量子力学3　　20110843 大山侑太　　NO.　DATE May. 28th. 2013

2つのスピン $(S=1/2)$

$$\begin{cases} \vec{S_1}, \ [S_{1i}, S_{1j}] = i\hbar\varepsilon_{ijk}S_{1k} \\ \vec{S_2}, \ [S_{2i}, S_{2j}] = i\hbar\varepsilon_{ijk}S_{2k} \\ \quad [S_{1i}, S_{2j}] = 0 \quad (\vec{S_1}, \vec{S_2} \text{は独立}) \end{cases}$$

状態

$$\begin{cases} \vec{S_1}^2 |S_1 m_1\rangle = \hbar^2 S_1(S_1+1)|S_1, m_1\rangle \quad (S_1 = 1/2) \\ S_{1z}|S_1 m_1\rangle = \hbar m_1 |S_1 m_1\rangle \\ \vec{S_2}^2 |S_2 m_2\rangle = \hbar^2 S_2(S_2+1)|S_2 m_2\rangle \quad (S_2 = 1/2) \\ S_{2z}|S_2 m_2\rangle = \hbar m_2 |S_2 m_2\rangle \end{cases}$$

全体の状態

$|S_1 m_1\rangle |S_2 m_2\rangle \equiv |S_1 m_1, S_2 m_2\rangle$

$|S_1 m_1\rangle \otimes |S_2 m_2\rangle$

$|A\rangle \otimes |B\rangle$ ← S_2の状態

S_1の状態

$$S_{1i}|A\rangle \otimes |B\rangle = S_1 \otimes 1 \ |A\rangle \otimes |B\rangle$$

$S_{1i} \otimes 1 \sim S_{1i}$と書いている

$$S_{2i}|A\rangle \otimes |B\rangle = 1 \otimes S_{2i} \ |A\rangle \otimes |B\rangle$$

$1 \otimes S_{2i} \sim S_{2i}$と書いている

$$\vec{S} = \vec{S_1} + \vec{S_2} = (\vec{S_1} \otimes 1 + 1 \otimes \vec{S_2})$$

$$S_i = S_{1i} + S_{2i}$$

$$\begin{aligned} [S_i, S_j] &= [S_{1i} + S_{2i}, S_{1j} + S_{2j}] \\ &= [S_{1i}, S_{1j}] + [S_{2i}, S_{2j}] \\ &= i\hbar\varepsilon_{ijk}(S_{1k} + S_{2k}) \\ &= i\hbar\varepsilon_{ijk}S_k \end{aligned}$$

$\vec{S}$: 角運動量

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$\vec{S}$の状態

$\vec{S}^2|Sm\rangle = \hbar^2 S(S+1)|Sm\rangle$

$S_z|Sm\rangle = \hbar m|Sm\rangle$

$S_\pm = S_x \pm i S_y$

$S_+|SS\rangle = 0$

$S_-|S\,-S\rangle = 0$

$$m = \underbrace{-S,\ -S+1,\ \cdots,\ S}_{2S+1\text{コ}}$$

状態に関して２通りの表現

$|S_1 m_1\rangle|S_2 m_2\rangle = |\underset{=1/2}{S_1}\, m_1,\ \underset{=1/2}{S_2}\, m_2\rangle \overset{\text{省略}}{=} |m_1\ m_2\rangle$

$\to |S, m\rangle$

$\leftarrow S, m$ の2コ

$$S_z|S_1 m_1, S_2 m_2\rangle = (S_{1z} + S_{2z})|S_1 m_1\rangle|S_2 m_2\rangle$$
$$= \underbrace{S_{1z}|S_1 m_1\rangle}_{=\hbar m_1|S_1 m_1\rangle}|S_2 m_2\rangle + |S_1 m_1\rangle \underbrace{S_{2z}|S_2 m_2\rangle}_{=\hbar m_2|S_2 m_2\rangle}$$
$$= \hbar(m_1+m_2)|S_1 m_1\rangle|S_2 m_2\rangle$$

