20110844
荻野孝浩

量子力学3 7/26(金)

①

$$D^j_{m'm} = e^{-i\alpha m'} d^j_{m'm} e^{-i\gamma m}$$

$D^j_{m'm}(R(\alpha\beta\gamma))$

$$d^j_{m'm} = \sum_k (-1)^{k-m+m'} \frac{\sqrt{(j+m)!(j-m)!(j+m')!(j-m')!}}{(j+m-k)!k!(j-m+m')!(j-k-m')!} \cos^{2j-2k+m-m'}\left(\frac{\beta}{2}\right) \sin^{2k-m+m'}\left(\frac{\beta}{2}\right)$$

$j\ l = 0, 1, 2, 3 \ldots$

$$Y_{lm}(\beta, \alpha) = C\{D^l_{m0}(\alpha, \beta)\}^* \quad \text{となると？}$$

$m = l$

$$Y_{ll}(\underset{\theta}{\beta}, \underset{\varphi}{\alpha}) = (-1)^l \frac{1}{\sqrt{2\pi}} \sqrt{\frac{(2l+1)!}{2}} \frac{1}{2^l l!} e^{il\alpha} \sin^l\beta$$

$$(D^l_{l0}(\alpha, \beta))^* = e^{i\alpha l} \sum_k (-1)^{k+l} \frac{\sqrt{l!\,l!\,(2l)!\,0!}}{(l-k)!k!(k+l)!(-k)!} \cos^{l-2k}\left(\frac{\beta}{2}\right) \sin^{2k+l}\left(\frac{\beta}{2}\right)$$

$$\overset{k=0}{=} (-1)^l e^{i\alpha l} \frac{\sqrt{(2l)!}}{l!} \underbrace{\cos^l\left(\frac{\beta}{2}\right) \sin^l\left(\frac{\beta}{2}\right)}_{\frac{1}{2^l}\sin^l\beta}$$

$$Y_{ll} = D^*_{ll} \frac{1}{\sqrt{2\pi}} \sqrt{\frac{2l+1}{2}} = \sqrt{\frac{2l+1}{4\pi}} D^*_{ll}$$

$$Y_{lm}(\beta, \alpha) = D^{l\,*}_{m0}(\alpha\beta\gamma) \sqrt{\frac{2l+1}{4\pi}}$$

$R_0 = R_0(\alpha\beta\gamma)$
$R_1 = R_1(\alpha\beta\gamma)$ として $R_0^{-1} R_1 = R_2$

$$D^l_{00}(R_0^{-1} R) = \Big(\underbrace{D^l(R_0^{-1})}_{\substack{\shortparallel \\ [D^l(R_0)]^\dagger}} D^l(R_1)\Big)_{00}$$

$$= (D^l_{m0}(R_0))^* D^l_{m0}(R_1)$$

$$Y_{lm} = \sqrt{\frac{2l+1}{4\pi}} D^{l*}_{m0}$$

$$\sqrt{\frac{4\pi}{2l+1}}\, Y^*_{l0}(R_2\hat{z}) = \sum_m Y_{lm}(R_0\hat{z})\, Y^*_{lm}(R_1\hat{z})\, \frac{2l+1}{4\pi}$$

$$\begin{array}{ccc} \| & \| & \| \\ \sqrt{\frac{2l+1}{4\pi}}\, P_l(\cos\beta_2) & (\beta_0\ \alpha_0) & (\beta_1\ \alpha_1) \end{array}$$

$$\sum_m Y_{lm}(\beta_0\ \alpha_0)\, Y^*_{lm}(\beta_1 \alpha_1) = \frac{4\pi}{2l+1} P_l(\cos\beta_2)$$

$$R_0^{-1}R_1 = R_2 \qquad R_1 = R_0 R_2 \qquad \hat{z}_1 = R_1\hat{z} = R_0 R_2 \hat{z}$$

$$\hat{z}_0 = R_0 \hat{z}$$

$\hat{z}_1$と$\hat{z}_0$のなす角β_2 = $\hat{z}$と$R_2\hat{z}$のなす角

球面調和関数の加法定理

$$\frac{4\pi}{2l+1}\sum_m Y_{lm}(\hat{r})\, Y^*_{lm}(\hat{r}') = P_l(\hat{r}\cdot\hat{r}')$$

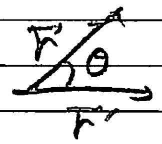
Legendre 多項式の母関数

$$\frac{1}{\sqrt{\zeta^2-2\zeta x+1}} = \sum_{n=0}^{\infty} P_n(x)\,\zeta^n$$

$\zeta \to 0 \sim 1$

$$\frac{1}{|\vec{r}-\vec{r}'|} = \frac{1}{\sqrt{(\vec{r}-\vec{r}')^2}} = \frac{1}{\sqrt{\vec{r}^2+\vec{r}'^2-2\vec{r}\cdot\vec{r}'}}$$

$$r_> = \max(|\vec{r}|, |\vec{r}'|)$$
$$r_< = \min(|\vec{r}|, |\vec{r}'|)$$
$$\vec{r}\cdot\vec{r}' = r_> r_< \cos\theta$$



$$\frac{1}{|\vec{r}-\vec{r}'|} = \frac{1}{\sqrt{r_>^2+r_<^2-2r_>r_<\cos\theta}} = \frac{1}{r_>}\frac{1}{\sqrt{1+\left(\frac{r_<}{r_>}\right)^2-2\frac{r_<}{r_>}\cos\theta}}$$

