量子力学3 演習6

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量子スピン系　山崎 智矢

$\boxed{1}$ $j_1 = 1,\ j_2 = \frac{1}{2}$ のCG係数

$m_1 : 1, 0, -1, \quad m_2 : \frac{1}{2}, -\frac{1}{2}$

$m_2 \backslash m_1$	1	0	−1	
1/2	3/2	1/2	−1/2	$j = \frac{1}{2}$
−1/2	1/2	−1/2	−3/2	$j = \frac{3}{2}$

考えられるケット $|j\,m\rangle$

$\Rightarrow \left|\frac{3}{2}\ \frac{3}{2}\right\rangle, \left|\frac{3}{2}\ \frac{1}{2}\right\rangle, \left|\frac{3}{2}\ -\frac{1}{2}\right\rangle$

$\left|\frac{3}{2}\ -\frac{3}{2}\right\rangle, \left|\frac{1}{2}\ \frac{1}{2}\right\rangle, \left|\frac{1}{2}\ -\frac{1}{2}\right\rangle$

考えられるケット $|j_1 m_1\ j_2 m_2\rangle$

$\left|1\,1\,\frac{1}{2}\,\frac{1}{2}\right\rangle, \left|1\,1\,\frac{1}{2}\,-\frac{1}{2}\right\rangle, \left|1\,0\,\frac{1}{2}\,-\frac{1}{2}\right\rangle, \left|1\,-1\,\frac{1}{2}\,-\frac{1}{2}\right\rangle$

$\left|1\,0\,\frac{1}{2}\,\frac{1}{2}\right\rangle, \left|1\,-1\,\frac{1}{2}\,\frac{1}{2}\right\rangle$

$$\left|\frac{3}{2}\ \frac{3}{2}\right\rangle = |1\,1\rangle \otimes \left|\frac{1}{2}\ \frac{1}{2}\right\rangle = \underline{\left|1\,1\,\frac{1}{2}\,\frac{1}{2}\right\rangle}$$

$$\left|\frac{3}{2}\ \frac{1}{2}\right\rangle = \frac{1}{\hbar\sqrt{\left(\frac{3}{2}+\frac{3}{2}\right)\left(\frac{3}{2}-\frac{3}{2}+1\right)}} J_- \left|\frac{3}{2}\ \frac{3}{2}\right\rangle$$

$$= \frac{1}{\sqrt{3}\hbar}(J_{1-} + J_{2-})|1\,1\rangle\left|\frac{1}{2}\ \frac{1}{2}\right\rangle$$

$$= \frac{1}{\sqrt{3}\hbar}\left\{\hbar\sqrt{(1+1)(1-1+1)}\,|1\,0\rangle\left|\frac{1}{2}\ \frac{1}{2}\right\rangle + \hbar\sqrt{\left(\frac{1}{2}+\frac{1}{2}\right)\left(\frac{1}{2}-\frac{1}{2}+1\right)}\,|1\,1\rangle\left|\frac{1}{2}\ -\frac{1}{2}\right\rangle\right\}$$

$$= \underline{\sqrt{\frac{2}{3}}\left|1\,0\,\frac{1}{2}\,\frac{1}{2}\right\rangle + \frac{1}{\sqrt{3}}\left|1\,1\,\frac{1}{2}\,-\frac{1}{2}\right\rangle}$$

$$\left|\frac{3}{2}\ -\frac{1}{2}\right\rangle = \frac{1}{\hbar\sqrt{\left(\frac{3}{2}+\frac{1}{2}\right)\left(\frac{3}{2}-\frac{1}{2}+1\right)}} J_- \left|\frac{3}{2}\ \frac{1}{2}\right\rangle$$

$$= \frac{1}{2\hbar}\left(J_{1-}+J_{2-}\right)\left(\sqrt{\frac{2}{3}}\left|1\,0\,\frac{1}{2}\,\frac{1}{2}\right\rangle + \frac{1}{\sqrt{3}}\left|1\,1\,\frac{1}{2}\,-\frac{1}{2}\right\rangle\right)$$

$$= \frac{1}{2\hbar}\sqrt{\frac{2}{3}}\left(J_{1-}|1\,0\rangle\right)\left|\frac{1}{2}\,\frac{1}{2}\right\rangle + \frac{1}{2\hbar}\sqrt{\frac{2}{3}}|1\,0\rangle\left(J_{2-}\left|\frac{1}{2}\,\frac{1}{2}\right\rangle\right)$$

$$+ \frac{1}{2\hbar}\frac{1}{\sqrt{3}}\left(J_{1-}|1\,1\rangle\right)\left|\frac{1}{2}\,-\frac{1}{2}\right\rangle + \cancel{\frac{1}{2\hbar}\frac{1}{\sqrt{3}}|1\,1\rangle\left(J_{2-}\left|\frac{1}{2}\,-\frac{1}{2}\right\rangle\right)}$$

