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前回の補足

$\langle S_1 M_1, S_2 M_2 | S, M\rangle$

$|S, M\rangle = \sum_{M_1 M_2} \underline{|S_1, M_1\rangle |S_2 M_2\rangle} C_{M_1 M_2}$，とかく
⊗ とかいたり
$|S_1 M_1, S_2 M_2\rangle$ といたり

$|S, M\rangle = |S_1 M_1\ S_2 M_2\rangle\langle S_1 M_1\ S_2 M_2 | S, M\rangle$　和をとる

$(\Sigma)\ |S_1 M_1, S_2 M_2\rangle\langle S_1 M_1, S_2 M_2| = 1$

完全性　completeness

$\vec{S} = \vec{S}_1 + \vec{S}_2$　sum (和)
$= \vec{S}_1 \otimes 1 + 1 \otimes \vec{S}_2$
$|S_1 M_1\rangle \otimes |S_2, M_2\rangle$
状態はかけ算に

o Another example

$$H = \frac{\vec{P}^2}{2m} = \frac{1}{2m}(P_x^2 + P_y^2 + P_z^2) \quad H_\alpha = \frac{P_\alpha^2}{2m}$$
$$= H_x + H_y + H_z$$

$$\psi(x, y, z) = \phi_{kx}(x)\phi_{ky}(y)\phi_{kz}(z)$$
変数分離

$$H\psi = (H_x + H_y + H_z)\phi_{kx}\phi_{ky}\phi_{kz}$$
$$= (H_x\phi_{kx})\phi_{ky}\phi_{kz} + \phi_{kx}(H_y\phi_{ky})\phi_{kz} + \phi_{kx}\phi_{ky}(H_z\phi_{kz})$$

$\varepsilon_{ki}\phi_{ki}(i)$

$\phi_{ki} = e^{k_i i}$　$i = x, y, z$ とおく

$$H_i\phi_{ki}(i) = \frac{P_i^2}{2m}\phi_{ki}(i) = \frac{\hbar^2}{2m}k_i^2\phi_{ki}(i)$$

$$= (\varepsilon_{kx} + \varepsilon_{ky} + \varepsilon_{kz})\phi_{kx}\phi_{ky}\phi_{kz}$$

$$\psi = e^{ik_x x}e^{ik_y y}e^{ik_z z}$$
$$= e^{i\vec{k}\cdot\vec{r}}$$

一般にして (generically)

$\vec{J} = \vec{J}_1 + \vec{J}_2$，$\vec{J}^2|j, m\rangle = \hbar^2 j(j+1)|j, m\rangle$
$J^z|j, m\rangle = \hbar m|j, m\rangle$

$[J_i^\alpha, J_j^\beta] = \delta_{ij}\, i\hbar\varepsilon_{\alpha\beta\gamma}J^\gamma$

$[J^\alpha, J^\beta] = i\hbar\varepsilon_{\alpha\beta\gamma}J^\gamma$

$j_1 = \frac{1}{2}$，$j_2 = \frac{1}{2}$
$m_1 = \pm\frac{1}{2}$，$m_2 = \pm\frac{1}{2}$　　$\frac{1}{2} \otimes \frac{1}{2} = (\otimes)$
$2 \times 2 = 4$

合成 $\left(\begin{matrix} j = 1, & m = 1, 0, 1 & 3つ \\ j = 0 & , m = 0 & 1つ \end{matrix}\right)$ 4つ

☆ $\vec{J} = \vec{J}_1 + \vec{J}_2$　$j_1 = 1$，$j_2 = 1$ のとき $(1 \otimes 1 = ?)$

$$J_+|j, m\rangle = \hbar\sqrt{(j+m+1)(j-m)}\,|j\, m+1\rangle$$

$$J_-|j\, m\rangle = \hbar\sqrt{(j-m+1)(j+m)}\,|j\, m-1\rangle$$

$|j_1 m_1\rangle, |j_2 m_2\rangle = |j_1, m_1, j_2, m_2\rangle$
$|m_1\rangle|m_2\rangle$ とかく　$|m_1 m_2\rangle$　$j_1 = j_2$

