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☆ 角運動量の合成

☆ spin $(S=\frac{1}{2})$ $\vec{S}=\vec{S}_1+\vec{S}_2$

$\vec{S}_1$ $S=\frac{1}{2}$ $\vec{S}_1^2=S(S+1)$
$=\frac{1}{2}(\frac{1}{2}+1)=\frac{3}{4}$

$|j,m\rangle=(S,M)$

$\vec{S}^2|S,M\rangle=\hbar^2S(S+1)|S,M\rangle$

$S_x^2+S_y^2+S_z^2=\frac{1}{2}(S_+S_-+S_-S_+)+S_z^2$

$S_z|SM\rangle=\hbar M|SM\rangle$

$|SM\rangle=|M\rangle$

$S_z|M\rangle=\hbar M|M\rangle$

$S=\frac{1}{2}$

$M=-S,-S+1,\cdots,S=-\frac{1}{2},\frac{1}{2}=\downarrow,\uparrow$

$\;\;\;-\frac{1}{2}\;\;\;\frac{1}{2}\;\;\;\cdots\;\;\;\frac{1}{2}$

S_1, S_2

$S_1^z|\uparrow\rangle_1=\frac{\hbar}{2}|\uparrow\rangle_1,\ S_1^z|\downarrow\rangle_1=-\frac{\hbar}{2}|\downarrow\rangle_1$

$S_2^z|\uparrow\rangle_2=\frac{\hbar}{2}|\uparrow\rangle_2,\ S_2^z|\downarrow\rangle_2=-\frac{\hbar}{2}|\downarrow\rangle_2$

$\vec{S}=\vec{S}_1+\vec{S}_2$

$\vec{S}^2=\vec{S}_1^2+\vec{S}_2^2+2\vec{S}_1\cdot\vec{S}_2$

$=\frac{3}{4}+\frac{3}{4}+2\vec{S}_1\cdot\vec{S}_2=\frac{3}{2}+2\vec{S}_1\cdot\vec{S}_2$

$S_\pm=S_1^\pm+S_2^\pm$

$S_z=S_1^z+S_2^z$

$\vec{S}=\vec{S}_1+\vec{S}_2\rightarrow[S_\alpha^i,S_\beta^j]=i\hbar\varepsilon^{ijk}S_\alpha^k\delta_{\alpha\beta}$

Angular momentum

$\Rightarrow[S_i,S_j]=i\hbar\varepsilon_{ijk}S_k$

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$$S^2|S,M\rangle = \hbar^2 S(S+1)|S,M\rangle$$
$$S_z|S,M\rangle = \hbar M|S,M\rangle$$

$$|\uparrow\uparrow\rangle = |\uparrow\rangle_1 |\uparrow\rangle_2$$

$$S^+|\uparrow\uparrow\rangle = (S_1^+ + S_2^+)|\uparrow\rangle_1|\uparrow\rangle_2$$
$$= \underbrace{S_1^+|\uparrow\rangle_1}_{=0}|\uparrow\rangle_2 + |\uparrow\rangle_1 \underbrace{S_2^+|\uparrow\rangle_2}_{=0}$$
$$= 0 + 0 = 0$$

$$|\uparrow\uparrow\rangle = |S,M\rangle = |S\rangle$$

$$S_z|\uparrow\uparrow\rangle = (S_1^z + S_2^z)|\uparrow\rangle_1|\uparrow\rangle_2$$
$$= \underbrace{S_1^z|\uparrow\rangle_1}_{=\frac{\hbar}{2}|\uparrow\rangle_1}|\uparrow\rangle_2 + |\uparrow\rangle_1 \underbrace{S_2^z|\uparrow\rangle_2}_{=\frac{\hbar}{2}|\uparrow\rangle_2}$$
$$= \frac{\hbar}{2}(|\uparrow\rangle_1|\uparrow\rangle_2 = \hbar|\uparrow\uparrow\rangle$$

$$|S\underset{=S}{M}\rangle = |\uparrow\uparrow\rangle$$

$$S^z|\uparrow\uparrow\rangle = \hbar|\uparrow\uparrow\rangle$$
$$= \hbar M|\uparrow\uparrow\rangle$$