$S_z|S\,m\rangle = \hbar m|S\,m\rangle \qquad m = m_1 + m_2$

$S_1 = 1/2,\ S_2 = 1/2$

$m_1 = -S_1 = -1/2,\ -1/2+1 = 1/2 = S_1$

$m_1, m_2 = \pm\frac{1}{2}$

$$|S_1 m_1, S_2 m_2\rangle = \begin{cases} |1/2\ 1/2,\ 1/2\ 1/2\rangle = |\uparrow\uparrow\rangle \\ |1/2\ 1/2,\ 1/2\ -1/2\rangle = |\uparrow\downarrow\rangle \\ |1/2\ -1/2\ \ 1/2\ 1/2\rangle = |\downarrow\uparrow\rangle \\ |1/2\ -1/2\ \ 1/2\ -1/2\rangle = |\downarrow\downarrow\rangle \end{cases}$$

$|\uparrow\uparrow\rangle = |1/2\ 1/2,\ 1/2\ 1/2\rangle$

$S_z|\uparrow\uparrow\rangle = \hbar(1/2+1/2)|\uparrow\uparrow\rangle = \hbar|\uparrow\uparrow\rangle$

$S_+|\uparrow\uparrow\rangle = (S_{1+}+S_{2+})|\uparrow\uparrow\rangle$

$= S_{1+}|\uparrow_1\rangle|\uparrow_2\rangle + |\uparrow_1\rangle S_{2+}|\uparrow_2\rangle$

$$S_{1+}|\uparrow_1\rangle = S_{1+}|\tfrac{1}{2}\ \tfrac{1}{2}\rangle = 0$$
$$S_{2+}|\uparrow_2\rangle = 0$$

$$S_+|\uparrow\uparrow\rangle = 0$$
$$S_z|\uparrow\uparrow\rangle = \hbar\cdot 1\ |\uparrow\uparrow\rangle = \hbar|\uparrow\uparrow\rangle$$

$$|S\,m\rangle \qquad S_+|S\,S\rangle = 0$$
$$m = -S, -S+1, \dots, S-1, S$$
$$|\uparrow\uparrow\rangle = |S=1, m=1\rangle = |1, 1\rangle$$

$$|1\,0\rangle \propto S_-|1\,1\rangle$$

$$|j, m-1\rangle = \frac{1}{\hbar\sqrt{(j+m)(j-m+1)}} J_-|j\,m\rangle$$

$j=1$, $m=1$

$$|1,0\rangle = \frac{1}{\hbar\sqrt{2}} S_-|1\,1\rangle$$
$$= \frac{1}{\hbar\sqrt{2}} S_-|1\,1\rangle$$
$$S=1, m=1$$
$$= \frac{1}{\hbar\sqrt{2}} S_-|\uparrow\uparrow\rangle$$
$$= \frac{1}{\hbar\sqrt{2}} (S_{1-}+S_{2-})|\uparrow_1\rangle|\uparrow_2\rangle$$
$$= \frac{1}{\hbar\sqrt{2}} \left(S_{1-}|\uparrow_1\rangle|\uparrow_2\rangle + |\uparrow_1\rangle S_{2-}|\uparrow\rangle\right)$$

$$|\downarrow\rangle = \frac{1}{\hbar} S_-|\uparrow\rangle$$
$$S_{1-}|\uparrow_1\rangle = \hbar|\downarrow_1\rangle$$
$$S_{2-}|\uparrow_2\rangle = \hbar|\downarrow_2\rangle$$

$$|1\,0\rangle = \frac{1}{\sqrt{2}}\left(|\downarrow_1\rangle|\uparrow_2\rangle + |\uparrow_1\rangle|\downarrow_2\rangle\right)$$
$$= \frac{1}{\sqrt{2}}\left(|\downarrow\uparrow\rangle + |\uparrow\downarrow\rangle\right)$$

