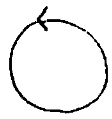
授業の記録【事実・知識】

$\vec{B}$ finite



$\Delta E \sim -\underbrace{\frac{e}{2m}}\vec{L}\cdot\vec{B}$

circular motion ~ magnetization
磁化

$\vec{L} \to \vec{L} + g\vec{S} \quad (g \sim 2)$

orbital magnetization 軌道磁化

$\vec{S}: [S_i, S_j] = i\hbar\varepsilon_{ijk}S_k, \quad \vec{S}^\dagger = \vec{S}$

$S^2 = \hbar^2 S(S+1), \quad S = \frac{1}{2}$

☆ spin and Pauri matrices
パウリ行列

$\vec{S}^2|S, M\rangle = \hbar^2 S(S+1)|S, M\rangle \quad S = \frac{1}{2}$

$S_z|S, M\rangle = \hbar M|S, M\rangle, \quad M = \underset{-\frac{1}{2}}{-S}, \underset{S}{-S+1}$

$\langle j, -m+1|J_+|jm\rangle = \hbar\sqrt{(j-m)(j+m+1)} \quad (*1)$

$\langle j, m-1|J_-|jm\rangle = \hbar\sqrt{(j+m)(j-m+1)} \quad (*2)$

$M = +\frac{1}{2}$. $\uparrow$ とかく
$M = -\frac{1}{2}$. $\downarrow$ とかく

$(*1) \to \langle\uparrow|S_+|\downarrow\rangle = \hbar\sqrt{(\frac{1}{2}-(-\frac{1}{2}))(\frac{1}{2}-\frac{1}{2}+1)} = \hbar, \quad S_+|\uparrow\rangle = 0$
$(j = \frac{1}{2}, m = -\frac{1}{2})$

$(*2) \to \langle\downarrow|S_-|\uparrow\rangle = \hbar\sqrt{(\frac{1}{2}+\frac{1}{2})(\frac{1}{2}-\frac{1}{2}+1)} = \hbar, \quad S_-|\downarrow\rangle = 0$
$(j = \frac{1}{2}, m = \frac{1}{2})$

$$\begin{pmatrix}\langle\uparrow| \\ \langle\downarrow|\end{pmatrix} S_z (|\uparrow\rangle, |\downarrow\rangle) = \begin{pmatrix}\langle\uparrow|S_z|\uparrow\rangle & \langle\uparrow|S_z|\downarrow\rangle \\ \langle\downarrow|S_z|\uparrow\rangle & \langle\downarrow|S_z|\downarrow\rangle\end{pmatrix} = \frac{\hbar}{2}\begin{pmatrix}1 & 0 \\ 0 & -1\end{pmatrix} \equiv \frac{\hbar}{2}\sigma_z$$

$\sigma_z = \begin{pmatrix}1 & 0 \\ 0 & -1\end{pmatrix}$ one of the Pauli matrices

メモ【考え・気持ち・自主学習】

$S_z \sim \frac{\hbar}{2}\sigma_z$

$S_z = \frac{\hbar}{2}\sigma_z \quad \vec{S} = \begin{pmatrix}S_x \\ S_y \\ S_z\end{pmatrix} = \frac{\hbar}{2}\begin{pmatrix}\sigma_x \\ \sigma_y \\ \sigma_z\end{pmatrix}$

$$\left.\begin{aligned} &S_\pm = S_x \pm iS_y \\ &S_x = \frac{1}{2}(S_+ + S_-), \quad S_y = \frac{1}{2i}(S_+ - S_-)\end{aligned}\right\} (**)$$

$$\begin{pmatrix}\langle\uparrow| \\ \langle\downarrow|\end{pmatrix} S_\pm (|\uparrow\rangle, |\downarrow\rangle) = \begin{pmatrix}\langle\uparrow|S_\pm|\uparrow\rangle & \langle\uparrow|S_\pm|\downarrow\rangle \\ \langle\downarrow|S_\pm|\uparrow\rangle & \langle\downarrow|S_\pm|\downarrow\rangle\end{pmatrix}$$

$\langle\uparrow|\downarrow\rangle = 0$

$$= \underset{S_+}{\hbar\begin{pmatrix}0 & 1 \\ 0 & 0\end{pmatrix}}, \quad \underset{S_-}{\hbar\begin{pmatrix}0 & 0 \\ 1 & 0\end{pmatrix}}$$

$**$

$S_x = \frac{1}{2}(S_+ + S_-) = \frac{\hbar}{2}\begin{pmatrix}0 & 1 \\ 1 & 0\end{pmatrix} = \frac{\hbar}{2}\sigma_x$

$S_y = \frac{1}{2i}(S_+ - S_-) = \frac{\hbar}{2i}\begin{pmatrix}0 & 1 \\ -1 & 0\end{pmatrix} = \frac{\hbar}{2}\sigma_y$