$$= \frac{1}{\sqrt{6}}\sqrt{(1+0)(1-0+1)}\,|1\,-1\rangle\left|\frac{1}{2}\,\frac{1}{2}\right\rangle + \frac{1}{\sqrt{6}}\sqrt{\left(\frac{1}{2}+\frac{1}{2}\right)\left(\frac{1}{2}-\frac{1}{2}+1\right)}\,|1\,0\rangle\left|\frac{1}{2}\,-\frac{1}{2}\right\rangle$$

$$+ \frac{1}{\sqrt{12}}\sqrt{(1+1)(1-1+1)}\,|1\,0\rangle\left|\frac{1}{2}\,-\frac{1}{2}\right\rangle + \cancel{\frac{1}{\sqrt{12}}\sqrt{\left(\frac{1}{2}-\frac{1}{2}\right)}}$$

$$= \frac{1}{\sqrt{3}}\left|1\,-1\,\frac{1}{2}\,\frac{1}{2}\right\rangle + \frac{1}{\sqrt{6}}\left|1\,0\,\frac{1}{2}\,-\frac{1}{2}\right\rangle + \frac{1}{\sqrt{6}}\left|1\,0\,\frac{1}{2}\,-\frac{1}{2}\right\rangle$$

$$= \frac{1}{\sqrt{3}}\left|1\,-1\,\frac{1}{2}\,\frac{1}{2}\right\rangle + \frac{2}{\sqrt{6}}\left|1\,0\,\frac{1}{2}\,-\frac{1}{2}\right\rangle$$

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$$\left|\frac{3}{2}\ -\frac{3}{2}\right\rangle = \frac{1}{\hbar\sqrt{(\frac{3}{2}-\frac{1}{2})(\frac{3}{2}+\frac{1}{2}+1)}} J_-\left|\frac{3}{2}\ -\frac{1}{2}\right\rangle$$

$$= \frac{1}{\sqrt{3}\hbar}(J_{1-}+J_{2-})\left(\frac{1}{\sqrt{3}}\left|1\ -1\ \frac{1}{2}\ \frac{1}{2}\right\rangle + \frac{2}{\sqrt{6}}\left|1\ 0\ \frac{1}{2}\ -\frac{1}{2}\right\rangle\right)$$

$$= \frac{1}{3\hbar}|1\ -1\rangle\left(J_{2-}\left|\frac{1}{2}\ \frac{1}{2}\right\rangle\right) + \frac{1}{\sqrt{3}\hbar}\cdot\frac{2}{\sqrt{6}}(J_{1-}|1\ 0\rangle)\left|\frac{1}{2}\ -\frac{1}{2}\right\rangle$$

$$= \frac{1}{3\cancel{\hbar}}\cancel{\hbar}\sqrt{(\tfrac{1}{2}+\tfrac{1}{2})(\tfrac{1}{2}-\tfrac{1}{2}+1)}\left|1\ -1\ \frac{1}{2}\ -\frac{1}{2}\right\rangle + \frac{2}{3\sqrt{2}}\sqrt{(1+0)(1-0+1)}\left|1\ -1\ \frac{1}{2}\ -\frac{1}{2}\right\rangle$$

$$= \frac{1}{3}\left|1\ -1\ \frac{1}{2}\ -\frac{1}{2}\right\rangle + \frac{2}{3}\left|1\ -1\ \frac{1}{2}\ -\frac{1}{2}\right\rangle$$

$$= \left|1\ -1\ \frac{1}{2}\ -\frac{1}{2}\right\rangle$$

次に$\left|\frac{1}{2}\ \frac{1}{2}\right\rangle$を考える。$m=\frac{1}{2}$となる$|j_1m_1\ j_2m_2\rangle$は

$\left|1\ 0\ \frac{1}{2}\ \frac{1}{2}\right\rangle$と$\left|1\ 1\ \frac{1}{2}\ -\frac{1}{2}\right\rangle$であるから

$\left|\frac{1}{2}\ \frac{1}{2}\right\rangle = a\left|1\ 0\ \frac{1}{2}\ \frac{1}{2}\right\rangle + b\left|1\ 1\ \frac{1}{2}\ -\frac{1}{2}\right\rangle$とおいて$a$、$b$を求める。

$\left\langle\frac{1}{2}\ \frac{1}{2}\middle|\frac{1}{2}\ \frac{1}{2}\right\rangle = 1$より $a^2+b^2=1$

$\left\langle\frac{3}{2}\ \frac{1}{2}\middle|\frac{1}{2}\ \frac{1}{2}\right\rangle = 0$より $\sqrt{\frac{2}{3}}a + \frac{1}{\sqrt{3}}b = 0$

