群の表現

対称操作 $R$, $R^r R = 1$

$H \mapsto RHR^{-1} = H'$

$RH = HR$, $[H, R] = 0$

$H |\psi_i\rangle = |\psi_i\rangle \in E$ 縮度度 $d_i = 1, \ldots, d$

$R |\psi_i\rangle = R |\psi_i\rangle = R |\psi_i\rangle \in E$

$R |\psi_i\rangle \equiv \{ |\psi_1\rangle, \ldots, |\psi_d\rangle \}$ の線型結合

$R |\psi_i\rangle = \sum |\psi_i\rangle \in D_{ij}(R) = (4D)_d$

縮型結合の係数

$(R |\psi_1\rangle, R |\psi_2\rangle, \ldots, R |\psi_d\rangle) = R (|\psi_1\rangle, \ldots, |\psi_d\rangle)$

$R |\psi\rangle = (4D)_d, \quad (4D)_d = D_{ij}(R)$

$R |\psi\rangle = 4D(R)$

$R_2 R_1 = \frac{3}{2} R_2 R_1$

$D(R)$, 群 $\mathbb{R}^n$ によるユニタリ表現 $\cdots$

$R$: unitary $R^r R = 1$

$R |\psi\rangle = 4D$ $\psi^r R^r = D^r \psi^r$

$\psi^r R^r \psi = \psi^r = E_d$ $D^r \psi^r \psi D = D^r \psi$ $D^r = E_d, \quad D^r = D^{-1}$

$\psi = (|\psi_1\rangle, \ldots, |\psi_d\rangle)$, $\psi^r = E_d$

$\langle \psi^r | \psi \rangle = \langle \psi | \psi \rangle$

$\psi^r = \begin{pmatrix} \langle \psi | \psi \rangle \\ \langle \psi_1 | \psi \rangle \\ \vdots \\ \langle \psi_d | \psi \rangle \end{pmatrix}$

* 回転群の表現

$R_\alpha$ の行列

$D(R) = \begin{pmatrix} \alpha & 0 \\ 0 & \alpha \end{pmatrix}$

無限小変換

$4:\mathfrak{g} : \mathfrak{g} \rightarrow \mathfrak{g}$ $\psi^r = \begin{pmatrix} 1 - i\alpha \mathfrak{g} \cdot \mathfrak{J} \\ 1 - i\alpha \mathfrak{g} \cdot \mathfrak{J} \end{pmatrix}$

$R_4 = 4 \mathfrak{g}$

$(\mathfrak{g} \cdot \mathfrak{J}) 4 = 4 \mathfrak{g}(\mathfrak{g} \cdot \mathfrak{J})$
\[ J_z, J_y, J_z \to J_x, J_z \text{と考えれば} J_z \text{の固有状態} \]

\[ J_z |\psi_m\rangle = m |\psi_m\rangle = 1 \psi_m J_{z} \]

\[ D_{w_m}^{j} (J_z) = m J_{z} \]

\[ \psi m \]

\[ D_{w_m}^{j} (J_z) = \sqrt{(j \pm m)(j \pm m + 1)} J_{z} \]

\[ D_{w_m}^{j} (J_z) = \sqrt{(j \pm m)(j \pm m + 1)} J_{z} \]

一般の回転 Euler 角 \((\alpha, \beta, \gamma)\) で定まる。

\[ R(z) = R_{x}(z) R_{y}(y) R_{x}(z) = e^{-i\alpha J_{x}} e^{-i\beta J_{y}} e^{-i\gamma J_{x}} \]

\[ D^{j} (R(z)) = D^{j} (e^{-i\alpha J_{x}} e^{-i\beta J_{y}} e^{-i\gamma J_{x}}) \]

\[ |\psi_{m}\rangle = |j m\rangle, \quad m = j \to \frac{e^{-i\alpha m}}{\sqrt{m}} |j m\rangle \]

\[ D_{w_m}^{j} (R(z)) = D_{w_m}^{j} (e^{-i\alpha m} e^{-i\beta J_{y}} e^{-i\gamma J_{x}}) \frac{e^{-i\alpha m}}{\sqrt{m}} |j m\rangle \]