$1, 0, -1$，$1, 0, -1$
$3 \otimes 3 = \underline{9つ}$

$$|j\, m\rangle = \sum_{m_1 + m_2 = m} |j_1 m_1\, j_2 m_2\rangle\underbrace{\langle j_1 m_1\, j_2 m_2 | j m\rangle}_{CG係数}$$

$J^z|j, m\rangle = \hbar m|j, m\rangle$

$(J_1^z + J_2^z)|j_1 m_1\, j_2 m_2\rangle = \hbar(m_1 + m_2)|j_1 m_1\, j_2 m_2\rangle$

$\langle j\, m | j_1 m_1\, j_2 m_2\rangle = 0$　$m \neq m_1 + m_2$

$|1, 1\rangle = |j_1 = 1\ m_1 = 1\ j_2 = 1\ m_2 = 1\rangle$
$= |1, 1\ 1, 1\rangle$

$J^+|11\rangle = (J_1^+ + J_2^+)|1\ 1\rangle$
$= (J_1^+|1\rangle_1)|1\rangle_2 + |1\rangle(J_2^+|1\rangle_2)$
$= 0$

合成

$|j = 2, m = 2\rangle = |1\rangle|1\rangle = |1\ 1\rangle$
$j \geq 2$　もとの基底

新しい基底　　もとの基底
$\underline{|2, 2\rangle = |1, 1, 1, 1\rangle}$

$J^-|2\,2\rangle = \hbar\sqrt{1 \times 4}\ |2, 1\rangle$
$= 2\hbar\ |2, 1\rangle$

$|2, 1\rangle = \frac{1}{2\hbar}J^-|2, 2\rangle$　もとの基底
$= \frac{1}{2\hbar}(J_1^- + J_2^-)|1, 1\rangle|1, 1\rangle$
$= \frac{1}{2\hbar}\left[\hbar\sqrt{2}|1\ 0\rangle|1\ 1\rangle + \hbar\sqrt{2}|1\ 1\rangle|1\ 0\rangle\right]$
$= \frac{1}{\sqrt{2}}\left[|1, 0, 1, 1\rangle + |1\ 1, 1, 0\rangle\right]$

$$\underline{|2, 1\rangle = \frac{1}{\sqrt{2}}\left[|1, 0, 1, 1\rangle + |1\ 1\ 1\ 0\rangle\right]}$$

$|j\, m\rangle = |j_1 m_1\, j_2 m_2\rangle\langle j_1 m_1\, j_2 m_2 | j\, m\rangle$
$j = 2, m = 1$
$\left.\begin{matrix}\langle 1, 0, 1, 1 | 2, 1\rangle \\ \langle 1\ 1, 1, 0 | 2, 1\rangle\end{matrix}\right\} = \frac{1}{\sqrt{2}}$

$$J^-|2,1\rangle = \hbar\sqrt{2\cdot 3}\,|2,0\rangle = \sqrt{6}\,\hbar\,|2,0\rangle$$

$$|2,0\rangle = \frac{1}{\sqrt{6}\hbar}J^-|2,1\rangle$$

$$= \frac{1}{\sqrt{6}\hbar}(J_1^- + J_2^-)\times\frac{1}{\sqrt{2}}(|1,0\rangle|1,1\rangle + |1,1\rangle|1,0\rangle)$$

$$= \frac{1}{2\sqrt{3}\hbar}\Big[(J_1^-|1,0\rangle)|1,1\rangle + |1,0\rangle(J_2^-|1,1\rangle) + (J_1^-|1,1\rangle)|1,0\rangle + |1,1\rangle(J_2^-|1,0\rangle)\Big]$$