$b = -\sqrt{2}a$

$a^2+2a^2 = 1$

$a^2 = \frac{1}{3}$ $a = \pm\frac{1}{\sqrt{3}}$, $b = \mp\sqrt{\frac{2}{3}}$

ここで$b>0$の方にすると $a=-\frac{1}{\sqrt{3}}$, $b=\sqrt{\frac{2}{3}}$

$$\left|\frac{1}{2}\,\frac{1}{2}\right\rangle = -\frac{1}{\sqrt{3}}\left|1\,0\,\frac{1}{2}\,\frac{1}{2}\right\rangle + \sqrt{\frac{2}{3}}\left|1\,1\,\frac{1}{2}\,-\frac{1}{2}\right\rangle$$

$$\left|\frac{1}{2}\,-\frac{1}{2}\right\rangle = \frac{1}{\hbar\sqrt{\left(\frac{1}{2}+\frac{1}{2}\right)\left(\frac{1}{2}-\frac{1}{2}+1\right)}}J_-\left|\frac{1}{2}\,\frac{1}{2}\right\rangle$$

$$= \frac{1}{\hbar}(J_{1-}+J_{2-})\left(-\frac{1}{\sqrt{3}}\left|1\,0\,\frac{1}{2}\,\frac{1}{2}\right\rangle + \sqrt{\frac{2}{3}}\left|1\,1\,\frac{1}{2}\,-\frac{1}{2}\right\rangle\right)$$

$$= -\frac{1}{\sqrt{3}}\sqrt{(1+0)(1-0+1)}\left|1\,-1\,\frac{1}{2}\,\frac{1}{2}\right\rangle$$

$$-\frac{1}{\sqrt{3}}\sqrt{\left(\frac{1}{2}+\frac{1}{2}\right)\left(\frac{1}{2}-\frac{1}{2}+1\right)}\left|1\,0\,\frac{1}{2}\,-\frac{1}{2}\right\rangle$$

$$+\sqrt{\frac{2}{3}}\cdot\sqrt{(1+1)(1-1+1)}\left|1\,0\,\frac{1}{2}\,-\frac{1}{2}\right\rangle$$

$$= -\sqrt{\frac{2}{3}}\left|1\,-1\,\frac{1}{2}\,\frac{1}{2}\right\rangle - \frac{1}{\sqrt{3}}\left|1\,0\,\frac{1}{2}\,-\frac{1}{2}\right\rangle + \frac{2}{\sqrt{3}}\left|1\,0\,\frac{1}{2}\,-\frac{1}{2}\right\rangle$$

$$= -\sqrt{\frac{2}{3}}\left|1\,-1\,\frac{1}{2}\,\frac{1}{2}\right\rangle + \frac{1}{\sqrt{3}}\left|1\,0\,\frac{1}{2}\,-\frac{1}{2}\right\rangle.$$

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以上をまとめると

$$\left|\tfrac{3}{2}\ \tfrac{3}{2}\right\rangle = \left|1\,1\,\tfrac{1}{2}\,\tfrac{1}{2}\right\rangle$$

$$\left|\tfrac{3}{2}\ \tfrac{1}{2}\right\rangle = \sqrt{\tfrac{2}{3}}\left|1\,0\,\tfrac{1}{2}\,\tfrac{1}{2}\right\rangle + \sqrt{\tfrac{1}{3}}\left|1\,1\,\tfrac{1}{2}\,-\tfrac{1}{2}\right\rangle$$

$$\left|\tfrac{3}{2}\ -\tfrac{1}{2}\right\rangle = \sqrt{\tfrac{1}{3}}\left|1\,-1\,\tfrac{1}{2}\,\tfrac{1}{2}\right\rangle + \sqrt{\tfrac{2}{3}}\left|1\,0\,\tfrac{1}{2}\,-\tfrac{1}{2}\right\rangle$$

$$\left|\tfrac{3}{2}\ -\tfrac{3}{2}\right\rangle = \left|1\,-1\,\tfrac{1}{2}\,-\tfrac{1}{2}\right\rangle$$

$$\left|\tfrac{1}{2}\ \tfrac{1}{2}\right\rangle = -\sqrt{\tfrac{1}{3}}\left|1\,0\,\tfrac{1}{2}\,\tfrac{1}{2}\right\rangle + \sqrt{\tfrac{2}{3}}\left|1\,1\,\tfrac{1}{2}\,-\tfrac{1}{2}\right\rangle$$

$$\left|\tfrac{1}{2}\ -\tfrac{1}{2}\right\rangle = -\sqrt{\tfrac{2}{3}}\left|1\,-1\,\tfrac{1}{2}\,\tfrac{1}{2}\right\rangle + \sqrt{\tfrac{1}{3}}\left|1\,0\,\tfrac{1}{2}\,-\tfrac{1}{2}\right\rangle$$

$|j_1 m_1\ j_2 m_2\rangle$ の前にかかっている係数が $j_1 = 1,\ j_2 = \frac{1}{2}$ のCG係数である。

//

$\boxed{2}$ (i) $|S_1 S_2 S_3\rangle = |S_1\rangle \otimes |S_2\rangle \otimes |S_3\rangle$