$\sigma_x = \begin{pmatrix}0 & 1 \\ 1 & 0\end{pmatrix} \quad \sigma_y = \frac{1}{i}\begin{pmatrix}0 & 1 \\ -1 & 0\end{pmatrix} = \begin{pmatrix}0 & -i \\ i & 0\end{pmatrix}$

Pauli matrices
パウリ行列

$\vec{\sigma} = \begin{pmatrix}\sigma_x \\ \sigma_y \\ \sigma_z\end{pmatrix} \quad \sigma_x = \begin{pmatrix}0 & 1 \\ 1 & 0\end{pmatrix} \quad \sigma_y = \begin{pmatrix}0 & -i \\ i & 0\end{pmatrix} \quad \sigma_z = \begin{pmatrix}1 & 0 \\ 0 & -1\end{pmatrix}$

Vector of matrices
行列のベクトル

$\sigma_x^2 = \begin{pmatrix}0 & 1 \\ 1 & 0\end{pmatrix}\begin{pmatrix}0 & 1 \\ 1 & 0\end{pmatrix} = \begin{pmatrix}1 & 0 \\ 0 & 1\end{pmatrix} = E_2$

$\sigma_y^2 = \begin{pmatrix}0 & -i \\ i & 0\end{pmatrix}\begin{pmatrix}0 & -i \\ i & 0\end{pmatrix} = \begin{pmatrix}-ii & 0 \\ 0 & i(-i)\end{pmatrix} = E_2$

$\sigma_z^2 = \begin{pmatrix}1 & 0 \\ 0 & -1\end{pmatrix}\begin{pmatrix}1 & 0 \\ 0 & -1\end{pmatrix} = \begin{pmatrix}1 & 0 \\ 0 & 1\end{pmatrix} = E_2$

$\sigma_x\sigma_y = \begin{pmatrix}0 & 1 \\ 1 & 0\end{pmatrix}\begin{pmatrix}0 & -i \\ i & 0\end{pmatrix} = \begin{pmatrix}i & 0 \\ 0 & -i\end{pmatrix} = i\sigma_z$

$\sigma_y\sigma_z = \begin{pmatrix}0 & -i \\ i & 0\end{pmatrix}\begin{pmatrix}1 & 0 \\ 0 & -1\end{pmatrix} = \begin{pmatrix}0 & i \\ i & 0\end{pmatrix} = i\sigma_x$

$\sigma_z\sigma_x = \begin{pmatrix}1 & 0 \\ 0 & -1\end{pmatrix}\begin{pmatrix}0 & 1 \\ 1 & 0\end{pmatrix} = \begin{pmatrix}0 & 1 \\ -1 & 0\end{pmatrix} = i\sigma_y$

$\sigma_x\sigma_y = i\sigma_z$ c.p. 循環置換

作成年月日: 年 月 日

$\sigma_x^\dagger = \sigma_x$

$\sigma_y^\dagger = \begin{pmatrix}0 & i\\ -i & 0\end{pmatrix}^* = \begin{pmatrix}0 & -i\\ i & 0\end{pmatrix} = \sigma_y$

$\sigma_z^\dagger = \sigma_z \qquad \therefore\ \vec\sigma^\dagger = \vec\sigma$ hermite

$\sigma_i\sigma_j = -\sigma_j\sigma_i \quad (i\neq j)$

$\sigma_i^2 = E_2$

$(i\neq j)$, $\sigma_i\sigma_j = i\varepsilon_{ijk}\sigma_k$

$(\sigma_x\sigma_y = i\varepsilon_{12k}\sigma_k = i\sigma_z)$

$\mathrm{Tr}\,\sigma_x = \mathrm{Tr}\begin{pmatrix}0&1\\1&0\end{pmatrix} = 0$

⋮

$\mathrm{Tr}\,\vec\sigma = \vec 0$

$\{\sigma_i,\sigma_j\} = \sigma_i\sigma_j+\sigma_j\sigma_i = \begin{cases} E_2+E_2 = 2E_2 & i=j\\ 0 & i\neq j\end{cases}$

$= 2\delta_{ij}E_2$

for general 2×2 matrix $A = \begin{pmatrix}a&b\\c&d\end{pmatrix}$

$E_2 \equiv \sigma_0 \quad \sigma_x = \sigma_1,\ \sigma_y=\sigma_2,\ \sigma_z=\sigma_3$

$A = \underset{\sim}{A_0}\sigma_0 + \underset{\sim}{A_1}\sigma_1 + \underset{\sim}{A_2}\sigma_2 + \underset{\sim}{A_3}\sigma_3$

$(a, b, c, d) \to (A_0, A_1, A_2, A_3)$

$A = \sum_{i=0}^{3} A_i\sigma_i$

$\mathrm{Tr}\,A\underbrace{\sigma_0}_{E_2} = \mathrm{Tr}A = \sum_{i=0}^{3} \mathrm{Tr}(A_0\sigma_0^2 + A_1\sigma_1\sigma_0 + A_2\sigma_2\sigma_0 + A_3\sigma_3\sigma_0)$