$M=1,\ S=1$

$$|S=1, M=1\rangle = |\uparrow\uparrow\rangle$$

$$J_-|j,m\rangle = \hbar\sqrt{(j+m)(j-m+1)}\,|j,m-1\rangle$$
$$J_+|j,m\rangle = \hbar\sqrt{(j+m+1)(j-m)}\,|j,m+1\rangle$$

$$S_-|1,1\rangle = \hbar\sqrt{2\cdot 1}\,|1,0\rangle$$

$$|1,0\rangle = \frac{1}{\hbar\sqrt{2}} S_-|\uparrow\uparrow\rangle$$

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$$S=\frac{1}{2} \quad S_1^-|\uparrow\rangle_1=\hbar\sqrt{\left(\frac{1}{2}+\frac{1}{2}\right)\left(\frac{1}{2}-\frac{1}{2}+1\right)}\,|\downarrow\rangle_1$$
$$=\hbar|\downarrow\rangle_1$$

$$S=\frac{1}{2},\quad M=\frac{1}{2}$$
$$S_2^-|\uparrow\rangle_2=\hbar|\downarrow\rangle_2$$

$$|1,0\rangle=\frac{1}{\hbar\sqrt{2}}S_-|\uparrow\uparrow\rangle$$
$$=\frac{1}{\hbar\sqrt{2}}(S_1^-+S_2^-)|\uparrow\rangle_1|\uparrow\rangle_2$$
$$=\frac{1}{\hbar\sqrt{2}}\left\{\underbrace{(S_1^-|\uparrow\rangle_1)}_{\hbar|\downarrow\rangle_1}|\uparrow\rangle_2+|\uparrow\rangle_1\underbrace{(S_2^-|\uparrow\rangle_2)}_{\hbar|\downarrow\rangle_2}\right\}$$

$$|1,0\rangle=\frac{1}{\sqrt{2}}\left(|\downarrow\rangle_1|\uparrow\rangle_2+|\uparrow\rangle_1|\downarrow\rangle_2\right)$$
$$=\frac{1}{\sqrt{2}}\left(|\downarrow\uparrow\rangle+|\uparrow\downarrow\rangle\right)$$

$$S_-|\underset{s}{1},\underset{m}{0}\rangle=\hbar\sqrt{(1+0)(1-0+1)}\,|1,-1\rangle$$
$$=\hbar\sqrt{2}\,|1,-1\rangle$$

$$|1,-1\rangle=\frac{1}{\hbar\sqrt{2}}S_-|1,0\rangle=\frac{1}{\hbar\sqrt{2}}S_-\frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle+|\downarrow\uparrow\rangle\right)$$
$$=\frac{1}{2\hbar}(S_1^-+S_2^-)\left(\underbrace{|\uparrow\downarrow\rangle}_{|\uparrow\rangle_1|\downarrow\rangle_2}+\underbrace{|\downarrow\uparrow\rangle}_{|\downarrow\rangle_1|\uparrow\rangle_2}\right)$$
$$=\frac{1}{2\hbar}\Big\{(S_1^-|\uparrow\rangle)|\downarrow\rangle_2+\underbrace{(S_1^-|\downarrow\rangle_1)}_{=0}|\uparrow\rangle_2$$
$$+|\uparrow\rangle_1\underbrace{(S_2^-|\downarrow\rangle_2)}_{=0}+|\downarrow\rangle_1\underbrace{(S_2^-|\uparrow\rangle_2)}_{=\hbar|\downarrow\rangle_2}$$

$$=\frac{1}{2\hbar}\left(\hbar|\downarrow\rangle_1|\downarrow\rangle_2+\hbar|\downarrow\rangle_1|\downarrow\rangle_2\right)$$

$$=|\downarrow\rangle_1|\downarrow\rangle_2=|\downarrow\downarrow\rangle$$

$$|1,1\rangle=|\uparrow\uparrow\rangle$$
$$|1,0\rangle=\frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle+|\downarrow\uparrow\rangle\right)$$