$S_i = \uparrow\downarrow \cdots$ 2通り よって $|S_1 S_2 S_3\rangle$ は $2^3 = 8$通り //

(ii) $\vec{S}^2 = S_1^2 + S_2^2 + S_3^2 + 2(\vec{S}_1 \cdot \vec{S}_2 + \vec{S}_2 \cdot \vec{S}_3 + \vec{S}_3 \cdot \vec{S}_1)$

$$= \frac{3}{4}\hbar^2 \times 3 + 2(\sim)$$

$\therefore$ $H_3 = J(\sim)$

$$= J\left(\frac{1}{2}\vec{S}^2 - \frac{1}{2} \cdot \frac{9}{4}\hbar^2\right)$$

$$= \underline{\frac{1}{2}\left(J\vec{S}^2 - \frac{9}{4}J\hbar^2\right)}$$

(iii) $\frac{1}{2} \otimes \frac{1}{2} \otimes \frac{1}{2}$

$$= \left\{\left(\frac{1}{2} + \frac{1}{2}\right) \oplus \left(\frac{1}{2} - \frac{1}{2}\right)\right\} \otimes \frac{1}{2}$$

$$= (1 \oplus 0) \otimes \frac{1}{2}$$

$$= \left(1 \otimes \frac{1}{2}\right) \oplus \left(0 \otimes \frac{1}{2}\right)$$

$$= \left\{\left(1 + \frac{1}{2}\right) \oplus \left(1 - \frac{1}{2}\right)\right\} \oplus \left(0 + \frac{1}{2}\right)$$

$$= \underline{\frac{1}{2} \oplus \frac{1}{2} \oplus \frac{3}{2}}$$

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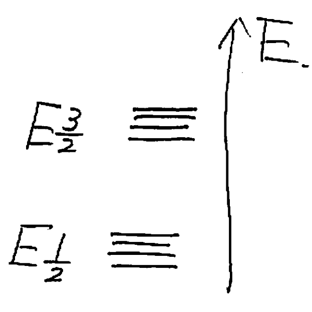
(iv) $\frac{1}{2}\otimes\frac{1}{2}\otimes\frac{1}{2}=\frac{1}{2}\oplus\frac{1}{2}\oplus\frac{3}{2}$

$\Rightarrow$ 合成ケットは $|\frac{3}{2}\,m\rangle,\ |\frac{1}{2}\,m\rangle_1,\ |\frac{1}{2},m\rangle_2$ がある。

$$H|\tfrac{3}{2}\,m\rangle=\frac{1}{2}J\vec{S}^2|\tfrac{3}{2}\,m\rangle-\frac{9}{8}J\hbar^2|\tfrac{3}{2}\,m\rangle$$
$$=\frac{1}{2}J\hbar^2\left(\frac{3}{2}\right)\left(\frac{3}{2}+1\right)|\tfrac{3}{2},m\rangle-\frac{9}{8}J\hbar^2|\tfrac{3}{2},m\rangle$$
$$=\frac{3}{4}J\hbar^2|\tfrac{3}{2},m\rangle=E_{\frac{3}{2}}|\tfrac{3}{2}\,m\rangle$$

$E_{\frac{3}{2}}=\frac{3}{4}J\hbar^2$ （4重項）$\to$ 4重縮退

$$H|\tfrac{1}{2}\,m\rangle=\frac{1}{2}J\vec{S}^2|\tfrac{1}{2}\,m\rangle-\frac{9}{8}J\hbar^2|\tfrac{1}{2}\,m\rangle$$
$$=\frac{1}{2}J\hbar^2\frac{1}{2}\left(\frac{1}{2}+1\right)|\tfrac{1}{2}\,m\rangle-\frac{9}{8}J\hbar^2|\tfrac{1}{2}\,m\rangle$$
$$=\left(\frac{3}{8}-\frac{9}{8}\right)J\hbar^2|\tfrac{1}{2}\,m\rangle$$
$$=-\frac{3}{4}J\hbar^2|\tfrac{1}{2}\,m\rangle$$
$$=E_{\frac{1}{2}}|\tfrac{1}{2}\,m\rangle$$



$E_{\frac{1}{2}}=-\frac{3}{4}J\hbar^2$ （4重項）$\to$ 4重縮退

(v) $\left|\frac{3}{2}\ \frac{3}{2}\right\rangle = |\uparrow_1\rangle \otimes |\uparrow_2\rangle \otimes |\uparrow_3\rangle = |\uparrow_1\rangle|\uparrow_2\rangle|\uparrow_3\rangle$.

$$\left|\frac{3}{2}\ \frac{1}{2}\right\rangle = \frac{1}{\hbar\sqrt{(\frac{3}{2}+\frac{3}{2})(\frac{3}{2}-\frac{3}{2}+1)}} S_- \left|\frac{3}{2}\ \frac{3}{2}\right\rangle$$

$$= \frac{1}{\sqrt{3}\hbar}(S_{1-} + S_{2-} + S_{3-})|\uparrow_1\rangle|\uparrow_2\rangle|\uparrow_3\rangle$$

$$= \frac{1}{\sqrt{3}\hbar}\left\{(S_{1-}|\uparrow_1\rangle)|\uparrow_2\rangle|\uparrow_3\rangle + |\uparrow_1\rangle(S_{2-}|\uparrow_2\rangle)|\uparrow_3\rangle + |\uparrow_1\rangle|\uparrow_2\rangle(S_{3-}|\uparrow_3\rangle)\right\}$$