$x \leftrightarrow \cos\theta$
$\zeta \leftrightarrow r_>/r_<$

$$= \frac{1}{r_>}\sum_{l=0}^{\infty} P_l(\cos\theta)\left(\frac{r_<}{r_>}\right)^l$$

$$= \sum_{l=0} \frac{r_<^l}{r_>^{l+1}} P_l(\cos\theta)$$

$$= \sum_{lm} \frac{4\pi}{2l+1} Y_{lm}(\hat{r}) Y_{lm}^*(\hat{r}') \frac{r_<^l}{r_>^{l+1}}$$

多重極展開

$\rho(\vec{r}') \longrightarrow \phi(\vec{r})$

$\rho(\vec{r}) = 0 \quad |\vec{r}| > r_c$

$$\phi(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \int_{r'<r_c} d^3r' \frac{\rho(\vec{r}')}{|\vec{r}-\vec{r}'|} \qquad r > r_c$$

$|\vec{r}| > |\vec{r}'|$

$$= \frac{1}{4\pi\varepsilon_0} \int d^3r' \rho(\vec{r}') \sum_{lm} \frac{r'^l}{r^{l+1}} \frac{4\pi}{2l+1} Y_{lm}(\hat{r}) Y_{lm}^*(\hat{r}')$$

$$= \sum_{lm} \frac{1}{4\pi\varepsilon_0} \frac{4\pi}{2l+1} \frac{q_{lm}}{r^{l+1}} Y_{lm}(\hat{r})$$

$$q_{lm} = \int d^3r' r'^l \rho(\vec{r}') Y_{lm}^*(\hat{r}')$$

~~多重極展開 q_{lm} : 多重極~~

Clebsch-Gordan 係数と回転群

$$|j_1 m_1, j_2 m_2\rangle = |j_1 m_1\rangle \otimes |j_2 m_2\rangle = \sum_{jm} |jm\rangle \underbrace{\langle jm | j_1 m_1 j_2 m_2\rangle}_{\text{CG係数}}$$

$$R|jm\rangle = \underbrace{|jm'\rangle D^j_{m'm}(R)}_{\text{スピン}j\text{表現}}$$

$$|j_1 m_1 j_2 m_2\rangle = |jm\rangle\langle jm | j_1 m_1 j_2 m_2\rangle$$

$$R|j_1 m_1, j_2 m_2\rangle = \underbrace{R|jm\rangle}_{|jm'\rangle D^j_{m'm}}\langle jm | j_1 m_1 j_2 m_2\rangle$$

$$\parallel$$

$$R|j_1 m_1\rangle \otimes R|j_2 m_2\rangle$$

$$\parallel$$

$$|j_1 m_1'\rangle D^{j_1}_{m_1' m_1} |j_2 m_2'\rangle D^{j_2}_{m_2' m_2}$$

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$$|j_1 m_1', j_2 m_2\rangle D^{j_1}_{m_1' m_1} D^{j_2}_{m_2' m_2} = |j m'\rangle D^{j}_{m' m} \langle j m | j_1 m_1, j_2 m_2\rangle$$

$\langle j_1 m_1' j_2 m_2' |$ と内積

$$D^{j_1}_{m_1' m_1} D^{j_2}_{m_2' m_2} = \underbrace{\langle j_1 m_1', j_2 m_2' | j m'\rangle}_{\substack{\parallel\, \mathrm{CG}(実) \\ \langle j m' | j_1 m_1', j_2 m_2'\rangle}} D^{j}_{m' m} \underbrace{\langle j m | j_1 m_1, j_2 m_2\rangle}_{\substack{\parallel\, \mathrm{CG}(実) \\ \langle j_1 m_1, j_2 m_2 | j m\rangle}}$$

$j_1 = l_1,\ j_2 = l_2 \qquad m_1 = m_2 = 0$

$$D^{l_1}_{m_1' 0} D^{l_2}_{m_2' 0} = \langle l_1 m_1', l_2 m_2' | l m'\rangle D^{l}_{m' m} \langle l m | l_1 0, l_2 0\rangle \qquad (m = 0)$$

$$D^{l_1}_{m_1' 0} D^{l_2}_{m_2' 0} \to \frac{4\pi}{\sqrt{(2l_1+1)(2l_2+1)}} Y^*_{l_1 m_1'} Y^*_{l_2 m_2'} \qquad D^{l}_{m' 0} \to \sqrt{\frac{4\pi}{2l+1}}\, Y^*_{l m'}$$

$$Y_{l_1 m_1}(\hat{r}) Y_{l_2 m_2}(\hat{r}) = \sum_{lm} \sqrt{\frac{(2l_1+1)(2l_2+1)}{(2l+1)4\pi}} \langle l 0 | l_1 0, l_2 0\rangle \times Y_{lm}(\hat{r}) \langle l_1 m_1, l_2 m_2 | l m\rangle$$

$$\int d\hat{r}\, Y^*_{lm}(\hat{r}) Y_{l_1 m_1}(\hat{r}) Y_{l_2 m_2}(\hat{r})$$

$$= \sqrt{\frac{(2l_1+1)(2l_2+1)}{(2l+1)4\pi}} \langle l 0 | l_1 0, l_2 0\rangle \langle l_1 m_1, l_2 m_2 | l m\rangle$$