量子力学3 5/26 201310873 田村 啓

Career Portfolio Worksheet X-31

授業の記録【事実・知識】

Spherical harmonics, spin について

j: integer

$J^2 = \hbar j(j+1)$

$\vec{L} = \vec{r}\times\vec{p}$, classical correspondent
古典対応

$[J_i, J_j] = \hbar\varepsilon_{ijk}J_k$

Atomic spectra 原子分光

と $\vec{L}$ の関わり

When B finite,
$\uparrow$ magnetic field

quantum mechanics

free particle + potential

$H = \frac{p^2}{2m} + V$, $\frac{\partial H}{\partial p} = \frac{p}{m} = \dot{x}$

$p = mv = m\dot{x}$

$\frac{\partial H}{\partial x} = \partial_x V = -F$

$= -\dot{p}$

classical mechanics

$\left.\begin{aligned}\dot{x} &= \frac{\partial H}{\partial p}\\ \dot{p} &= \frac{\partial H}{\partial x}\end{aligned}\right\} \to$ Newton canonical eq.
正準方程式

$H(x, p)$

→ quantization

$\hat{H}(\hat{x}, \hat{p})$, $[\hat{x}, \hat{p}] = i\hbar$

$i\hbar\frac{\partial}{\partial t}\psi = H(\hat{x}, \hat{p})\psi$

$H = \frac{\vec{p}^2}{2m}$ free particle

$\downarrow$ Lorentz force

$H = \frac{1}{2m}(\vec{p} - e\vec{A})^2 + e\phi$

メモ【考え・気持ち・自主学習】

$\vec{A}$: vector potential
ϕ: scalar 〃

$\vec{B} = \mathrm{rot}\vec{A}$, $\vec{E} = -e\nabla\phi - \frac{\partial A}{\partial t}$

$\vec{B}, \vec{E}$: $\vec{A}, \phi$ で表される

$\to \frac{\partial H}{\partial r_i} = -\dot{p}_i$

$\frac{\partial H}{\partial p_i} = \dot{r}_i$

$\to m\ddot{\vec{r}} = \vec{F} = e(\vec{E} + \vec{v}\times\vec{B})$

$\phi = 0$
$H = \frac{1}{2m}(\vec{p} - e\vec{A})^2 + V(r)$
potential

↑ effects of magnetic field

uniform magnetic field 一様磁場

z ↑ ↑↑↑ B $\vec{B} = \begin{pmatrix}0\\0\\B\end{pmatrix} = \mathrm{rot}\vec{A}$

A: not unique (gauge choice)

* symmetric gauge (対称ゲージ)

$\vec{A} = \frac{1}{2}\vec{B}\times\vec{r}$ $A_i = \frac{1}{2}\varepsilon_{ijk}B_j r_k$

確認

$(\mathrm{rot}\vec{A})_i = \varepsilon_{iab}\partial_a A_b = \varepsilon_{iab}\partial_a \frac{1}{2}\varepsilon_{bjk}B_j r_k$

$= \frac{1}{2}\varepsilon_{iab}\varepsilon_{bja}B_j = \frac{1}{2}\varepsilon_{bia}\varepsilon_{bja}B_j$

$= \frac{1}{2}\cdot 2\delta_{ij}B_j$

$= B_i \to \vec{B} = \mathrm{rot}\vec{A}$ //

$H = \frac{1}{2m}(\vec{p} - e\vec{A})^2$

$= \frac{\vec{p}^2}{2m} \underbrace{- \frac{e}{2m}(\vec{p}\cdot\vec{A} + \vec{A}\cdot\vec{p})}_{H_P} + \underbrace{\frac{1}{2m}(e\vec{A})^2}_{H_D}$