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$$|1,-1\rangle = |\downarrow\downarrow\rangle$$

$\overset{\shortparallel}{S}\ \ \overset{\shortparallel}{M} = -S \qquad S_-|\downarrow\downarrow\rangle = 0$

$|S,M\rangle \quad S=1,\ M=1,0,-1 \quad$ ③

$$\left.\begin{array}{l} |\uparrow\uparrow\rangle = |\uparrow\rangle_1|\uparrow\rangle_2 \\ |\downarrow\downarrow\rangle = |\downarrow\rangle_1|\downarrow\rangle_2 \\ |\uparrow\downarrow\rangle = |\uparrow\rangle_1|\downarrow\rangle_2 \\ |\downarrow\uparrow\rangle = |\downarrow\rangle_1|\uparrow\rangle_2 \end{array}\right\} (4)$$

$$\frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) = |x\rangle$$

$$\langle 1,1|x\rangle = \frac{1}{\sqrt{2}}\langle\uparrow\uparrow|\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$$

$$= \frac{1}{\sqrt{2}}\left(\langle\uparrow\uparrow|\uparrow\downarrow\rangle - \langle\uparrow\uparrow|\downarrow\uparrow\rangle\right)$$

$$= \frac{1}{\sqrt{2}}\left({}_1\langle\uparrow|\uparrow\rangle_1\,{}_2\langle\uparrow|\overset{0}{\downarrow}\rangle_2 + {}_1\langle\uparrow|\overset{0}{\downarrow}\rangle_1 + {}_2\langle\uparrow|\downarrow\rangle_2\right) = 0$$

$$\therefore \langle 1,-1|x\rangle = 0$$

$\langle\alpha,\beta|\alpha',\beta'\rangle \quad \alpha,\beta = \uparrow\downarrow$

$= {}_1\langle\alpha|\,{}_2\langle\beta|\,(|\alpha'\rangle_1|\beta'\rangle_2)$

$= {}_1\langle\alpha|\alpha'\rangle_1\,{}_2\langle\beta|\beta'\rangle_2 \quad = \{|\uparrow\uparrow\rangle, |\uparrow\downarrow\rangle, |\downarrow\uparrow\rangle, |\downarrow\downarrow\rangle\}$

$\quad\ \overset{\shortparallel}{\delta_{\alpha\alpha'}} \qquad \overset{\shortparallel}{\delta_{\beta\beta'}} \qquad$ 規格直交系

$$\langle 1,0|x\rangle = \frac{1}{2}(\langle\uparrow\downarrow| + \langle\downarrow\uparrow|)(|\uparrow\downarrow\rangle - \langle\downarrow\uparrow|)$$

$$= \frac{1}{2}\left(\langle\uparrow\downarrow|\overset{1}{\uparrow\downarrow}\rangle - \langle\downarrow\overset{1}{\uparrow}|\downarrow\uparrow\rangle - \langle\uparrow\downarrow|\overset{0}{\downarrow\uparrow}\rangle + \langle\overset{0}{\downarrow\uparrow}|\uparrow\downarrow\rangle\right)$$

$$= \frac{1}{2}(1-1) = 0$$

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$$(|\uparrow\uparrow\rangle, \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle+|\downarrow\uparrow\rangle), |\downarrow\downarrow\rangle, \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle-|\downarrow\uparrow\rangle), |\downarrow\downarrow\rangle)$$

$$=(|\uparrow\uparrow\rangle, |1,0\rangle, |1,-1\rangle, |x\rangle)$$

$$=(|\uparrow\uparrow\rangle, |\uparrow\downarrow\rangle, |\downarrow\uparrow\rangle, |\downarrow\downarrow\rangle)\begin{pmatrix}1&0&0&0\\0&\frac{1}{\sqrt{2}}&\frac{1}{\sqrt{2}}&0\\0&\frac{1}{\sqrt{2}}&-\frac{1}{\sqrt{2}}&0\\0&0&0&1\end{pmatrix}$$ Clobsch-Gordon 係数