Schwinger Boson 族 \[ a, a^{\dagger} = 1, \quad [a, a] = 0, \quad [a^{\dagger}, a^{\dagger}] = 0 \]

\[ a |0\rangle = 0, \quad |n\rangle = \frac{1}{\sqrt{n!}} (a^{\dagger})^{n} |0\rangle \quad \Rightarrow \quad a^{\dagger} a |n\rangle = n |n\rangle \]

粒子対応子

\[ a^{\dagger} a = n \]
\[ [\alpha_+, \alpha_-] = 0, \quad [\alpha_-, \alpha_+] = 0, \quad [\alpha_+, \alpha_+] = \gamma^0 \gamma^0 \gamma^0 
\]

\[ a = \begin{pmatrix} a_+ \\ a_- \end{pmatrix}, \quad a^+ = \begin{pmatrix} a_+ \\ a_- \end{pmatrix} \]

\[ J = a^+ \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} a 
\]

\[ J_x = a^+ \gamma^1 (\sigma^0) a, \quad J_y = a^+ \gamma^1 (\sigma^0) a, \quad J_z = a^+ \gamma^1 (\sigma^0) a 
\]

\[ [\gamma^i, \gamma^j] = [\alpha^+ \gamma^i \sigma^0 \alpha^-, \alpha^+ \gamma^j \sigma^0 \alpha^-] = \alpha^+ \gamma^i \sigma^0 \frac{\alpha^+ \gamma^i \sigma^0 \alpha^-}{2} \]

\[ J_i J_j = \alpha^+ \gamma^i \sigma^0 \alpha^- \]

\[ J = J_x \gamma^i + J_y \gamma^j + J_z \gamma^k 
\]

\[ J_x = a^+ \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} a 
\]

\[ J_x = a^+ \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} a 
\]

\[ J_x = a^+ \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} a 
\]

\[ J_x = a^+ \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} a 
\]

\[ J_x = a^+ \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} a 
\]

\[ J_x = a^+ \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} a 
\]
\[ n = n_+ + n_- \]
\[ \mathbf{J}^2 = \frac{1}{4} n^2 + \frac{1}{2} n = \frac{1}{2} n (n+1) \]
\[ \hat{J} = \frac{1}{2} (\hat{n} + \hat{n}_-) \]
\[ \mathbf{J}^2 = \frac{1}{2} \hat{n} + \frac{1}{2} \hat{n}_- \]
\[ n_+ = \hat{n} + m \]
\[ n_- = \hat{n} - m \]

\[ |n_+, n_-\rangle = \frac{1}{\sqrt{n_+!}} (\hat{a}_+)^{n_+} \frac{1}{\sqrt{n_-!}} (\hat{a}_-)^{n_-} |0\rangle = \frac{(\hat{a}_+)^{n_+} (\hat{a}_-)^{n_-}}{\sqrt{n_+! n_-!}} |0\rangle \]

\[ \hat{n}_\pm = \hat{a}_\pm \hat{a}_\mp \]
\[ \hat{n}_+ |n_+, n_-\rangle = n_+ |n_+ n_-\rangle \]
\[ \hat{n}_- |n_+, n_-\rangle = n_- |n_+ n_-\rangle \]

\[ |j m\rangle = |n_+ = j + m, n_- = j - m\rangle = \frac{(\hat{a}_+)^j (\hat{a}_-)^m}{\sqrt{(j+m)! (j-m)!}} |0\rangle \]

\[ J^2 |j m\rangle = j (j+1) |j m\rangle = j (j+1) |j m\rangle \]

\[ J_z |j m\rangle = m |j m\rangle \]

\[ \langle j m' | e^{i \beta J_z} | j m\rangle = \delta_{m' m} \]