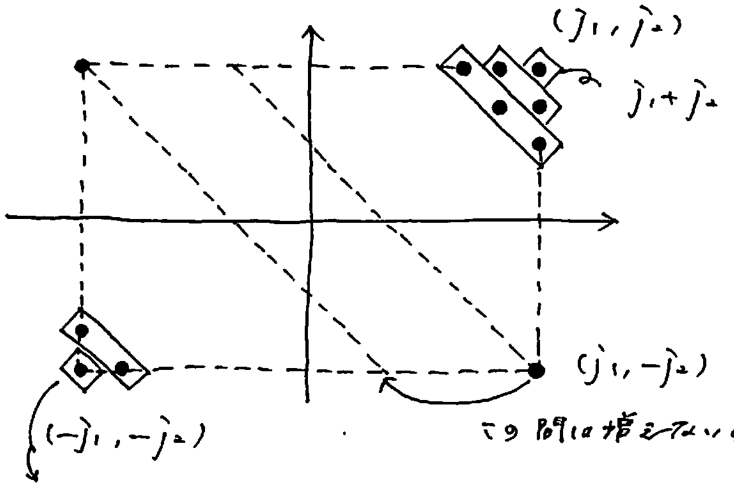
first 右上の点, $|j_1 j_1 j_2 j_2\rangle$ $J^+|j_1 j_1 j_2 j_2\rangle = 0$ (どっちも上げられない)

second 横長を仮定 $j_1 \geq j_2$

$$|j, j-1\rangle \propto J^-|j j\rangle = (J_1^- + J_2^-)|j_1, j_1, j_2, j_2\rangle$$

$$J^-|j, j\rangle \propto |j, j-1\rangle = \Sigma * (J_1^- + J_2^-)|j_1, j_1, j_2, j_2\rangle$$
$$= \Sigma * |j_1, j_1-1, j_2, j_2\rangle + \Sigma * |j_1, j_1, j_2, j_2-1\rangle$$

$$|j, m-1\rangle \propto J^-|j, m\rangle = (J_1^- + J_2^-)\, \Sigma * |j_1, m_1, j_2, m_2\rangle$$
$$= \Sigma * |j_1, m_1-1, j_2, m_2\rangle + \Sigma * |j_1, m_1, j_2, m_2-1\rangle$$



$j_1 + j_2 - 2j = -(j_1 + j_2)$

i.e. $j_{min} = (j_1 + j_2) - 2j_2 = j_1 - j_2$

➡ $j_1 \otimes j_2 = (j_1 + j_2) \oplus (j_1 + j_2 - 1) \oplus \cdots \oplus (j_1 - j_2 + 1) \oplus (j_1 - j_2)$

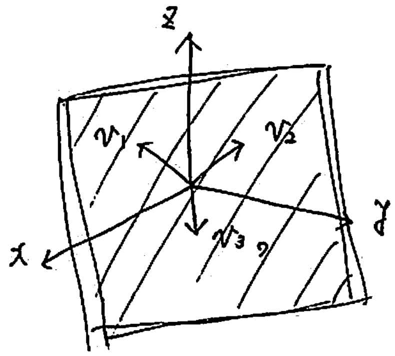
(右辺) state counting $\sum_{j=j_{min}}^{j_{max}} (2j+1) = \left(\sum_{j=1}^{j_{max}} - \sum_{j=1}^{j_{min}-1}\right)(2j+1)$

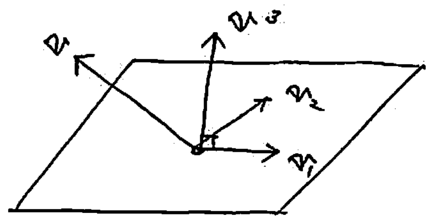
$$= \{(j_{max})^2 + 2(j_{max})\} - \{(j_{min}-1)^2 + 2(j_{min}-1)\}$$
$$= (j_{max} + j_{min} - 1)(j_{max} - j_{min} + 1) + 2(j_{max} - j_{min} + 1$$
$$= (j_{max} + j_{min} + 1)(j_{max} - j_{min} + 1)$$
$$= \{(j_1 + j_2) + (j_1 - j_2) + 1\}\{(j_1 + j_2) - (j_1 - j_2) + 1$$
$$= (2j_1 + 1)(2j_2 + 1)$$ (左辺)に一致

$1 \otimes 1 = 2 \oplus 1 \oplus 0$

$|v_1\rangle$, $|v_2\rangle$ を知っているとき.

$$\left.\begin{aligned} \langle v_1|v_3\rangle &= 0 \\ \langle v_2|v_3\rangle &= 0 \end{aligned}\right\} \text{となる } |v_3\rangle \text{ を どう作るか?}$$





規格直交化.

$\langle v_i|v_j\rangle = \delta_{ij}$

$P = |v_1\rangle\langle v_1| + |v_2\rangle\langle v_2|$

$P^2 = P$: projection

Pv : $v_1 v_2$ 平面上.

$v - Pv \perp v_1, v_2$

$$|v\rangle - |v_1\rangle\langle v_1|v\rangle - |v_2\rangle\langle v_2|v\rangle = |*\rangle$$

$$\begin{aligned} \langle v_1|*\rangle &= \langle v_1|v\rangle - \underbrace{\langle v_1|v_1\rangle}_{=1}\langle v_1|v\rangle - \underbrace{\langle v_1|v_2\rangle}_{=0}\langle v_2|v\rangle \\ &= \langle v_1|v\rangle - \langle v_1|v\rangle - 0\cdot\langle v_2|v\rangle \\ &= 0 \end{aligned}$$

同様に. $\langle v_2|*\rangle = 0$

generically $P = \sum_{i=1}^{q} |v_i\rangle\langle v_i|$, $P^2 = P$

$\langle v|(1-P)v\rangle = 0$