$$= \frac{1}{\sqrt{3}}\left(|\downarrow_1\rangle|\uparrow_2\rangle|\uparrow_3\rangle + |\uparrow_1\rangle|\downarrow_2\rangle|\uparrow_3\rangle + |\uparrow_1\rangle|\uparrow_2\rangle|\downarrow_3\rangle\right)$$

$$\left|\frac{3}{2}\ -\frac{1}{2}\right\rangle = \frac{1}{\hbar\sqrt{(\frac{3}{2}+\frac{1}{2})(\frac{3}{2}-\frac{1}{2}+1)}} S_- \left|\frac{3}{2}\ \frac{1}{2}\right\rangle$$

$$= \frac{1}{2\hbar}(S_{1-} + S_{2-} + S_{3-})\frac{1}{\sqrt{3}}\left(|\downarrow_1\rangle|\uparrow_2\rangle|\uparrow_3\rangle + |\uparrow_1\rangle|\downarrow_2\rangle|\uparrow_3\rangle + |\uparrow_1\rangle|\uparrow_2\rangle|\downarrow_3\rangle\right)$$

$$= \frac{1}{2\sqrt{3}}\left[\begin{aligned}&|\downarrow_1\rangle|\downarrow_2\rangle|\uparrow_3\rangle + |\downarrow_1\rangle|\uparrow_2\rangle|\downarrow_3\rangle + |\downarrow_1\rangle|\downarrow_2\rangle|\uparrow_3\rangle \\ &+ |\uparrow_1\rangle|\downarrow_2\rangle|\downarrow_3\rangle + |\downarrow_1\rangle|\uparrow_2\rangle|\downarrow_3\rangle + |\uparrow_1\rangle|\downarrow_2\rangle|\downarrow_3\rangle\end{aligned}\right]$$

$$= \frac{1}{\sqrt{3}}\left(|\uparrow_1\rangle|\downarrow_2\rangle|\downarrow_3\rangle + |\downarrow_1\rangle|\uparrow_2\rangle|\downarrow_3\rangle + |\downarrow_1\rangle|\downarrow_2\rangle|\uparrow_3\rangle\right)$$

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$$\left|\frac{3}{2}\ -\frac{3}{2}\right\rangle = \frac{1}{\hbar\sqrt{\left(\frac{3}{2}-\frac{1}{2}\right)\left(\frac{3}{2}+\frac{1}{2}+1\right)}} S_- \left|\frac{3}{2}\ -\frac{1}{2}\right\rangle$$

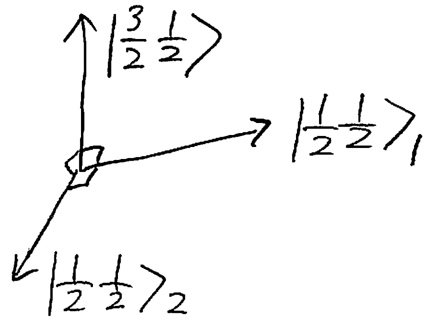
$$= \frac{1}{\sqrt{3}\hbar}\frac{\hbar}{\sqrt{3}}\Big(|\downarrow_1\rangle|\downarrow_2\rangle|\downarrow_3\rangle + |\downarrow_1\rangle|\downarrow_2\rangle|\downarrow_3\rangle + |\downarrow_1\rangle|\downarrow_2\rangle|\downarrow_3\rangle \Big)$$

$$= \underline{|\downarrow_1\rangle|\downarrow_2\rangle|\downarrow_3\rangle}$$

次に $\left|\frac{1}{2}\ \frac{1}{2}\right\rangle_1$, $\left|\frac{1}{2}\ \frac{1}{2}\right\rangle_2$ を考える。

$\left|\frac{3}{2}\ \frac{1}{2}\right\rangle$, $\left|\frac{1}{2}\ \frac{1}{2}\right\rangle_1$, $\left|\frac{1}{2}\ \frac{1}{2}\right\rangle_2$ は基底 $\underbrace{|\downarrow_1\rangle|\uparrow_2\rangle|\uparrow_3\rangle}_{=\alpha}$, $\underbrace{|\uparrow_1\rangle|\downarrow_2\rangle|\uparrow_3\rangle}_{=\beta}$, $\underbrace{|\uparrow_1\rangle|\uparrow_2\rangle|\downarrow_3\rangle}_{=\gamma}$ の線形結合で表される。

さらに $\left|\frac{3}{2}\ \frac{1}{2}\right\rangle$, $\left|\frac{1}{2}\ \frac{1}{2}\right\rangle_1$, $\left|\frac{1}{2}\ \frac{1}{2}\right\rangle_1$ は互いに直交する。



$$\left|\frac{3}{2}\ \frac{1}{2}\right\rangle = \frac{1}{\sqrt{3}}(\alpha+\beta+\gamma)$$

$$\left|\frac{1}{2}\ \frac{1}{2}\right\rangle_1 = A(-\alpha-\beta+2\gamma)$$

A：定数

$${}_1\left\langle\frac{1}{2}\ \frac{1}{2}\middle|\frac{1}{2}\ \frac{1}{2}\right\rangle_1 = 1 \text{ より } A = \frac{1}{\sqrt{6}}$$

$$\therefore \left|\frac{1}{2}\ \frac{1}{2}\right\rangle_1 = \frac{1}{\sqrt{6}}(-\alpha-\beta+2\gamma)$$