$= A_0\,\mathrm{Tr}\,\sigma_0 = 2A_0$

$\mathrm{Tr}\,A\sigma_1 = \sum_{i=0}^{3} \mathrm{Tr}(A_0\sigma_0\sigma_1 + A_1\sigma_1^2 + A_2\sigma_2\sigma_1 + A_3\sigma_3\sigma_1)$

$= A_1\,\mathrm{Tr}\,\sigma_1^2 = 2A_1$

$A_0 = \frac{1}{2}\mathrm{Tr}A \qquad A_i = \frac{1}{2}\mathrm{Tr}A\sigma_i \quad (i=0,1,2,3)$

2×2 matrix

$A = A_0\sigma_0 + \vec A\cdot\vec\sigma$

A, B : 2×2 matrices

$A = (\vec A\cdot\vec\sigma),\quad B = (\vec B\cdot\vec\sigma)$

$AB = (\vec A\cdot\vec\sigma)(\vec B\cdot\vec\sigma) = \vec A\cdot\vec B\,\sigma_0 + i\vec\sigma\cdot(\vec A\times\vec B)$

product of the matrices 行列の積

$(\vec A\cdot\vec\sigma)(\vec B\cdot\vec\sigma) = A_i\sigma_i B_j\sigma_j$

$= A_iB_j\sigma_i\sigma_j$

$= \sum_{i=j} A_iB_i\sigma_i^2 + \sum_{i\neq j}A_iB_j\sigma_i\sigma_j$

$= \sum_{i=1}^{3} A_iB_i\sigma_0 + \sum_{i\neq j}A_iB_j\underbrace{\sigma_i\sigma_j}_{i\varepsilon_{ijk}\sigma_k}$

$= \vec A\cdot\vec B\,\sigma_0 + \sum_{ijk}\underbrace{A_iB_j\, i\varepsilon_{ijk}\sigma_k}_{(\vec A\times\vec B)_k}$

$= (\vec A\cdot\vec B)\sigma_0 + i\vec\sigma\cdot(\vec A\times\vec B)$

ex) $H = -\mu\vec S\cdot\vec B \qquad \vec B /\!/ \hat z$

$H = -\mu \underset{\frac{\hbar\sigma_z}{2}}{\underset{\shortparallel}{S_z}} B = -\frac{\hbar\mu B}{2}\begin{pmatrix}1&0\\0&-1\end{pmatrix} = \begin{pmatrix}-\frac{\hbar}{2}\mu B & 0\\ 0 & \frac{\hbar}{2}\mu B\end{pmatrix}$

Another way

$\mathrm{Tr}H = -\mu\,\mathrm{Tr}\,\vec S\cdot\vec B$

$= -\mu\hbar\vec B\cdot\mathrm{Tr}\,\vec\sigma = 0$

eigen values E_1, E_2

$\mathrm{Tr}H = E_1+E_2 = 0 \qquad E_2 = -E_1$

$H^2 = \left(\frac{\mu\hbar}{2}\vec B\cdot\vec\sigma\right)^2 = \left(\frac{\mu\hbar}{2}\right)^2(\vec B\cdot\vec\sigma)(\vec B\cdot\vec\sigma)$

$= \left(\frac{\hbar\mu}{2}\right)^2(\vec B\cdot\vec B\,\sigma_0 + i\sigma\cdot\underset{0}{\underset{\shortparallel}{\vec B\times\vec B}})$

$= \left(\frac{\hbar}{2}\mu\right)^2|\vec B|^2\sigma_0$

$E_1 = \frac{\mu\hbar}{2}|\vec B|$

* time reversal symmetry – anti symmetry operation

$\Theta = i\sigma_2 K \qquad K$ complex conjuate

$(i\sigma_2)^\dagger = -i\sigma_2^\dagger = -i\sigma_2$

$(i\sigma_2)^\dagger i\sigma_2 = \sigma_2^2 = E_2 \qquad i\sigma_2$: unitary

Θ : time reversal operator.

$\Theta\sigma_1\Theta^{-1} = i\sigma_2(\sigma_1)^*(-i\sigma_2) = \sigma_2\overset{\sigma_1}{\overset{\shortparallel}{\sigma_1^*}}\sigma_2$

$= -\sigma_1$

⋮

$\Theta\vec\sigma\Theta^{-1} = -\vec\sigma \qquad \Theta\vec S\Theta^{-1} = -\vec S$

$\Theta\vec L\Theta^{-1} = -\vec L \quad (\Theta\vec r\Theta^{-1} = \vec r,\ \Theta\vec p\Theta^{-1} = -\vec p$ より$)$

$\Theta\vec L\cdot\vec S\Theta^{-1} = (-\vec L)\cdot(-\vec S) = \vec L\cdot\vec S$

spin orbit interaction スピン軌道相互作用