$$S_z|x\rangle = (S_1^z+S_2^z)\frac{1}{\sqrt{2}}(|\uparrow\rangle_1|\downarrow\rangle_2-|\downarrow\rangle_1|\uparrow\rangle_2)$$

$$=\frac{1}{\sqrt{2}}(\underbrace{\overset{\frac{\hbar}{2}}{S_1^z}|\uparrow\rangle_1|\downarrow\rangle_2+|\uparrow\rangle_1\overset{\frac{\hbar}{2}}{S_2^z}|\downarrow\rangle_2}_{=0}\underbrace{-S_1^z|\downarrow\rangle_1|\uparrow\rangle_2-|\downarrow\rangle S_2^z|\uparrow\rangle_2}_{=0})$$

$$=0$$

$$S_+|x\rangle = (S_1^+ + S_2^+)\frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle-|\downarrow\uparrow\rangle)$$

$$=\frac{1}{\sqrt{2}}(\overset{=0}{S^+|\uparrow\downarrow\rangle}-\overset{=\hbar|\uparrow\uparrow\rangle}{S_1^+|\downarrow\uparrow\rangle}+\overset{=\hbar|\uparrow\uparrow\rangle}{S_2^+|\uparrow\downarrow\rangle}-\overset{=0}{S_2^+|\downarrow\uparrow\rangle})$$

$$=(\hbar-\hbar)|\uparrow\uparrow\rangle=0$$ M: max $S=M=0$

$|1,1\rangle = |\uparrow\uparrow\rangle$

$|1,0\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle+|\downarrow\uparrow\rangle)$

$|1,-1\rangle = |\downarrow\downarrow\rangle$ $S=1, M=1,0,-1$

$\underset{=|x\rangle}{|0,0\rangle} = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle-|\downarrow\uparrow\rangle)$

$S=M=0$

$S=\frac{1}{2}$ splus(2) $\Rightarrow S=1,0$

$\frac{1}{2}\otimes\frac{1}{2} = 1\oplus 0$

state

$|M_1\rangle_1|M_2\rangle_2 = |M_1\rangle\otimes|M_2\rangle$

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$\left(|\uparrow\uparrow\rangle, \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle+|\downarrow\uparrow\rangle), \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle-|\downarrow\uparrow\rangle), |\downarrow\downarrow\rangle\right)$

$= (|1,1\rangle, |1,0\rangle, |0,0\rangle, |1,-1\rangle)$

$= (|\uparrow\uparrow\rangle, |\uparrow\downarrow\rangle, |\downarrow\uparrow\rangle, |\downarrow\downarrow\rangle) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

$= (|\frac{1}{2}\frac{1}{2}, \frac{1}{2}\frac{1}{2}\rangle, |\frac{1}{2}\frac{1}{2}, \frac{1}{2}-\frac{1}{2}\rangle, |\frac{1}{2}-\frac{1}{2}, \frac{1}{2}\frac{1}{2}\rangle, |\frac{1}{2}-\frac{1}{2}, \frac{1}{2}-\frac{1}{2}\rangle$

$|S_1 M_1, S_2 M_2\rangle$

$\quad \frac{1}{2} \quad \pm\frac{1}{2} \quad \frac{1}{2} \quad \pm\frac{1}{2}$

$|S_1 M_1, S_2 M_2\rangle = |\overset{2S_1+1}{S_1} M_1\rangle \otimes |\overset{2S_2+1}{S_2} M_2\rangle$

base charge

$\downarrow$

$|S M\rangle$

全角运動量

$(2S_1+1)(2S_2+1) = 2 \cdot 2 = 4$

$S \quad S = 1, 0 \quad 3+1=4, 2S+1=3, 2\cdot 0+1=1$

$\frac{1}{2} \otimes \frac{1}{2} = 1 \oplus 0$

$2S+1=3$ 三重項

$2S+1=2\cdot 0+1=1$ 一重項