ここで $\left|\frac{1}{2}\,\frac{1}{2}\right\rangle_2 = (d\alpha + e\beta + f\gamma)$ とおく

d, e, f は定数.

$${}_2\left\langle\frac{1}{2}\,\frac{1}{2}\middle|\frac{1}{2}\,\frac{1}{2}\right\rangle_2 = d^2 + e^2 + f^2 = 1$$

$$\left\langle\frac{3}{2}\,\frac{1}{2}\middle|\frac{1}{2}\,\frac{1}{2}\right\rangle_2 = d + e + f = 0.$$

$$\Rightarrow f = 0,\quad d = \frac{1}{\sqrt{2}},\quad e = -\frac{1}{\sqrt{2}}$$

∴よって

$$\left|\frac{1}{2}\,\frac{1}{2}\right\rangle_1 = \frac{1}{\sqrt{6}}\Big(-|\downarrow_1\rangle|\uparrow_2\rangle|\uparrow_3\rangle - |\uparrow_1\rangle|\downarrow_2\rangle|\uparrow_3\rangle + 2|\uparrow_1\rangle|\uparrow_2\rangle|\downarrow_3\rangle\Big)$$

$$\left|\frac{1}{2}\,\frac{1}{2}\right\rangle_2 = \frac{1}{\sqrt{2}}\Big(|\downarrow_1\rangle|\uparrow_2\rangle|\uparrow_3\rangle - |\uparrow_1\rangle|\downarrow_2\rangle|\uparrow_3\rangle\Big)$$

$$\left|\frac{1}{2}\,-\frac{1}{2}\right\rangle_1 = \frac{1}{\hbar}S_-\left|\frac{1}{2}\,\frac{1}{2}\right\rangle_1$$

$$= \frac{1}{\sqrt{6}}\begin{pmatrix} -|\downarrow_1\rangle|\downarrow_2\rangle|\uparrow_3\rangle + 2|\downarrow_1\rangle|\uparrow_2\rangle|\downarrow_3\rangle \\ -|\downarrow_1\rangle|\downarrow_2\rangle|\uparrow_3\rangle + 2|\uparrow_1\rangle|\downarrow_2\rangle|\downarrow_3\rangle \\ -|\downarrow_1\rangle|\uparrow_2\rangle|\downarrow_3\rangle - |\uparrow_1\rangle|\downarrow_2\rangle|\downarrow_3\rangle \end{pmatrix}$$

$$= \frac{1}{\sqrt{6}}\Big(|\downarrow_1\rangle|\uparrow_2\rangle|\downarrow_3\rangle + |\uparrow_1\rangle|\downarrow_2\rangle|\downarrow_3\rangle - 2|\downarrow_1\rangle|\downarrow_2\rangle|\uparrow_3\rangle\Big)$$

$$\left|\frac{1}{2}\,-\frac{1}{2}\right\rangle_2 = \frac{1}{\hbar}S_-\left|\frac{1}{2}\,\frac{1}{2}\right\rangle_2$$

$$= \frac{1}{\sqrt{2}}\Big(-|\downarrow_1\rangle|\downarrow_2\rangle|\uparrow_3\rangle + |\downarrow_1\rangle|\downarrow_2\rangle|\uparrow_3\rangle + |\downarrow_1\rangle|\uparrow_2\rangle|\downarrow_3\rangle - |\uparrow_1\rangle|\downarrow_2\rangle|\downarrow_3\rangle\Big)$$

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$$\left|\tfrac{1}{2}\ -\tfrac{1}{2}\right\rangle_2 = \frac{1}{\sqrt{2}}\left(|\downarrow_1\rangle|\uparrow_2\rangle|\downarrow_3\rangle - |\uparrow_1\rangle|\downarrow_2\rangle|\downarrow_3\rangle\right)$$

以上から H の固有関数は

$$\left|\tfrac{3}{2}\ \tfrac{3}{2}\right\rangle = |\uparrow_1\rangle|\uparrow_2\rangle|\uparrow_3\rangle$$

$$\left|\tfrac{3}{2}\ \tfrac{1}{2}\right\rangle = \frac{1}{\sqrt{3}}\left(|\downarrow_1\rangle|\uparrow_2\rangle|\uparrow_3\rangle + |\uparrow_1\rangle|\downarrow_2\rangle|\uparrow_3\rangle + |\uparrow_1\rangle|\uparrow_2\rangle|\downarrow_3\rangle\right)$$

$$\left|\tfrac{3}{2}\ -\tfrac{1}{2}\right\rangle = \frac{1}{\sqrt{3}}\left(|\uparrow_1\rangle|\downarrow_2\rangle|\downarrow_3\rangle + |\downarrow_1\rangle|\uparrow_2\rangle|\downarrow_3\rangle + |\downarrow_1\rangle|\downarrow_2\rangle|\uparrow_3\rangle\right)$$