$H_D \ll H_P$

$H \sim \frac{\vec{p}^2}{2m} + H_P$

作成年月日: 年 月 日

$(\vec{p}\cdot\vec{A})f = -i\hbar\vec{\nabla}\cdot(\vec{A}f)$

$= -i\hbar(\vec{\nabla}\cdot\vec{A})f - i\hbar\vec{A}\cdot\vec{\nabla}f$

$(\vec{A}\cdot\vec{p})f = -i\hbar\vec{A}\cdot\vec{\nabla}f$

$\vec{A} = \frac{1}{2}\vec{B}\times\vec{r}$

for symmetric gauge

$\vec{\nabla}\cdot\vec{A} = \partial_i A_i = \partial_i \frac{1}{2}(\varepsilon_{iab}B_a r_b)$

$= \frac{1}{2}\varepsilon_{iai}B_a$

$= 0$

よって

$(\vec{p}\cdot\vec{A})f = 0 + (\vec{A}\cdot\vec{p})f$

$H_p = -\frac{e}{2m}\cdot 2\vec{A}\cdot\vec{p}$

$= -\frac{e}{m}\cdot\frac{1}{2}(\vec{B}\times\vec{r})\cdot\vec{p}$

$= -\frac{e}{2m}\underbrace{(\vec{r}\times\vec{p})}_{=\vec{L}}\cdot\vec{B}$

$= -\frac{e}{2m}\vec{B}\cdot\vec{L}$ ← scalar

$H = \frac{\vec{p}^2}{2m} + H_p$ * independent of the coordinate

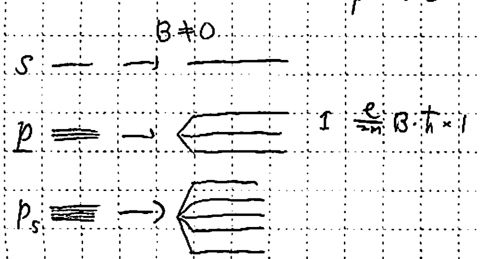
$\vec{B} = \begin{pmatrix} 0 \\ 0 \\ B \end{pmatrix}$

$H_p = -\frac{e}{2m}BL_z$

$\vec{L}^2 = \hbar^2 L(L+1)$

L		
0	s	→ 1
1	p	→ 3
2	d	→ 5

notation for spectroscopy



H_p : perturbation 摂動論

split by magnetic field

Zeeman effect

$H = \frac{\vec{p}^2}{2m} + H_p$

$H_p = -\frac{e}{2m}\vec{B}\cdot\vec{L}$

$\to H_p' = -\frac{e}{2m}\vec{B}\cdot(\vec{L} + g\vec{S})$

$\vec{L} \to \vec{L} + g\underbrace{\vec{S}}_{\text{spin}}$ to be consist with experiment

spin hypothesis

g factor (~2)

$\vec{S} = \begin{pmatrix} S_x \\ S_y \\ S_z \end{pmatrix}$ $[L_i, S_j] = 0$ independent

Dirac eq. (Special relativity)

non relativistic limit

quantum mechanics

$|\frac{v}{c}| \ll 1$

→ spin

$\vec{S}$ $\vec{S}^2 = \hbar S(S+1)$ $S = \frac{1}{2}$ half odd integer

$|\frac{v}{c}| \ll 1$ lowest order → Zeeman effect

next leading term → spin-orbit interaction

$H_p = -\frac{e}{2m}\vec{B}\cdot(\vec{L} + g\vec{S})$

$= -\frac{e}{2m}\vec{B}\cdot\vec{L} - \underbrace{\frac{e}{2m}g\vec{S}}\cdot\vec{B}$

$= -\frac{e}{2m}\vec{B}\cdot\vec{L} - \underbrace{\vec{\mu}}_{\text{スピン}}\cdot\vec{B}$

$\vec{\mu} = \frac{e}{2m}g\vec{S}$

$= g\mu_B\vec{S}/\hbar$ $\mu_B = \frac{e\hbar}{2m}$ Bohr magnetron ボーア磁子

$\vec{S} = \frac{\hbar}{2}\vec{\sigma}$

$\vec{\sigma}$: Pauli matrix

time reversal ↔ anti-unitary

(symmetry ↔ unitary)

ユニタリ + 複素共役 → 反ユニタリ

↓

クラマース縮退