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$$|1\,-1\rangle = \frac{1}{\hbar\sqrt{1\cdot 2}} S_- |1\,0\rangle$$

$$= \frac{1}{\hbar\sqrt{2}} S_- \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle\right)$$

$$= \frac{1}{\hbar 2}\left(S_{1-} + S_{2-}\right)\left(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle\right)$$

$$= \frac{1}{\hbar 2}\left(S_{1-}|\uparrow_1\rangle|\downarrow_2\rangle + |\downarrow_1\rangle S_{2-}|\uparrow_2\rangle\right)$$

$$(\because S_{1-}|\downarrow_1\rangle = 0,\ S_{2-}|\downarrow_2\rangle = 0)$$

$$= \frac{1}{2}\left(|\downarrow\downarrow\rangle + |\downarrow\downarrow\rangle\right)$$

$$= |\downarrow\downarrow\rangle$$

$|1\,1\rangle = |\uparrow\uparrow\rangle$

$|1\,0\rangle = 1/\sqrt{2}\,(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)$

$|1\,-1\rangle = |\downarrow\downarrow\rangle$

$S=1,\ \underbrace{m=-1,0,1}_{[2\cdot 1+1=3\text{コ}]}$

$|\uparrow\downarrow\rangle$ と $|\downarrow\uparrow\rangle$

$m=0 \rightarrow 2$コ

$m=0$ の状態は2つある

$|\psi\rangle$, $\langle 1\,0\,|\,\psi\rangle = 0$ となる $|\psi\rangle$ を考える。

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right) \quad (m=0)$$

$$S_+|\psi\rangle = \frac{1}{\sqrt{2}}\left(S_{1+} + S_{2+}\right)\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$$

$$= \frac{1}{\sqrt{2}}\left(|\uparrow\uparrow\rangle - |\uparrow\uparrow\rangle\right)$$

$$= 0$$

$|\psi\rangle : m=0$

$$S_+|\psi\rangle = 0 \rightarrow S=0$$

$S=0,\ m=0$

$$|0,0\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$$

$|S_1 m_1, S_2 m_2\rangle$

$\quad \parallel$

$|1/2\ m_1, 1/2\ m_2\rangle$: $|\uparrow\uparrow\rangle, |\uparrow\downarrow\rangle, |\downarrow\uparrow\rangle, |\downarrow\downarrow\rangle$ 4通り

$S=1,\ m=1,0,1$

$S=0,\ m=0$ $\quad |S m\rangle$ 4通り

基底変換

$$|S m\rangle = |S_1 m_1 S_2 m_2\rangle\langle S_1 m_1 S_2 m_2 | S m\rangle$$

$$|1 1\rangle = (|\uparrow\uparrow\rangle)\cdot 1$$

$$|1 0\rangle = (|\uparrow\downarrow\rangle, |\downarrow\uparrow\rangle)\frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}$$

$$|1 -1\rangle = (|\downarrow\downarrow\rangle)\cdot 1$$

$$|0 0\rangle = (|\uparrow\downarrow\rangle, |\downarrow\uparrow\rangle)\frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix}$$

$$(|11\rangle, |10\rangle, |00\rangle, |1-1\rangle) = (|\uparrow\uparrow\rangle, |\uparrow\downarrow\rangle, |\downarrow\uparrow\rangle, |\downarrow\downarrow\rangle)\begin{pmatrix}1 & 0 & 0 & 0\\ 0 & 1/\sqrt{2} & 1/\sqrt{2} & 0\\ 0 & 1/\sqrt{2} & -1/\sqrt{2} & 0\\ 0 & 0 & 0 & 1\end{pmatrix}$$