$$\left|\tfrac{3}{2}\ -\tfrac{3}{2}\right\rangle = |\downarrow_1\rangle|\downarrow_2\rangle|\downarrow_3\rangle$$

$$\left|\tfrac{1}{2}\ \tfrac{1}{2}\right\rangle_1 = \frac{1}{\sqrt{6}}\left(-|\downarrow_1\rangle|\uparrow_2\rangle|\uparrow_3\rangle - |\uparrow_1\rangle|\downarrow_2\rangle|\uparrow_3\rangle + 2|\uparrow_1\rangle|\uparrow_2\rangle|\downarrow_3\rangle\right)$$

$$\left|\tfrac{1}{2}\ -\tfrac{1}{2}\right\rangle_1 = \frac{1}{\sqrt{6}}\left(|\downarrow_1\rangle|\uparrow_2\rangle|\downarrow_3\rangle + |\uparrow_1\rangle|\downarrow_2\rangle|\downarrow_3\rangle - 2|\downarrow_1\rangle|\downarrow_2\rangle|\uparrow_3\rangle\right)$$

$$\left|\tfrac{1}{2}\ \tfrac{1}{2}\right\rangle_2 = \frac{1}{\sqrt{2}}\left(|\downarrow_1\rangle|\uparrow_2\rangle|\uparrow_3\rangle - |\uparrow_1\rangle|\downarrow_2\rangle|\uparrow_3\rangle\right)$$

$$\left|\tfrac{1}{2}\ -\tfrac{1}{2}\right\rangle_2 = \frac{1}{\sqrt{2}}\left(|\downarrow_1\rangle|\uparrow_2\rangle|\downarrow_3\rangle - |\uparrow_1\rangle|\downarrow_2\rangle|\downarrow_3\rangle\right) \quad //$$

$\boxed{3}$ (i) $\vec{S} = \vec{S_1} + \vec{S_2}$

$$\vec{S}^2 = S_1^2 + S_2^2 + 2\vec{S_1}\cdot\vec{S_2}$$

$$= 2\hbar^2 S(S+1) + 2\cdot\frac{H^S}{J_1}$$

$$\underline{H^S = \frac{1}{2}J_1\vec{S}^2 - \hbar^2 J_1 S(S+1)}$$

(ii) $S\otimes S = 2S \oplus 2S-1 \oplus \cdots \oplus 0$

$$\vec{S}^2 = \hbar^2 S'(S'+1)$$

$$H^S|s_1 s_2\rangle = \left[\frac{1}{2}J_1\hbar^2 S'(S'+1) - \hbar^2 J_1 S(S+1)\right]|s_1 s_2\rangle$$

$$= E^S|s_1 s_2\rangle$$

エネルギー $\underline{E^S = \frac{1}{2}J_1\hbar^2 S'(S'+1) - \hbar^2 J_1 S(S+1)}$ //

<u>縮退度は $2S'+1$ $(S' = 2S, 2S-1, \cdots, 0)$</u>

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(iii) $H_m^S = J_m(\vec{S}_1 \cdot \vec{S}_2)^m$

$$\vec{S}^2 = S_1^2 + S_2^2 + 2\vec{S}_1 \cdot \vec{S}_2$$

$$\left(\frac{1}{2}\vec{S}^2 - \hbar^2 S(S+1)\right)^m = (\vec{S}_1 \cdot \vec{S}_2)^m = \frac{H_m^S}{J_m}$$

$$\Rightarrow E_m^S = J_m\left(\frac{1}{2}\hbar^2 S'(S'+1) - \hbar^2 S(S+1)\right)^m$$

縮退度は $2S'+1$ $(S' = 2S, 2S-1, \cdots, 0)$

$\boxed{4}$ (i) $H_N = J\sum_{i,j=1}^{N} \vec{S}_i \cdot \vec{S}_j$　$\vec{S}_i^2 = \frac{3}{4}\hbar^2$

$$\vec{S}^2 = \sum_i^N \vec{S}_i^2 + 2\sum_{ij}^N \vec{S}_i \cdot \vec{S}_j$$

$$= \frac{3}{4}N\hbar^2 + 2 \cdot \frac{H_N}{J}$$

$$\longrightarrow H_N = \frac{J}{2}\left(\vec{S}^2 - \frac{3}{4}N\hbar^2\right)$$

(ii) $S_i = \pm\frac{1}{2} \to 2$通り

$$\underbrace{2\times 2\times\cdots\times 2}_{Nコ} = \underline{2^N個}$$

(iii) Nコの↑のうちmコを↓に変えたものであるので

$$S_z|s_1, s_2, \cdots, s_N\rangle = \hbar\left(\frac{N-m}{2} - \frac{m}{2}\right)|s_1\cdots s_N\rangle$$

$$= \hbar\left(\frac{N}{2} - m\right)|s_1\cdots s_N\rangle$$

$$D_M = {}_N C_m = \frac{N!}{(N-m)!\,m!}$$

$$= \frac{N!}{\left\{N-\left(\frac{N}{2}-M\right)\right\}!\left(\frac{N}{2}-M\right)!}$$

$$= \underline{\frac{N!}{\left(\frac{N}{2}+M\right)!\left(\frac{N}{2}-M\right)!}\ コ.}$$