$$= \frac{1}{2\sqrt{3}\hbar}\Big[\sqrt{2}\hbar|1,-1,1,1\rangle + \sqrt{2}\hbar|1,0,1,0\rangle + \sqrt{2}\hbar|1,0,1,0\rangle + \sqrt{2}\hbar|1,1,1,-1\rangle\Big]$$

$$= \frac{1}{\sqrt{6}}\Big[|1,-1,1,1\rangle + 2|1,0,1,0\rangle + |1,1,1,-1\rangle\Big]$$

$$\underline{|2,0\rangle = \frac{1}{\sqrt{6}}\Big[|1,-1,1,1\rangle + 2|1,0,1,0\rangle + |1,1,1,-1\rangle\Big]}$$

$$\langle 1,-1,1,1|2,0\rangle = \langle 1,1,1,-1|2,0\rangle = \frac{1}{\sqrt{6}}$$
$$\langle 1,0,1,0|2,0\rangle = \frac{2}{\sqrt{6}}$$

$|2,1\rangle = \frac{1}{\sqrt{2}}\Big[|1,1,1,0\rangle + |1,0,1,1\rangle\Big]$
であったから

$\frac{1}{\sqrt{2}}\Big[|1,1,1,0\rangle - |1,0,1,1\rangle\Big]$

もできるはず。 $m = m_1 + m_2 = 1 + 0 = 1$ これを
$|?,1\rangle$ $j>1$ とおき

$$J^+|?,1\rangle = (J_1^+ + J_2^+)|?,1\rangle$$
$$= \frac{1}{\sqrt{2}}\Big[|1,1\rangle(J_2^+|1,0\rangle) - (J_1^+|1,0\rangle)|1,1\rangle\Big]$$
$$= \frac{1}{\sqrt{2}}\Big[\sqrt{2}\hbar|1,1,1,1\rangle - \sqrt{2}\hbar|1,1,1,1\rangle\Big]$$
$$= 0$$

$|?,1\rangle$ は上がらない

$\Rightarrow j = 1$

$$\underline{|1,1\rangle = \frac{1}{\sqrt{2}}\Big[|1,1,1,0\rangle - |1,0,1,1\rangle\Big]}$$

$$\langle 1,1,1,0|1,1\rangle = -\langle 1,0,1,1|1,1\rangle = \frac{1}{\sqrt{2}}$$

$$J^-|2,0\rangle = \hbar\sqrt{(2+1)\cdot 2}\,|2,-1\rangle = \sqrt{6}\,\hbar\,|2,-1\rangle$$

$$|2,-1\rangle = \frac{1}{\sqrt{6}\hbar}J^-|2,0\rangle$$

$$= \frac{1}{6\hbar}(J_1^- + J_2^-)\Big[|1,-1,1,1\rangle + 2|1,0,1,0\rangle + |1,1,1,-1\rangle\Big]$$

$$= \frac{1}{6\hbar}\Big[|1,-1\rangle(J_2^-|1,1\rangle) + 2\{(J_1^-|1,0\rangle)|1,0\rangle + 2|1,0\rangle(J_2^-|1,0\rangle)\} + (J_1^-|1,1\rangle)|1,-1\rangle\Big]$$

$$= \frac{1}{6\hbar}\Big[\sqrt{2}\hbar|1,-1,1,0\rangle + 2\sqrt{2}\hbar|1,-1,1,0\rangle + 2\sqrt{2}\hbar|1,0,1,-1\rangle + \sqrt{2}\hbar|1,0,1,-1\rangle\Big]$$

$$= \frac{1}{\sqrt{2}}\Big[|1,-1,1,0\rangle + |1,0,1,-1\rangle\Big]$$

$$\underline{|2,-1\rangle = \frac{1}{\sqrt{2}}\Big[|1,-1,1,0\rangle + |1,0,1,-1\rangle\Big]}$$

$$\langle 1,-1,1,0|2,-1\rangle = \langle 1,0,1,-1|2,-1\rangle = \frac{1}{\sqrt{2}}$$