$\langle S_1 m_1, S_2 m_2 | S m\rangle$

行 列

$\langle S_1 m_1, S_2 m_2 | S m\rangle$

クレブシュ・ゴルダン係数

Clebsch-Gordan

実 $|S m\rangle = |S_1 m_1, S_2 m_2\rangle\langle S_1 m_1, S_2 m_2 | S m\rangle$

逆 $|S_1 m_1, S_2 m_2\rangle = |S m\rangle\langle S m | S_1 m_1 S_2 m_2\rangle$

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$$|s\,m\rangle = |s'\,m'\rangle \underbrace{\langle s'm'|s_1 m_1, s_2 m_2\rangle\langle s_1 m_1, s_2 m_2|s\,m\rangle}_{\delta_{s's}\,\delta_{m'm}}$$

$$\langle s'm'|s_1 m_1 s_2 m_2\rangle\langle s_1 m_1 s_2 m_2|s\,m\rangle = \delta_{s's}\,\delta_{mm'}$$

$$|s_1 m_1, s_2 m_2\rangle = |s_1 m_1', s_2 m_2'\rangle\langle s_1 m_1', s_2 m_2'|s\,m\rangle\langle s\,m|s_1 m_1 s_2 m_2\rangle$$

$$\langle s_1 m_1' s_2 m_2'|s\,m\rangle\langle s\,m|s_1 m_1, s_2 m_2\rangle = \delta_{m_1 m_1'}\,\delta_{m_2 m_2'}$$

$s_1 = 1/2,\ s_2 = 1/2$ の時を考えた

$\vec{J} = \vec{J}_1 + \vec{J}_2$ 合成角運動量

$\vec{J}_1, \vec{J}_2$, $[\vec{J}_1, \vec{J}_2] = 0$

$$\vec{J}_1^2|j_1, m_1\rangle = \hbar^2 j_1(j_1+1)|j_1 m_1\rangle$$
$$J_{1z}|j_1 m_1\rangle = \hbar m_1|j_1, m_1\rangle$$
$$\vec{J}_2^2|j_2, m_2\rangle = \hbar^2 j_2(j_2+1)|j_2, m_2\rangle$$
$$J_{2z}|j_2, m_2\rangle = \hbar m_2|j_2, m_2\rangle$$

$$\left(\begin{aligned}[J_{1i}, J_{1j}] &= i\hbar\varepsilon_{ijk}J_{1k} \\ [J_{2i}, J_{2j}] &= i\hbar\varepsilon_{ijk}J_{2k}\end{aligned}\right. \Rightarrow [J_i, J_j] = i\hbar\varepsilon_{ijk}J_k$$

$$\vec{J}^2|j\,m\rangle = \hbar^2 j(j+1)|j\,m\rangle$$
$$J_z|j\,m\rangle = \hbar m|j\,m\rangle$$

$$\{|j\,m\rangle\} \Leftrightarrow \{|j_1 m_1\rangle|j_2 m_2\rangle\}$$
$$\qquad\qquad\qquad\qquad \| $$
$$\qquad\qquad\qquad |j_1 m_1 j_2 m_2\rangle$$

線型変換で移りかわる。

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$$|jm\rangle = |j_1 m_1, j_2 m_2\rangle \langle j_1 m_1, j_2 m_2 | jm\rangle$$

$$|j_1 m_1, j_2 m_2\rangle = |j m\rangle \underbrace{\langle j m | j_1 m_1, j_2 m_2\rangle}$$

Clebsch-Gordan係数

実 $\langle jm | j_1 m_1, j_2 m_2\rangle\langle j_1 m_1, j_2 m_2 | j' m'\rangle = \delta_{jj'}\delta_{mm'}$

$\langle j_1 m_1, j_2 m_2 | jm\rangle\langle jm | j_1 m_1' j_2 m_2'\rangle = \delta_{m_1 m_1'}\delta_{m_2 m_2'}$

全行列の次元 $(2j_1+1)(2j_2+1)$

$m_1 : \underbrace{-j_1, \cdots, j_1}_{2j_1+1コ}$ $\quad m_2 : \underbrace{-j_2 \to j_2}_{2j_2+1コ}$

j ?

$m : \underbrace{-j \to j}_{2j+1コ}$

$$\sum_{j=j_{min}}^{j_{max}=j_1+j_2} (2j+1) = |j_1 - j_2|$$

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