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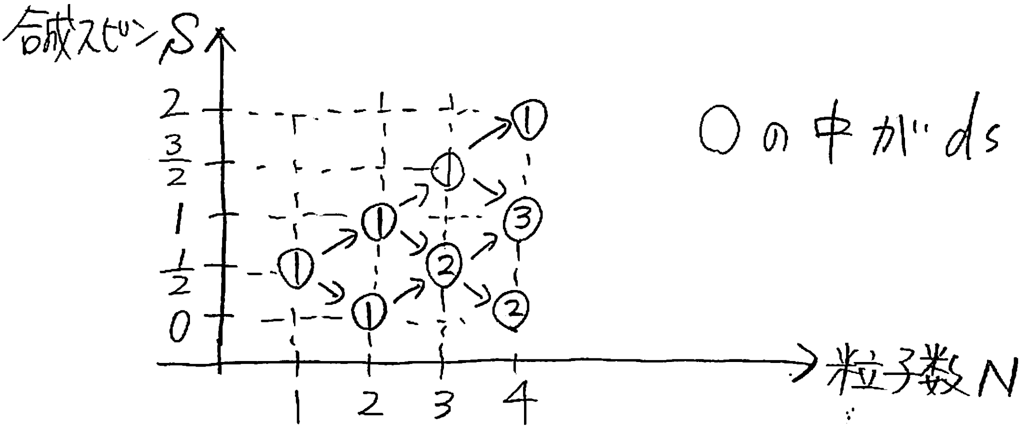
$$
\text{(iv)}\begin{cases} \vec{S}^2 |s_1 \cdots s_N\rangle = \hbar^2 S(S+1) |s_1 \cdots s_N\rangle \\ S_z |s_1 \cdots s_N\rangle = \hbar S |s_1 \cdots s_N\rangle \end{cases}
$$

電子数と合成スピンの関係は



○の中が d_S の値

$$d_{\frac{N}{2}} = D_{\frac{N}{2}} = \frac{N!}{N!} = 1$$

$$d_{\frac{N}{2}-m} \left(1 \leq m \leq \frac{N}{2}\right) = D_{\frac{N}{2}-m} = \frac{N!}{\left(\frac{N}{2}+m\right)!\left(\frac{N}{2}-m\right)!} - \frac{N!}{\left(\frac{N}{2}+m+1\right)!\left(\frac{N}{2}-m-1\right)!}$$

$$= \frac{N!}{\left(\frac{N}{2}+S\right)!\left(\frac{N}{2}-S\right)!} - \frac{N!}{\left(\frac{N}{2}+S+1\right)!\left(\frac{N}{2}-S-1\right)!}$$

となっている。

(v) $$\sum_{0}^{\frac{N}{2}}(2S+1)d_S = (N+1)D_{\frac{N}{2}} + \sum_{0}^{\frac{N}{2}-1}(D_S - D_{S+1})(2S+1)$$

$$= ND_{\frac{N}{2}} + D_{\frac{N}{2}} - D_0 + D_1 + 3D_1 - 3D_2 + 5D_2 - 5D_3 + \cdots$$

$$\cdots + ND_{\frac{N}{2}-2} - ND_{\frac{N}{2}-1} - 3D_{\frac{N}{2}-2}$$

$$+ 3D_{\frac{N}{2}-1} + ND_{\frac{N}{2}-1} - D_{\frac{N}{2}-1} - ND_{\frac{N}{2}}$$

$$+ D_{\frac{N}{2}}$$

$$= 2\sum_{S=1}^{\frac{N}{2}} D_S + D_0$$

D_0 は↑と↓がそれぞれ $\frac{N}{2}$ コある状態の個数。

↓

これは全状態の数を表すので 2^N と等しい。

$$\sum_{0}^{N/2}(2S+1)d_S = 2^N$$

d_S は $|SS\rangle$ の数を表すのでそれに $2S+1$ をかけると $|Sm\rangle$ の全状態数を表す。

よって $\sum_{0}^{N/2}(2S+1)d_S$ は $|S_1 \cdots S_N\rangle$ の全状態数を表す。

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(vi) $H_N|s_1 \cdots s_N\rangle = \frac{J}{2}\vec{S}^2|s_1 \cdots s_N\rangle - \frac{3}{8}JN\hbar^2|s_1 \cdots s_N\rangle$

$$= \left[\frac{J}{2}\hbar^2 S(S+1) - \frac{3}{8}JN\hbar^2\right]|s_1 \cdots s_N\rangle$$

$$E_N = \frac{J}{2}\hbar^2 S(S+1) - \frac{3}{8}JN\hbar^2$$

縮退度 $(2S+1)d_S$ $\quad (S = 0, \cdots, \frac{N}{2})$