$$J^-|1,1\rangle = \sqrt{2}\hbar\,|1,0\rangle$$

$$|1,0\rangle = \frac{1}{\sqrt{2}\hbar}(J_1^- + J_2^-)|1,1\rangle$$

$$= \frac{1}{2\hbar}\Big[(J_1^-|1,1\rangle)|1,0\rangle + |1,1\rangle(J_2^-|1,0\rangle) - (J_1^-|1,0\rangle)|1,1\rangle - |1,0\rangle(J_2^-|1,1\rangle)\Big]$$

$$= \frac{1}{2\hbar}\Big[\sqrt{2}\hbar|1,0,1,0\rangle + \sqrt{2}\hbar|1,1,1,-1\rangle - \sqrt{2}\hbar|1,-1,1,1\rangle - \sqrt{2}\hbar|1,0,1,0\rangle\Big]$$

$$= \frac{1}{\sqrt{2}}\Big[|1,1,1,-1\rangle - |1,-1,1,1\rangle\Big]$$

$$\underline{|1,0\rangle = \frac{1}{\sqrt{2}}\Big[|1,1,1,-1\rangle - |1,-1,1,1\rangle\Big]}$$

$$\langle 1,1,1,-1|1,0\rangle = -\langle 1,-1,1,1\rangle = \frac{1}{\sqrt{2}}$$

$$J^-|2,-1\rangle = \hbar\sqrt{(2+1+1)(2-1)}\,|2,-2\rangle = 2\hbar|2,-2\rangle$$

$$|2,-2\rangle = \frac{1}{2\hbar}J^-|2,-1\rangle$$

$$= \frac{1}{2\sqrt{2}\hbar}(J_1^- + J_2^-)\Big[|1,-1,1,0\rangle + |1,0,1,-1\rangle\Big]$$

$$= \frac{1}{2\sqrt{2}\hbar}\Big[|1,-1\rangle(J_2^-|1,0\rangle) + (J_1^-|1,0\rangle)|1,-1\rangle\Big]$$

$$= \frac{1}{2\sqrt{2}\hbar}\Big[|1,-1\rangle\sqrt{2}\hbar|1,-1\rangle + \sqrt{2}\hbar|1,-1\rangle|1,-1\rangle\Big]$$

$$= |1,-1,1,-1\rangle$$

$$\underline{|2,-2\rangle = |1,-1,1,-1\rangle}$$

$$J^-|2,-2\rangle = \hbar\sqrt{(2+2+1)(2-2)}\,|2,-3\rangle = 0$$

$$J^-|1,0\rangle = \sqrt{2}\hbar\,|1,-1\rangle$$

$$|1,-1\rangle = \frac{1}{\sqrt{2}\hbar}(J_1^- + J_2^-)|1,0\rangle$$

$$= \frac{1}{2\hbar}\Big[(J_1^-|1,1\rangle)|1,-1\rangle + |1,-1\rangle J_2^-|1,1\rangle\Big]$$

$$= \frac{1}{2\hbar}\Big[\sqrt{2}\hbar|1,0,1,-1\rangle - \sqrt{2}\hbar|1,-1,1,0\rangle\Big]$$

$$= \frac{1}{\sqrt{2}}\Big[|1,0,1,-1\rangle - |1,-1,1,0\rangle\Big]$$

$$\underline{|1,-1\rangle = \frac{1}{\sqrt{2}}\Big[|1,0,1,-1\rangle - |1,-1,1,0\rangle\Big]}$$

$|2,0\rangle$, $|1,0\rangle$ に直交するのは

$$|0,0\rangle = \frac{1}{\sqrt{3}}\left[|1,-1,1,1\rangle - |1,0,1,0\rangle + |1,1,1,-1\rangle\right]$$

$$1 \otimes 1 = 2 \oplus 1 \oplus 0$$